Novel high-fidelity tyre model for motorcycles to be characterised by quasi-static manoeuvres - Rationale and numerical validation

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ABSTRACT
Although diverse tyre model formulations exist in literature, they are suited for characterisation using a dedicated test bench, preventing parameters' estimation in driving conditions.

This study defined a novel motorcycle tyre model, characterisable through driving manoeuvres using simple instrumentation consisting in an inertial measurement unit, steering position sensor and wheel speed sensors. Acquired signals were used to estimate instantaneous tyre forces, moments, slip angle and other properties, which were employed to calculate tyre model's parameters.

Tyre model development and validation were performed in simulation environment: we used a Magic Formula tyre-equipped motorcycle model to perform a set of manoeuvres, which were employed to characterise the proposed tyre model. Lastly, a set of quasi-static manoeuvres was conducted using the same motorcycle model equipped with the two tyre models, and results were compared.

Comparison results showed a close reproduction of real tyre forces, moments and slips by the proposed tyre model for quasi-static manoeuvres, accurately reproducing motorcycle dynamics. Therefore, steering torque was correctly predicted for different lateral acceleration values.

These results show that the proposed tyre model can be characterised, for both longitudinal and lateral dynamics, using this limited set of manoeuvres and simple instrumentation; the correctly predicted steering torque could allow the use of this tyre model for handling description.

KEYWORDS
Motorcycle tyre, motorcycle dynamics, tyre model, tyre forces and moments estimation, simulation, quasi-static manoeuvre

1. Introduction

The state of motion of road vehicles changes mostly due to tyre-road interaction, through the forces and moments generated by pneumatic tyres. Tyre properties influence not only friction limits, but also the behaviour of the vehicle when operating far from these limits [1]: consequently, a faithful description of tyre behaviour is needed to accurately predict the behaviour of the vehicle.

This is also true for two-wheeled vehicles, as bicycles and motorcycles [2]: tyre slip greatly determines vehicle dynamics, stability, handling and control [3]. Due to the

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naturally unstable nature of two-wheeled vehicles, tyre properties can potentially lead to dangerous behaviour [4]: the importance of a correct tyre modelling is therefore evident for assistance device development and tuning, sensor fusion, crash reconstruction and motorcycle and tyre design and improvement.

Several tyre model formulations exist, spanning models derived from theory [5,6] and empirical models: of the latter category, the most famous and widely used is the Magic Formula tyre model [7]. Such model is able to describe in-depth the strongly non-linear behaviour of pneumatic tyres, with an error on forces and moments generated lower than 5% [8]. Moreover, compared to less complex formulations as the model by Dugoff [9], it correctly considers the influence of vertical load on calculated forces and also calculates the moments along the three axes.

This detailed model is composed of more than 50 parameters, which require a wide amount of data to be determined [10]: the characterisation of the Magic Formula tyre model is therefore conducted using a tyre test bench machine, as shown in literature [4,8,11–13]. This device allows complete freedom on imposing tyre input quantities (as load, slip and camber) and allows accurate measurement of forces and moments along the three axes; however such test bench is not commonly available to subjects interested in tyre models as Universities or suppliers. In the particular case of motorcycle tyres, as is the topic of this paper, the generation of large camber angles is needed: this requires a test bench specific for this application, like the one shown by Cossalter et al. [4]. Moreover, the fitting process is complex and the significant number of coefficients in the Magic Formula tyre model often leads to over-fitting even when relatively large data is available [14]. Road surface determines friction generation between the tyre and the road: given that in test benches the road surface is unrepresentative [15], a friction similarity approach [16] must be used, potentially leading to moderately different behavior [10]. To avoid this problem, skid trailers are often employed, with lower reliability [15]. Lastly, it is possible to measure tyre forces during driving using wheel force transducers; however, this is expensive equipment, requiring significant installation time and efforts [15], and the measurements are subject to noise due to road unevenness [17].

An alternative, lean and cost-effective approach for car tyre characterisation is proposed by the author in Lugo et al. [18] : general purpose and relatively affordable instruments as wheel speed sensors, steering angle sensor and an Inertial Measurement Unit (IMU) are used to estimate tyre inputs and outputs during a set of quasi-static driving manoeuvres, used to characterise a dedicated, novel tyre model formulation. The estimation takes place through simple resolution, for each instant, of the system that describes the dynamics of the simplified car model: therefore Kalman filters are not required, which would add significant tuning time. This tyre model is then validated, first by using a simulated dataset, and then with a real, experimental test, showing close reproduction of car states, tyre forces and moments also in transient manoeuvres. These positive results demonstrate the potential of such approach; however the tyre model cannot be used for motorcycle tyres due to the fundamental differences in forces and moments generation dynamics compared to car tyres. For example, lateral load transfer significantly influences car behaviour, and estimation of both its entity and its effects is challenging; on the other side motorcycle tyres generate most of their force due to camber and not due to slip [10], and camber also generates a twisting moment [19] that is independent to the pneumatic trail. Moreover, the roll angle is influenced by the crown radius of the tyre, or alternatively overturning moment [20]: all these differences require a novel tyre model formulation dedicated to motorcycles, to be characterised with an approach conceptually similar to the one described in Lugo et al. [18].

The aim of this study is to define a new high-fidelity motorcycle tyre model, for use in dynamic simulations, which can be characterised in real driving conditions by simple manoeuvres using reduced instrumentation, with limited knowledge of motorcycle parameters being required. This would allow to obtain a tyre model that reproduces the behaviour of the real component even for those subjects that do not have access to a tyre test bench, instead using a lean and cost effective approach that requires tools commonly available on instrumented vehicles. Moreover, it would allow to characterise that specific tyre with its specific pressure and wear values, mounted on that specific motorcycle and tested in real friction conditions.

The approach has been developed in simulation environment, and results of a full-fledged motorcycle model equipped with the proposed tyre model are compared to the ones deriving from the Magic Formula tyre model, taken as proxy of a real tyre. Motorcycle behaviour is closely described, as are tyre inputs (slip and camber) and outputs (especially forces, while some moments show an error).

The remainder of the paper is organised as follows: the general methodology of the investigation is introduced in Section 2. Then, Section 3 describes the proposed tyre model formulation in detail. Estimation models and the characterisation process are shown in Section 4, and Section 5 presents the results. Lastly, Section 6 provides discussion of results, along with potential extensions and applications of the tyre model.

2. General methodology

Simulation software BikeSim© (Mechanical Simulations, Ann Arbor, MI, US), is used in this paper both as a substitute of a real, experimental test and as the environment in which the motorcycle is modelled.

Of the various motorcycle models included in the program, the *Sports, Small: Chain Drive* model has been chosen due to its average displacement and mass properties: the high-fidelity model takes into account, among other things, rider's (modelled as lower and upper body) translation and rotation due to inertial forces, steering damping and compliance and nonlinear suspension behaviour. No changes have been made to default parameters, in order to make the experiment reproducible.

The motorcycle model is equipped, by default, with different front and rear tyres, using a *Magic Formula* formulation [7]: the coefficients of each tyre are used to calculate the outputs of the tyre (longitudinal force F_x, lateral force F_y and yaw moment M_z) as a function of its inputs (longitudinal slip s, slip angle α, camber γ and vertical load F_z), for both uncombined and combined slip conditions. Other tyre characteristics are defined outside of the Magic Formula parameters file: these include tyre radius, vertical stiffness, wheel mass and spin inertia, undeflected crown radius and rolling resistance coefficients. Lastly, a linear *relaxation length* formulation is used to model the transient build-up of previously cited outputs only due to slips (and not due to camber, which ideally generates instantaneous outputs [11]): longitudinal and lateral relaxation lengths have different values.

The study is articulated in two different steps:

(1) *Estimation phase*: the motorcycle model, equipped with the default, Magic Formula tyres as proxy for real tyres, is used to conduct the manoeuvres needed to characterise the tyre model. A set of quasi-static manoeuvres is chosen and simulated; tyre inputs and outputs are estimated for each time step from measured

signals and the resulting, simulated dataset is used to describe the input-output relationships of the tyre.

(2) *Validation phase*: a set of manoeuvres is conducted, first, with the motorcycle model equipped with the reference, Magic Formula tyres, and then with the custom tyre model, with the formulation proposed in this paper and coefficients determined in the previous phase. Results with the two tyre models are compared, and the accuracy of the custom tyre model is evaluated as its ability to correctly reproduce the behaviour of reference tyre.

The proposed tyre model formulation is described in detail in the following section.

3. Tyre model

As described in Section 1, a complex, traditional tyre model formulation (as the Magic Formula tyre model that equips the default BikeSim© motorcycle model), while being able to correctly reproduce complex phenomena and inputs combinations, proves unsuitable for a characterisation using driving data. In fact, its many parameters can only be fitted by measuring all tyre outputs for all the sets of tyre inputs: a tyre test bench allows such freedom, but this is not possible in driving conditions, for the following reasons:

- While conducting a driving manoeuvre, many inputs are *coupled*, meaning that they tend to vary together throughout the manoeuvre: this is also true for quasi-static manoeuvres, like a corner with progressively decreasing radius. For such a manoeuvre the roll angle, and thus the two camber angles, will progressively increase; at the same time the slip angle of each tyre will vary, for example monotonically increasing in modulus (towards the outside of the curve) in case the lateral force due to camber alone is not able to overcome its share of the centrifugal force of the motorcycle [19, p.66–68]. This means that a certain value of camber angle will correspond to one and only one possible value of slip angle, as long as the manoeuvre is quasi-static. Given that the aim of the paper is to only use quasi-static manoeuvres for estimation, due to the many parameters that, in such conditions, do not affect motorcycle behaviour (for example dampers, moments of inertia and the relaxation lengths of the tyres), it is not possible to vary one tyre input without varying the others.
- The driving test is conducted using cost-effective and general purpose instrumentation, due to the expensive nature of wheel force transducers and optical slip sensors [21], among others. This means that tyre inputs and outputs are not directly measured, but are instead estimated using simple vehicle models populated with sensor readings. Additionally, some quantities (for example, the yaw moment of the tyre due to camber) are obtained using statistical relationships obtained from literature data: these are simple and often linear relationships, incompatible with the complex, trigonometric nature of the Magic Formula tyre model.

To summarise, the completely different nature of the experimental test used for tyre characterisation (a driving test, instead of the widely used tyre bench test) poses peculiar challenges and potential rewards to the characterisation process, which requires a novel, custom tyre model, tailored to these unique needs and limited information available.

The tyre model formulation used, for both longitudinal and lateral dynamics description, must satisfy the following two requirements:

- It can only use the tyre inputs and outputs that can be estimated using the available instruments.
- At the same time, it must make the best possible use of this limited information to describe the behaviour of the tyre, and so also of the motorcycle, during a non-transient manoeuvre, which is the aim of the paper.

So, it is fundamental to consider which inputs and outputs can be estimated, and which ones we are interested to describe. Regarding the latter, the target is to obtain a tyre model that correctly describes all tyre inputs and outputs throughout the simulated manoeuvre:

- the complete set of inputs: longitudinal slip s, slip angle α, camber angle γ and vertical load F_z;
- the complete set of outputs: longitudinal force F_x, lateral force F_y, overturning moment M_x and yaw moment M_z.

3.1. Longitudinal dynamics formulation

Regarding longitudinal dynamics, we are interested in describing the relationship between longitudinal slip, vertical load and corresponding longitudinal force. To do so, we must be able to correctly estimate these quantities in the estimation phase, to later use them to obtain tyre input-output relationship via regression.

Amongst the available sensors, wheel speed sensors provide readings of the angular velocity of the tyre ω, while the IMU provides the estimated longitudinal speed v_x: this information can be combined, if rolling radius is known, to estimate longitudinal slip from its definition [7].

Furthermore, the IMU also provides the longitudinal acceleration a_x: this information, coupled with required knowledge of the mass of the vehicle m, wheelbase l and centre of gravity coordinates in lateral plane $x_{\mathrm{G}}, z_{\mathrm{G}}$, allows estimation of the vertical load acting on each tyre.

Lastly, the longitudinal force produced by each tyre can be estimated using the mass and longitudinal acceleration of the vehicle, with wheel inertia potentially included (if known) to increase estimation accuracy (especially if interested in describing manoeuvres with high values of longitudinal acceleration).

Summing up, we are able to estimate the two tyre inputs (longitudinal slip and vertical load) and the only output (longitudinal force) that are present in a longitudinal manoeuvre: these are the variables that will be used to describe the behaviour of the tyre via regression.

As previously mentioned, during a driving manoeuvre tyre inputs are coupled: in the case of longitudinal dynamics, we can make the example of a front braking manoeuvre, in which a certain value of front slip angle will produce a certain deceleration, causing the front wheel load to increase to a definite amount. This consideration seems to advise to express the longitudinal force of the tyre as a function of slip angle only, given that the information about its vertical load is *condensed* into the slip angle value:

$$F_x = F_x(s, F_z(s)) = F_x(s). \tag{1}$$

However, this is only true as long as the rear brake is not involved: in fact, if a front braking manoeuvre and a rear braking manoeuvre (due to brake bias being known in these manoeuvres) are used to describe the behaviour of the tyre during braking, in case formulation shown in Equation (1) was used it would not correctly describe a combined braking manoeuvre, for which the slip-load relationship of the tyre would change. So, in order to allow the tyre model to describe also combined braking manoeuvres, the proposed formulation is the following:

$$F_x = \mu_x(s) F_z, \tag{2}$$

where the adherence coefficient μ_x is expressed as the ratio of the estimated longitudinal and vertical force. This second formulation correctly takes into account the load transfer, induced by the slip of one tyre, on the other tyre, with the hypothesis that this additional load transfer will not change the adherence coefficient of the tyre, due to the effect of vertical load being taken into account with a proportionality hypothesis. This increases the robustness of the model, due to the fact that Equation (1) already describes, theoretically, an acceleration manoeuvre and an uncombined braking manoeuvre with no error (additionally to the error made on the estimated quantities s, F_z and F_x). Compared to that formulation, Equation (2) takes into account this additional load transfer, if present, by making the assumption of low *tyre load sensitivity* due to this additional load transfer.

The $\mu_x(s)$ relationship is obtained by polynomial regression, using estimated tyre inputs and outputs during the estimation manoeuvre.

Regarding longitudinal dynamics, other aspects that define tyre behaviour are the effective rolling radius R_{rol} and rolling resistance F_{rol}: this is assumed to be independent on speed and proportional to tyre load [10]:

$$F_{\text{rol}} = -\frac{c_{\text{rol}}}{R_{\text{rol}}} F_z, \tag{3}$$

where c_{rol} is the rolling resistance coefficient of the tyre.

3.2. Lateral dynamics formulation

Regarding lateral dynamics, we are interested in describing the relationship between the slip angle, the camber angle and the vertical load, and the corresponding lateral force, overturning moment and yaw moment.

A steering angle sensor provides measurement of the steering angle; the IMU directly measures the angular speeds and the translational velocities along the three axes of the vehicle, while using estimation algorithms to provide the orientation and speed components: this information can be used to estimate the slip angle, camber angle, vertical load, lateral force, overturning moment and yawing moment of the tyre, as will be shown in detail in Section 4.

In the remainder of the subsection, we will show that it is possible to estimate the three tyre inputs (slip angle, camber angle and vertical load) and the outputs (lateral force, overturning moment and yaw moment) present in a lateral manoeuvre: these variables will be used to describe the behaviour of the tyre via regression.

3.2.1. Lateral force

Similarly to what described regarding the coupling of longitudinal slip and vertical load in longitudinal dynamics, also tyre inputs related to lateral dynamics are coupled during a quasi-static manoeuvre, as stated in Section 3 for slip angle and camber angle. This means that, while it is possible to estimate total tyre lateral force F_y using a simplified motorcycle model, it is not possible to divide this total force in the portion due to slip, F_{y_α}, and due to camber, F_{y_γ}. The reason is that the two inputs are both present, and it is not possible to vary one without varying the other as would be possible when using a tyre test bench. Moreover, the two contributes can either have same or different sign, and this often changes throughout the manoeuvre: in fact, lateral force due to camber is always centripetal, while lateral force due to slip is only centripetal if camber force alone is insufficient to compensate its share of centrifugal force of the vehicle. Thus, F_{y_α} can be centrifugal for low lateral acceleration values, while it always becomes centripetal for high acceleration values [19].

However, while there is not a way to divide the total lateral force into slip and camber contributes (given that the two can have concordant or discordant sign, and that slip force can even be null in case slip angle is zero, as happens in the particular case of camber force being exactly equal and opposite to centrifugal force) it is indeed possible to express a fixed ratio between cornering stiffness and camber stiffness, defined as [17]:

$$K_\alpha = -\frac{F_{y_\alpha}}{\alpha} \tag{4a}$$

and

$$K_\gamma = -\frac{F_{y_\gamma}}{\gamma} \tag{4b}$$

respectively. In fact, for low to medium values of slip angle and camber angle (and so, of the lateral acceleration of the motorcycle) both F_{y_α} and F_{y_γ} are approximately proportional to their independent variable [4]: this means that the ratio of their slopes (the corresponding *stiffness*) is approximately constant. We will call this ratio *side force stiffness ratio*, denoted by:

$$C = \frac{K_\gamma}{K_\alpha}. \tag{5}$$

Cornering and camber stiffnesses will be estimated for different values of lateral acceleration, in order to make the tyre model *adaptive* with respect to variation of the relevant inputs, which are tyre slip and camber. This means that the two stiffnesses are not a constant value, but are expressed as a function of tyre inputs instead: this input could either be tyre slip or tyre camber. However, we'll use tyre camber as independent variable due to its monotone behaviour with the lateral acceleration of the motorcycle, that is linked to roll angle in quasi-static manoeuvres. Due to the coupled nature of slip and camber during a quasi-static manoeuvre, the tyre slip will be a function of the camber itself, so that by expressing the cornering or camber stiffness as a function of the camber we are implicitly taking into account the reduction of such stiffnesses due to the slip angle. As a consequence, no additional error will be introduced by just considering camber as the independent variable in place of the couple as long as the manoeuvre is quasi-static and so as long as the two variables are coupled. If we express lateral tyre force as sum of slip and camber contributes, and expressing such contributes using the stiffnesses defined by Equations (4), and by using Equation (5) to express the camber stiffness as a portion of the cornering stiffness we obtain:

$$F_y = F_{y_\alpha} + F_{y_\gamma} = -K_\alpha(\gamma)\alpha - K_\gamma(\gamma)\gamma = -K_\alpha(\gamma)(\alpha + C\gamma). \tag{6}$$

The aim of the tyre model is to describe both the longitudinal and the lateral dynamics independently, and the lateral acceleration present in a cornering manoeuvre does not cause load transfer, contrary to what experimented by car tyres. However, even while reproducing lateral manoeuvres with constant longitudinal speed there is a small load transfer caused by aerodynamic drag, which makes front and rear tyre load depend on the speed at which the manoeuvre is executed. Thus, it is necessary to include a vertical force dependency to take into account the load transfer due to aerodynamic drag, which could be significant in case of high speed manoeuvres. So, the final expression of lateral force is as follows:

$$F_y = -K_\alpha(\gamma)(\alpha + C\gamma) = -k_\alpha(\gamma)F_z(\alpha + C\gamma), \tag{7}$$

where k_α is the cornering stiffness per unit of vertical load. The influence of the vertical load is assumed to be proportional, but this simple hypothesis should produce a negligible error caused by load transfer due to drag being small, and due to proportional influence on cornering (and so also camber) stiffness being much closer to experimental evidence [4] than by totally neglecting such influence. Following this proportionality hypothesis, Equation (5) can be restated as $C = \frac{k_\gamma}{k_\alpha}$.

3.2.2. Overturning moment

The next step is to define a formulation for the overturning moment: BikeSim© default Magic Formula tyre represents the pneumatic tyre as a torus with circular (when undeformed) cross-section; the crown ground contact point is calculated considering the crown radius, vertical stiffness and instantaneous vertical load values. However, the crown radius is not an easily available or measurable parameter: so, the custom tyre model will be modelled as a lenticular tyre. This makes it possible to implement the tyre model also in simplified motorcycle models that, unlike the ones in BikeSim©, do not model the tyre as a torus. While cornering, the ground contact point is different for a lenticular tyre compared to a toroidal tyre, leading to lower roll: the overturning moment M_x compensates for that, increasing the roll. Tyre kinematics suggests the overturning moment to be proportional to both tyre load and to the tangent of camber angle [20]:

$$M_x = Q_x \tan\gamma = q_x F_z \tan\gamma, \tag{8}$$

where q_x is effectively the crown radius deformed with static load (so that $= q_x \tan\gamma$ is the tyre load moment arm), with the difference that its value is estimated from the difference between the effective roll angle and the ideal roll angle of a motorcycle equipped with lenticular wheels. The only difference, compared to the formulation of the reference tyre model, is the neglection of crown radius deformation due to *differential* vertical loads with respect to the one used in the estimation manoeuvre: given that description of combined dynamics is not the focus of this study, the only load transfer experienced in lateral manoeuvres is the one due to drag force; however the effect on the deformed crown radius of the differential load transfer due to difference in speed between the estimation manoeuvre and the manoeuvre that we are interested in reproducing is small, so that this contribution can effectively be neglected.

3.2.3. Yaw moment

Lastly, and similarly to lateral force, the yaw moment can be expressed as the sum of two separate contributes [4,8,13]: one due to slip angle, and one due to camber:

$$M_z = M_{z_\alpha} + M_{z_\gamma}. \tag{9}$$

The yaw moment due to the slip angle emerges from the (approximately triangular) lateral stress distribution described by *brush model* theory: the lateral force due to slip is not applied in contact patch centre, but is instead applied at a certain distance (rearward, for small to medium slip angles). Such distance is called *pneumatic trail* [22], indicated with t_{p}. Thus, it is possible to apply the force in the nominal contact patch centre by adding the following moment:

$$M_{z_\alpha} = -t_{\mathrm{p}} F_{y_\alpha}. \tag{10}$$

The following pneumatic trail expression [23] is used:

$$t_{\mathrm{p}} = t_{\mathrm{p0}}(1 - \frac{K_{\alpha 0}}{3\mu F_z}|\tan\alpha|), \tag{11}$$

where $K_{\alpha 0} = K_\alpha\big|_{\alpha=0}$, in the region where $F_{y_\alpha} \propto \alpha$ [4]; and μ is the tyre-road friction coefficient. Following the proposed formulation $K_{\alpha 0} = k_{\alpha 0} F_z$, Equation (11) reduces to

$$t_{\mathrm{p}} = t_{\mathrm{p0}}(1 - \frac{k_{\alpha 0}}{3\mu}|\tan\alpha|). \tag{12}$$

For small slip angles, Equation (12) becomes $t_{\mathrm{p}} \approx t_{\mathrm{p0}}$, and so t_{p0} is the pneumatic trail when the tyre slips moderately: in these circumstances, the slip region becomes very small, and the stress distribution can be approximated by a right triangle [7]; resulting force arm is:

$$t_{\mathrm{p0}} = \frac{l}{6}, \tag{13}$$

with l being the length of the contact patch, which can be approximated as [24]:

$$l = 2\sqrt{R_{\mathrm{unloaded}}^2 - R_{\mathrm{loaded}}^2}, \tag{14}$$

with R representing tyre radius. R_{loaded} can be expressed as:

$$R_{\mathrm{loaded}} = R_{\mathrm{unloaded}} - \frac{F_z}{K_z}, \tag{15}$$

with the vertical stiffness of the tyre K_z assumed constant.

Equation (10) shows the importance of having an accurate description of the cornering stiffness, and more generally of the portion of lateral force that is due to slip. In fact, an accurate estimation of the lateral force produced by the tyre does *not* guarantee a correct description of the lateral force due to slip, given that only this portion of lateral force is applied with a moment arm equal to the pneumatic trail. In fact, this also requires a correct value of the *side force stiffness ratio* C used to divide the total lateral force into the two slip and camber contributes. This inevitable error adds to the one due to the hypothesis that cornering and camber stiffnesses have a constant ratio throughout the manoeuvre.

The yaw moment due to camber, also called *twisting moment*, arises from the fact that, due to camber angle, the turn centre point of the circular trajectory described by the wheel is farther from the motorcycle than the intersection point of the axis of the wheel with the road plane [19]. If the longitudinal slip is globally zero, then the local longitudinal slip will be zero in the middle plane of the wheel, positive in the external zone and negative in the internal zone: resulting longitudinal force will still be zero, however this stress distribution originates a twisting moment that tends to reduce the curvature radius of the wheel. This moment is approximately proportional to the camber angle, based on both theoretical models [25] and experimental data [8,10,19, 26], especially at low to moderate camber angles; furthermore it is also approximately proportional to tyre load [10]; consequently, the formulation used in the proposed tyre model is:

$$M_{z_\gamma} = -Q_z\gamma = -q_z F_z \gamma. \tag{16}$$

By explicitly including the vertical force into the previous equation, we are making the tyre model adaptive to the load transfer due to drag force.

3.3. Formulation summary

The tyre model formulation proposed in this paper is summarised in the following equations (for the general, front or rear tyre):

$$F_x(s, F_z) = \mu_x F_z \tag{17a}$$

$$F_y(\alpha, \gamma, F_z) = -k_\alpha F_z(\alpha + C\gamma) \tag{17b}$$

$$M_x(\gamma, F_z) = q_x F_z \tan\gamma \tag{17c}$$

$$M_z(\alpha, \gamma, F_z) = [t_\mathrm{p} k_\alpha \alpha - q_z\gamma] F_z \tag{17d}$$

$$\mu_x = \mu_x(s), \quad k_\alpha = k_\alpha(\gamma), \quad t_\mathrm{p} = t_\mathrm{p}(\alpha, F_z). \tag{17e}$$

The proposed tyre model uses a novel formulation, with completely different expressions compared to the usual Magic Formula, in order to make the best use of the limited information available regarding inputs and outputs of the tyre and due to many tyre inputs being coupled during quasi-static manoeuvres. This would be a significant limiting factor in case of a conventional tyre model (as Pacejka's Magic Formula), while it favours the characterisation of the proposed tyre model: in fact, its custom formulation allows to use just one input to implicitly take into account all the others (for example, camber and slip angle influence on cornering stiffness). Moreover, its non-linear nature is evident from Equations (17): additionally, even though some equations may appear linear in one variable, they are effectively nonlinear. For example F_x is effectively non-linear with F_z, due to vertical load being coupled with tyre slip while braking. Last, it can be noticed that the tyre model successfully satisfies the target of using all the estimated inputs (s, α, γ, F_z) to calculate all the required outputs (F_x, F_y, M_x, M_z).

4. Estimation models and parameters evaluation

The previous section described the hypotheses and consequent equations that constitute the proposed tyre model; such equations contain constants (for example, q_x) and functions of input variables (for example, $k_\alpha(\gamma)$): these constants and functions must be estimated for the specific tyre, through appropriate quasi-static driving manoeuvres as is the aim of the study. Such manoeuvres are conducted with the motorcycle model equipped with the reference, Magic Formula tyres in order to have a simulated dataset; measured signals are then given as input to a simplified motorcycle model to estimate tyre inputs and outputs. These values are then used to characterise the custom tyre model via regression.

4.1. Longitudinal dynamics estimation

4.1.1. Drag and rolling resistance coefficients

First, drag and rolling resistance coefficients are estimated by conducting a coasting manoeuvre, starting at a speed high enough for aerodynamic resistance to be significant and ending at a speed low enough for it to be negligible compared to rolling resistance. An accurate drag estimation is only necessary if high speed manoeuvres are of interest.

The total longitudinal force acting on the motorcycle is simply calculated as $F_x = ma_x$; this force is equal to front and rear tyre rolling resistances plus drag force. Rolling resistance is assumed to be independent of speed as Equation (3) shows, while drag is assumed to be quadratic with speed:

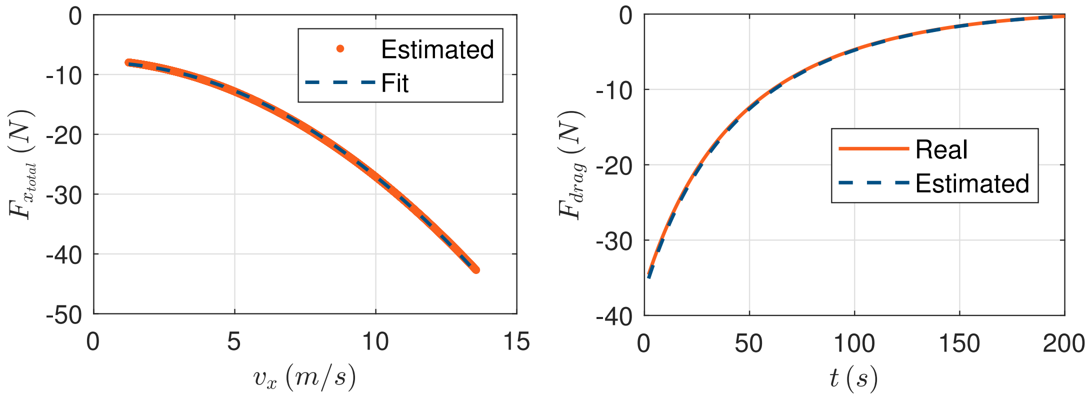
$$F_{\text{drag}} = -c_{\text{drag}} v_x^2. \tag{18}$$

Front and rear rolling resistance coefficients are assumed to be equal in order to reduce the number of unknowns.

Figure 1a shows the regression used to obtain the two coefficients, indicating good fit, while Figure 1b shows that by using the previously obtained drag resistance coefficient we are able to correctly estimate drag force during the manoeuvre.

4.1.2. Effective rolling radius estimation

An accurate estimation of the rolling radius of the tyre is crucial in order to estimate tyre slip; this parameter is estimated, for the front and the rear tyre, from the previous



(a) Regression providing rolling and aerodynamic resistance coefficients.

(b) Estimated drag force accuracy.

Figure 1. Rolling resistance and aerodynamic drag estimation from a coasting maneuver.

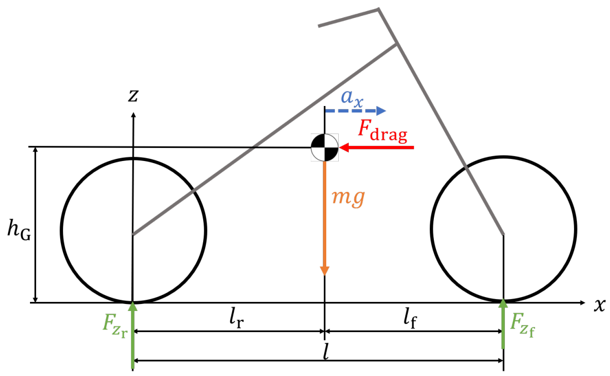


Figure 2. Motorcycle load transfer model.

coasting manoeuvre. In such conditions, tyre slip is assumed to be zero, and the tyre rolling radius is estimated from the definition of slip ratio:

$$R_{\text{rol}} = \frac{v_x}{\omega}, \tag{19}$$

where v_x is the longitudinal speed of the motorcycle and ω is the angular speed of the tyre. Rolling radius is then taken as the time average of the previous instantaneous estimate.

4.1.3. Input/output relationship estimation

First, an estimation method for the two tyre inputs s and F_z and the output F_x is developed.

Tyre slip is estimated through its SAE definition, using the previously estimated rolling radius:

$$s = \frac{\omega R_{\text{rol}} - v_x}{v_x}. \tag{20}$$

Figure 2 presents the simple load transfer model used, shown in the particular case of roll angle ϕ equal to zero. The load transfer is estimated as the rigid load transfer due to inertial and aerodynamic forces (the latter applied at centre of gravity height), and is added to the static load:

$$F_{z_\mathrm{f}} = mg\frac{l_r}{l} - \Delta F_z \qquad F_{z_\mathrm{r}} = mg\frac{l_f}{l} + \Delta F_z \tag{21a}$$

$$\Delta F_z = (ma_x - F_\mathrm{drag})\frac{h_\mathrm{G}\cos\phi}{l}, \tag{21b}$$

where l represents the wheelbase, $l_\mathrm{f,r}$ are the distances from the vehicle centre of gravity to front and rear axles, h_G is centre of gravity height and F_drag is the drag force calculated using the previously estimated parameter and the instantaneous speed.

Lastly, tyre longitudinal force must be estimated; this is done by considering the four different sources of longitudinal force that act on the motorcycle:

- Aerodynamic drag F_drag (always negative).
- Rolling resistance F_rol (always negative).
- Tyre force due spin inertia: each wheel must adapt its angular speed to the increasing or decreasing vehicle speed; this requires a tyre-road longitudinal force due to the spin inertia of the wheel I_y. This force is equal to:

  $$F_\mathrm{iner} = -\frac{I_y}{R_\mathrm{rol}}\dot{\omega}, \tag{22}$$

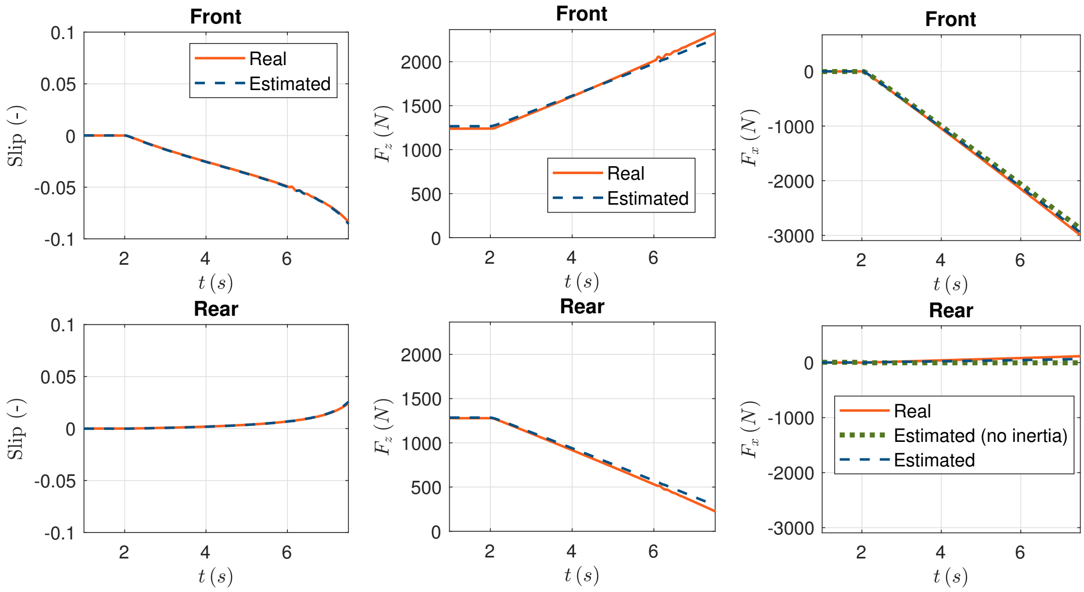
  where $\dot{\omega}$ is the angular acceleration of the wheel: the minus sign is due to the fact that a negative (rearward) longitudinal force originates to increase the angular speed of the wheel.
- Driving/Braking force: acts on the rear tyre when accelerating, or is shared between front and rear tyre while braking; braking manoeuvres used in the estimation phase use just one brake, so that the value of brake bias ρ is known ($\rho = 1$ for a front braking manoeuvre, $\rho = 0$ for a rear braking manoeuvre). For each instant, the driver is braking if and only if the total tyre force $F_{x_\mathrm{f}} + F_{x_\mathrm{r}} = ma_x - F_\mathrm{drag}$ is lower (with sign) than the total rolling resistance $F_{\mathrm{rol}_\mathrm{f}} + F_{\mathrm{rol}_\mathrm{r}}$.

With the previous considerations, the following longitudinal force estimation algorithm is defined and applied for each instant:

$$F_{x_\mathrm{f}} = \begin{cases} \rho(ma_x - F_\mathrm{drag} - (F_{\mathrm{rol}_\mathrm{f}} + F_{\mathrm{rol}_\mathrm{r}})) + F_{\mathrm{rol}_\mathrm{f}} + (1-\rho)F_{\mathrm{iner}_\mathrm{f}} - \rho F_{\mathrm{iner}_\mathrm{r}} & \text{braking} \\ F_{\mathrm{rol}_\mathrm{f}} + F_{\mathrm{iner}_\mathrm{f}} & \text{otherwise} \end{cases} \tag{23}$$

$$F_{x_\mathrm{r}} = \begin{cases} (1-\rho)(ma_x - F_\mathrm{drag} - (F_{\mathrm{rol}_\mathrm{f}} + F_{\mathrm{rol}_\mathrm{r}})) + F_{\mathrm{rol}_\mathrm{r}} - (1-\rho)F_{\mathrm{iner}_\mathrm{f}} + \rho F_{\mathrm{iner}_\mathrm{r}} & \text{braking} \\ ma_x - (F_\mathrm{drag} + F_{\mathrm{rol}_\mathrm{f}} + F_{\mathrm{iner}_\mathrm{f}}) & \text{otherwise.} \end{cases} \tag{24}$$

In a real test, the throttle and brake inputs channels are available from the CAN bus and can be used to detect throttle and brake usage, instead of the force-based conditions.



(a) Slip ratios. (b) Vertical loads. (c) Longitudinal forces.

Figure 3. Tyre slip ratio and forces estimation (Front braking manoeuvre).

The estimation equations described are applied to three manoeuvres used to characterise the tyre model: a second gear acceleration manoeuvre, a front braking manoeuvre and a rear braking manoeuvre. Figure 3 shows the estimation quantities for the front braking manoeuvre, compared to the real values.

In Figure 3a the very accurate tyre slip estimation is shown: notice that when front braking force becomes intense, the rear tyre slips forward. This is due to the fact that its vertical load is reducing thus reducing its force-producing potential, while its angular speed is decreasing fast to match the reducing speed of the vehicle: the resulting positive force F_{iner_r} makes it slip.

Figure 3b shows that tyre load is estimated with small error, due to the geometric load transfer due to pitch being neglected.

Lastly, Figure 3c shows the estimated tyre longitudinal force: the estimated value is close to the real one throughout the manoeuvre, with small error even when close to forward flip over. The estimation results with wheels inertia being neglected is also shown: the error increases but remains moderate; so in case wheel inertia was not known (for example, derived from a *CAD* model) the estimation method still provides a close estimate.

The two estimated inputs and the estimated output, for both tyres and in the three manoeuvres, can be used to determine the relationship in Equation (17a): the fitting process is shown in Figure 4, using a fifth order (uneven) polynomial for $\mu_x(s)$.

4.2. Lateral dynamics estimation

4.2.1. Tyre inputs and outputs estimation

Similarly to what already shown for longitudinal dynamics, it is necessary to develop a method for the estimation of tyre inputs (α, γ, F_z) and outputs (F_y, M_x, M_z) for lateral dynamics.

A single track model can be used to estimate tyre slip angles for both a car and a

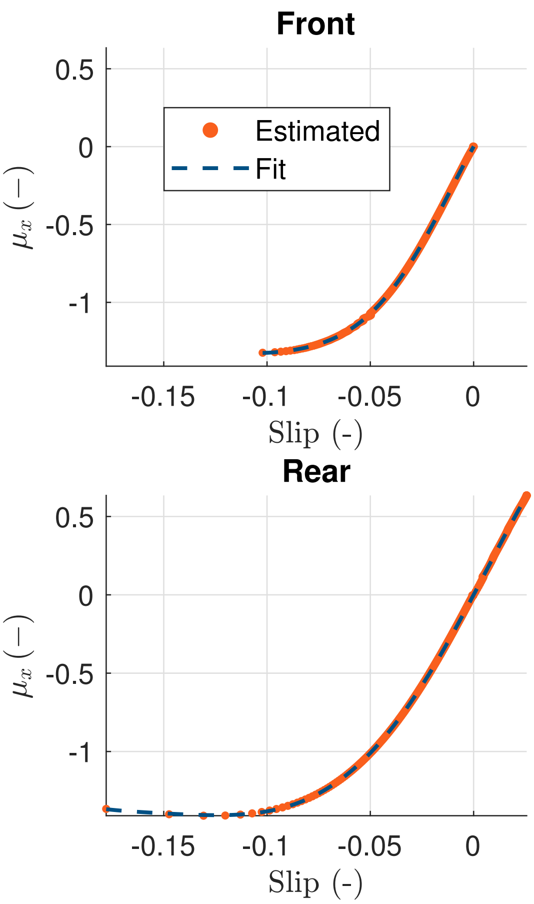
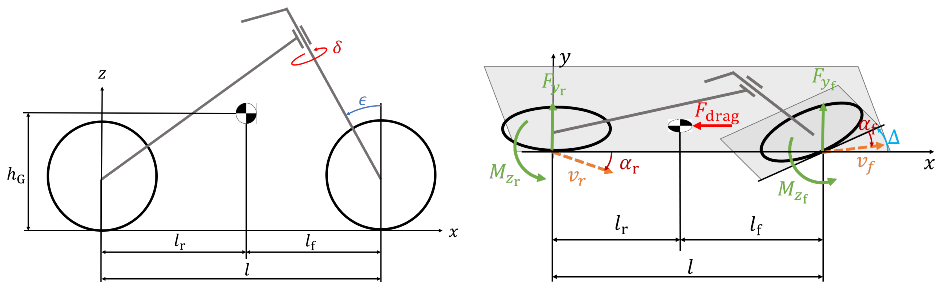


Figure 4. Polynomial regression used to obtain the longitudinal adherence coefficient expression.



(a) Side view. (b) Top view.

Figure 5. Simplified motorcycle model used for the estimation in lateral dynamics manoeuvres.

motorcycle; moreover, we can use a simplified, quasi-static expression as the manoeuvres used in the estimation phase are all quasi-static. The simplified model used for the estimation is shown in Figure 5. The expressions proposed by Sierra et al. [17] are adapted to describe motorcycle dynamics by adding the term $h_G \cos\phi\dot{\phi}$, due to the fact that motorcycle center of mass and tyres are at different heights:

$$\alpha_f = \frac{v_{y_f}}{v_{x_f}} - \Delta = \frac{v_{y_G} + h_G \cos\phi\dot{\phi} + l_f\dot{\psi}}{v_x} - \Delta \tag{25a}$$

$$\alpha_r = \frac{v_{y_r}}{v_{x_r}} = \frac{v_{y_G} + h_G \cos\phi\dot{\phi} - l_r\dot{\psi}}{v_x}, \tag{25b}$$

where $\dot{\psi} = \frac{\dot{\psi}_{IMU}}{\cos\phi}$ is the yaw rate of the motorcycle in ground frame of reference with $\dot{\psi}_{IMU}$ being the yaw rate measured by the IMU, $\dot{\phi}$ is the roll rate measured by the IMU, and Δ is the kinematic steering, that if pitch is neglected can be calculated as [19]:

$$\Delta = \arctan\left(\frac{\sin\delta\cos\epsilon}{\cos\phi\cos\delta - \sin\phi\sin\delta\sin\epsilon}\right), \tag{26}$$

where δ is the steering wheel angle and ϵ is the caster angle.

Camber angles are calculated as shown in the following equations [19, p.35]:

$$\gamma_f = \arcsin\left(\cos\delta\sin\phi + \cos\phi\sin\delta\sin\epsilon\right) \qquad \gamma_r = \phi. \tag{27}$$

Tyre vertical load, the last input to be estimated, is calculated using Equations (21) whose accuracy has already been validated as shown by Figure 3b. The term $h_G \cos\phi$, in fact, considers the influence of roll on the center of gravity height, extending its validity to lateral dynamics manoeuvres.

Tyre inputs estimation accuracy is shown in Figure 6 for the quasi-static lateral manoeuvre used for tyre characterisation: as is evident, the proposed method for tyre slip angle and camber angle estimation using signals measured by the IMU and the steering wheel sensor does provide an accurate estimate of tyre inputs, when a simulated dataset is used.

Tyre lateral force is estimated using the lateral and yaw equilibrium equations, in the simplified case of a quasi static manoeuvre: the equations shown by Sierra et al.

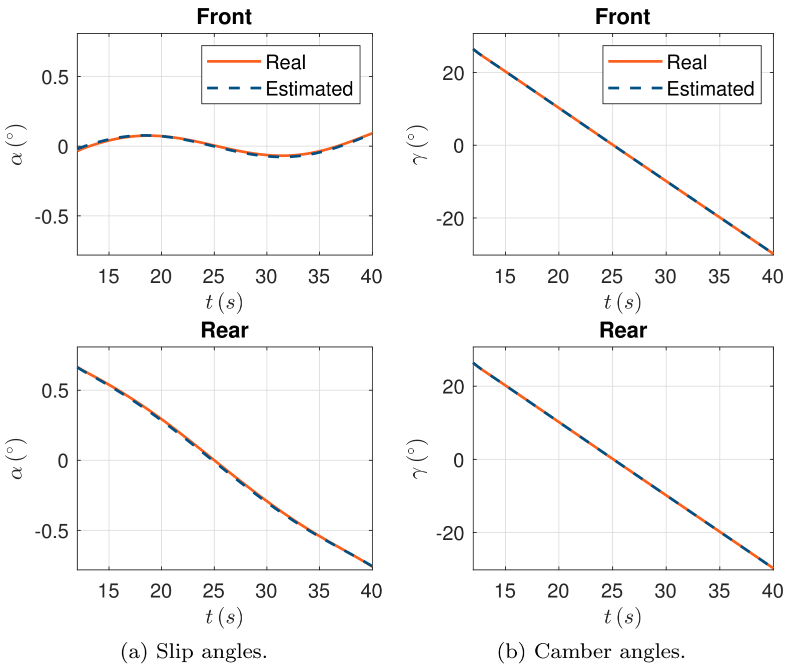


Figure 6. Tyre slip angle and camber estimation (quasi-static lateral manoeuvre).

[17] are simplified by neglecting the time derivatives. Additionally, we also consider the yawing moment (due to slip and camber) generated by the two tyres, alongside with the additional yawing moment (indicated with $M_{z_{\rm long}}$) generated by the longitudinal force produced by the tyres not being aligned with the centre of gravity of the motorcycle due to roll angle. The resulting equations are:

$$\begin{cases} F_{y_{\rm f}} = \frac{l_{\rm r}\dot{\psi}v_x - (M_{z_{\rm f}} + M_{z_{\rm r}} + M_{z_{\rm long}})}{l} \\ F_{y_{\rm r}} = \frac{l_{\rm f}\dot{\psi}v_x + M_{z_{\rm f}} + M_{z_{\rm r}} + M_{z_{\rm long}}}{l} \end{cases}, \tag{28}$$

where $M_{z_{\rm f}}$ and $M_{z_{\rm r}}$ are calculated using Equation (17d), and

$$M_{z_{\rm long}} = -h_{\rm G}\sin\phi(ma_x - F_{\rm drag}). \tag{29}$$

Considering that, for our formulation:

$$k_\alpha F_z \alpha = F_{y_\alpha} = \frac{\alpha}{\alpha + C\gamma} F_y, \tag{30}$$

we can obtain the estimated lateral forces by solving the system (28) for the two lateral tyre forces:

$$\begin{pmatrix} F_{y_{\rm f}} \\ F_{y_{\rm r}} \end{pmatrix} = \begin{bmatrix} l - t^*_{\rm p_f} & -t^*_{\rm p_r} \\ t^*_{\rm p_f} & l + t^*_{\rm p_r} \end{bmatrix}^{-1} \begin{pmatrix} l_{\rm r}m\dot{\psi}v_x - (M_{z_{\gamma_{\rm f}}} + M_{z_{\gamma_{\rm r}}} + M_{z_{\rm long}}) \\ l_{\rm f}m\dot{\psi}v_x + M_{z_{\gamma_{\rm f}}} + M_{z_{\gamma_{\rm r}}} + M_{z_{\rm long}} \end{pmatrix}, \tag{31}$$

where $t^*_{p_{\rm f,r}} = \frac{\alpha_{\rm f,r}}{\alpha_{\rm f,r} + C_{\rm f,r}\gamma_{\rm f,r}} t_{p_{\rm f,r}}$.

Equation (31) provides the estimated lateral tyre forces for each instant; however, it requires an estimation of the side force stiffness ratio C of each tyre and of the yaw moments due to camber $M_{z_\gamma} = -q_z F_z \gamma$. $C_{f,e}$ and $q_{z_{\mathrm{f,r}}}$ are obtained using the experimental data from [4,14] and [4,10,13,14], respectively: the data are relative to various sport motorcycle front and rear tyres, representative of the reference tyres used in the simulations. The values have been obtained as the average between the various tyres presented, for various load, slip and camber values, inside the linear behaviour region that justifies the proportionality hypothesis shown in Equations (5) and (16). With these constants being known, the terms in Equation (31) can be calculated for each time step obtaining an estimate of tyre lateral force; doing so, we also obtain an estimation of the yawing moment of each tyre, using Equation (17d).

Lastly, it is necessary to estimate the overturning moment M_x. In fact, the reference, Magic Formula tyre considers a toroidal tyre, while the proposed tyre model uses a lenticular formulation. Thus, the lenticular tyre requires an additional moment that compensates for the lateral shift of the point of application of the vertical force. If this moment is not included, the vehicle equipped with lenticular tyres will corner with lower roll angles [19, p.128] compared to the one equipped with more realistic, toroidal tyres. By writing the equilibrium equations for the motorcycle in the front view we obtain, after a few passages, the following estimation equation:

$$M_{x_\mathrm{f}} + M_{x_\mathrm{r}} = M_{x_\mathrm{tot}} = m[v\dot{\psi}(\cos\phi_\mathrm{len} - \cos\phi) - gh_\mathrm{G}(\sin\phi_\mathrm{len} - \sin\phi)], \tag{32}$$

where

$$\phi_\mathrm{len} = -\arctan\frac{\dot{\psi}v_x}{g} \tag{33}$$

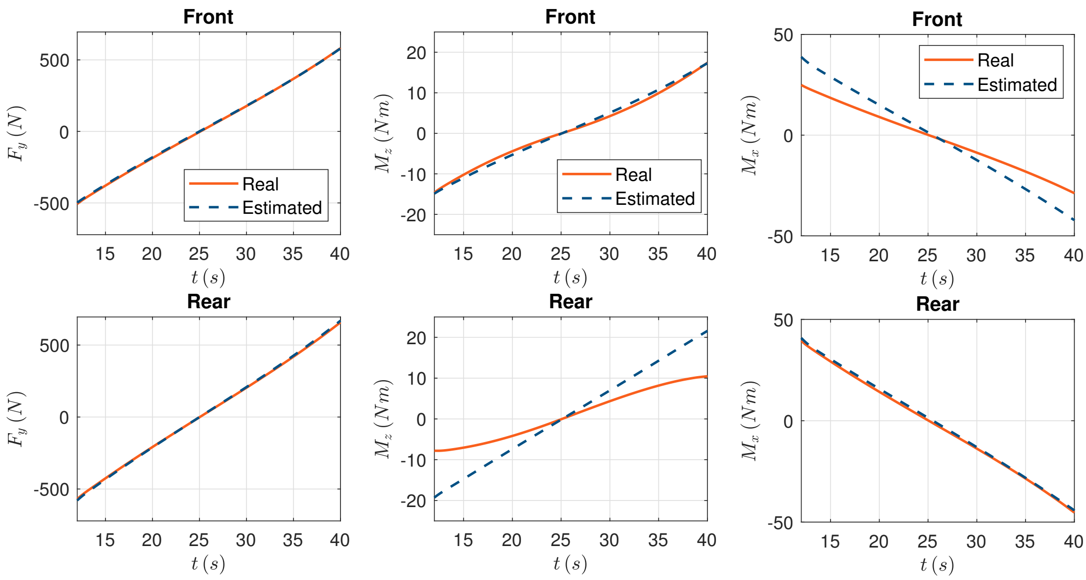
is the roll angle of the theoretical motorcycle model equipped with lenticular wheels, without overturning moments. The moment acting on the single tyre is obtained by dividing the total moment proportionally to the share of vertical load acting on each tyre:

$$M_{x_\mathrm{f}} = \frac{F_{z_\mathrm{f}}}{mg}M_{x_\mathrm{tot}} \qquad M_{x_\mathrm{r}} = \frac{F_{z_\mathrm{r}}}{mg}M_{x_\mathrm{tot}}. \tag{34}$$

Figure 7 compares the estimated output quantities to the real ones: the lateral force is correctly estimated, for both the front and the rear tyre. The total yaw moment, sum of the two single contributes due to slip and due to camber, is correctly estimated at the front while it is overestimated at the rear: however, the small nature of the absolute error is proven by the fact that the induced error on the estimated lateral force is negligible. On the contrary, tyre overturning moment is correctly estimated at the rear while being overestimated at the front.

4.2.2. Tyre parameters estimation from regression

The estimated inputs and outputs can be used to determine tyre parameters via regression, to characterise its equations. The test used is a manoeuvre in which the roll angle changes slowly, in order for the manoeuvre to be quasi-static. Roll angle is varied in a value interval that sweeps the lateral acceleration range of interest: the chosen range is $(-5, 5)m/s^2$ range, corresponding to a maximum roll angle around 25°. Due to the not perfectly quasi-static nature of every real manoeuvre, derivatives of tyre inputs are not zero: due, for example, to the relaxation behaviour of the reference tyre



(a) Lateral forces. (b) Yaw moments. (c) Overturning moments.

Figure 7. Tyre outputs estimation (quasi-static lateral manoeuvre).

this will cause slightly different outputs being produced for the same set of inputs, depending on the fact that roll angle (and so slip and camber angles) is increasing or decreasing. In order to make the characterisation process more robust, the same manoeuvre will be conducted twice, with opposite sign of its derivatives, and both results will be used in the regression.

Equations (17b–17d) provide the lateral dynamics description of the proposed model: the function $k_\alpha(\gamma)$ and the parameter q_x must be determined for each tyre in order to conclude the tyre model characterisation.

First, we estimate the cornering stiffness function for each tyre (and so also the camber stiffness functions, being the latter proportional to the former with the previously determined proportionality factor C). The cornering stiffness per unit of vertical load k_α is obtained from Equation (30):

$$k_\alpha = \frac{F_y}{(\alpha + C\gamma)F_z}. \tag{35}$$

Equation (35) is evaluated at each time step using the estimated tyre inputs and outputs and the known value of C: this can possibly lead to numerical division by zero for low camber values (and consequently low slip angle values, the two being coupled); this problem is solved by excluding the data points with camber angle lower than a minimum threshold ($< 2.5°$). Regression results for $k_\alpha(\gamma)$ are shown in Figure 8: a sixth order (even) polynomial is used to fit the estimated data, that cover the whole range of interest. The polynomial function obtained is used by the tyre model to calculate tyre lateral force and tyre yaw moment as shown in Equations (17b) and (17d) respectively.

Lastly, the overturning moment can be characterised. In order to do so, as shown by Equation (8), we need a value for q_x of each tyre. However, based on the load-based total moment division shown in Equation (34), we are considering the same q_x value for the two tyres. Considering that $\gamma_\mathrm{f} \approx \gamma_\mathrm{r} = \phi$, we can obtain such value as the slope

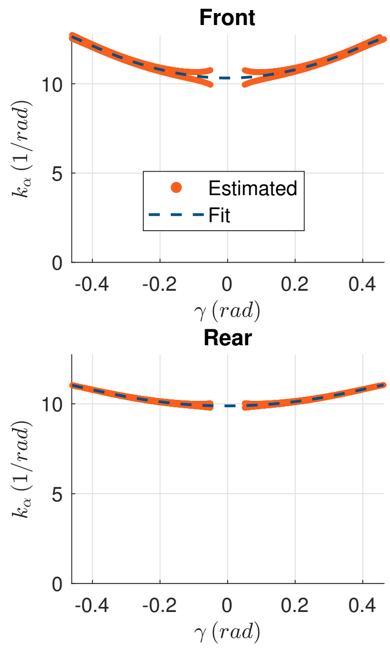


Figure 8. Polynomial regression used to obtain the cornering stiffness expression.

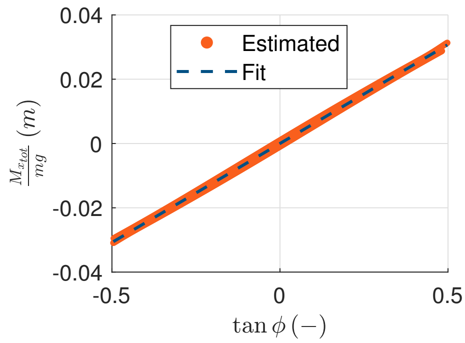


Figure 9. Linear regression to obtain the overturning moment coefficient.

of the linear regression (forced through point $(0, 0)$) shown in Figure 9.

Tyre model Equations (17) are now fully characterised, and the proposed tyre model can be used in simulations and compared to the reference, Magic Formula tyre model.

5. Results

The accuracy of the proposed tyre model is evaluated by comparing the simulation results of the motorcycle model equipped with the reference Magic Formula tyre model, considered a proxy for experimental data, with the same motorcycle model equipped with the custom tyre model. Both longitudinal and lateral dynamics manoeuvres are executed to assess its behaviour.

The tyre model is implemented in MATLAB-Simulink© using Equations (17): at each simulation time step the model receives tyre inputs from the BikeSim© high-fidelity motorcycle model; then it calculates tyre outputs which are fed back into the

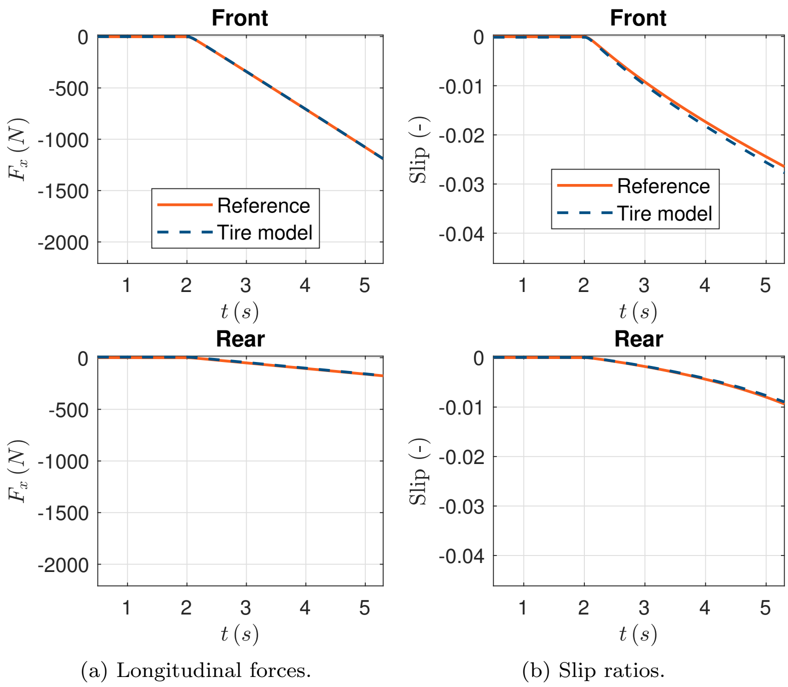


Figure 10. Tyre longitudinal force and slip ratio results (combined braking).

motorcycle model.

5.1. *Longitudinal dynamics*

First, two longitudinal manoeuvres are conducted:

- an acceleration manoeuvre in second gear, with progressively increasing throttle (shown in Appendix A);
- a combined braking manoeuvre, with progressively increasing braking forces.

Figure 10 shows the results for the combined braking manoeuvre. For these and all the following results, the solid, orange line indicates the reference, Magic Formula tyre, while the dashed, blue line indicates the proposed tyre model.: Longitudinal tyre forces almost coincide with reference, while slip ratios contain a small error in both absolute and relative terms, being slightly overestimated at the front and slightly underestimated at the rear.

5.2. *Lateral dynamics*

To test the description of the lateral dynamics operated by the tyre model the following manoeuvre is performed: the motorcycle, moving at a target speed of $20m/s$, progressively reduces its turning radius up to a minimum of $100m$, reaching a maximum roll angle of $23°$ and a lateral acceleration of $0.5g$. Figure 11 shows the inputs and outputs of each tyre during the simulation, along with other important variables: in general, the behaviour tyre model is very close to the one of the reference tyre, with errors on secondary variables, such as tyre moments. Regarding the most evident and significant variables the reference tyre and the proposed tyre model are almost identical: motorcycle roll is the same, and also its slip angle is very close between the

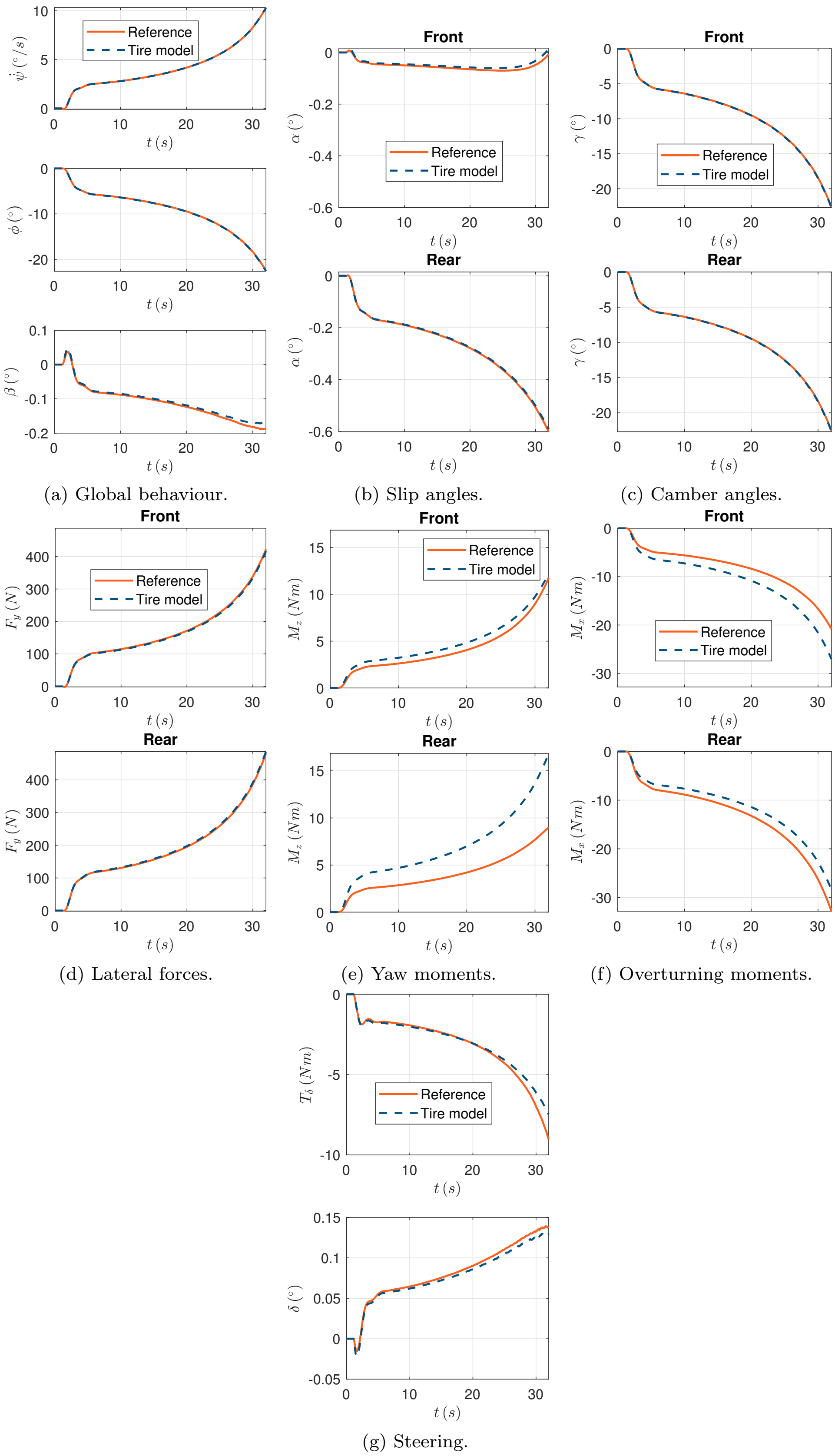


Figure 11. Tyre and motorcycle behavior during the lateral manoeuvre.

two, especially for low to medium lateral acceleration values. Tyre slip and camber angle are well reproduced for both front and rear tyre, as are tyre lateral forces. Front yaw moment contains a small error, while rear yaw moment is overestimated. Total overturning moment is also correct, as reflected by the correct description of motorcycle roll and camber angles. Lastly, steering torque is well predicted especially for low to medium acceleration values and the steering angle contains an error around 0.01°.

6. Discussion

The previous results show that the target of the paper, consisting in defining a motorcycle tyre model formulation that is characterisable through quasi-static manoeuvres, has been achieved. The physical quantities, which are measurable inputs provided by IMU, wheel speed sensors and steering angle sensor, are sufficient to define the parameters of a suitably defined tyre model. Such tyre model correctly described tyre inputs and outputs during both longitudinal and lateral manoeuvres. This constitutes a good starting point to verify if these variables, measured by commonly available sensors, are adequate for the estimation in a real test.

Results for the longitudinal manoeuvres show that, given a certain throttle or braking input, the single tyre forces are well reproduced due to the correct modelling and characterisation of the rolling resistance of each tyre. Such forces are produced with a longitudinal slip value close to the one shown by the reference tyre: this is a direct consequence of the fact that the equations used for the estimation of tyre slip ratio, longitudinal and vertical force are suitable, and these estimated quantities are used to obtain $\mu(s)$ with small regression error. Moreover, the combined braking manoeuvre (where an additional load transfer is present, for a given tyre slip ratio, compared to the estimation manoeuvres where only one tyre was braked) shows that the hypothesis that this additional load transfer does not change the friction coefficient of the tyre is realistic.

Results for the lateral manoeuvre show that the states of the motorcycle are well reproduced: the proposed tyre model is lenticular, however the additional overturning moment applied allows a correct roll angle description; this is also shown by the fact that the sum of front and rear overturning moments is correct. The vehicle slip angle is very close to reference, consequence of the fact that the proposed tyre model provides a description of the slip angle of each tyre close to the Magic Formula. Lateral forces are well described, indicating that the error on yaw moments is small enough not to affect the front/rear force distribution. The small error on the front slip angle causes a limited error on the steering angle, in the order of 0.01°, much lower than the resolution of real sensors. Lastly, an important result is that the steering torque is well predicted: in a quasi-static manoeuvre, where gyroscopic moments are modest, it is determined by all the forces and moments acting on the front tyre, so the fact that it is close to reference proves the good accuracy of the description of forces and moments by the tyre model and it also shows that the proposed tyre model is able to describe the handling of the motorcycle and driver's action.

Some quantities show a certain degree of error: the error on yaw moments is due to the fact that both C (influencing F_{y_α} and thus M_{z_α}) and q_z (influencing M_{z_γ}) are obtained from statistical data and are not estimated for the specific tyre: such error could be reduced by using a more extensive database, especially for the rear tyre where experimental results relative to only two rear tyres were available in literature for q_z calculation, translating into a more significant error on the rear yaw moment.

Regarding the overturning moment, the correctly estimated total overturning moment could be better divided between front and rear tyre by also taking tyre width into account, as a proxy for the crown radius: this would reduce front overturning moment while increasing the one at the rear, potentially reducing error.

In order to characterise the tyre models, equations for the estimation of tyre inputs (slip ratio, slip angle, camber, load) and outputs (longitudinal and lateral force, overturning and yaw moment) have been developed: these equations could be used for other purposes, for example active safety or virtual sensing.

It is important to consider that the simulated dataset has been used as a proxy for real experimental data: in the case of real data, disturbances due to road slope, wind, sensor accuracy and noise can reduce the accuracy of results. Special attention will be required in order to avoid the introduction of excessive additional error.

Further development could consist in: transient description, which should be aided by the adaptive nature of the proposed formulation with the potential addition of a relaxation length (as in Magic Formula model); combined dynamics; and making the model adaptive to different tyre-road friction coefficients. Results are encouraging, and suggest testing this approach with a real, experimental dataset.

To summarise and conclude, a motorcycle tyre model formulation has been defined with the target of being characterisable through driving manoeuvres, using simple and cost-effective instrumentation. Such formulation, along with the characterisation equations, unlocks the possibility to describe the very tyres equipping the motorcycle, implicitly taking into account the specific wear level, inflation pressure and tyre-road friction. This prevents the need for a dedicated tyre bench test, increasing the number of subjects interested in having a faithful tyre model that are able to satisfy their needs.

Data availability statement

The data that support the findings of this study are available from the corresponding author, Bartolozzi M, upon reasonable request.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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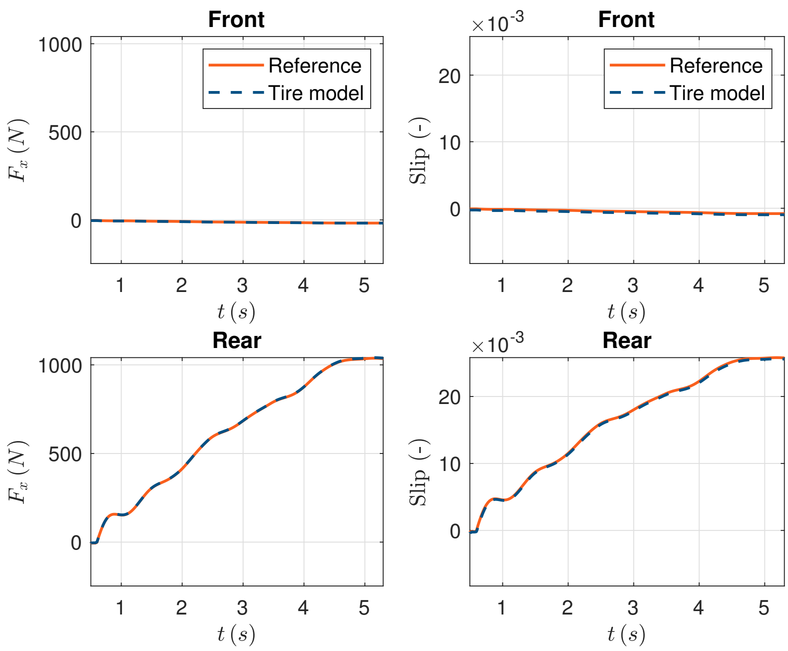
Appendix A. Acceleration manoeuvre

Figure A1 shows the results for the acceleration manoeuvre; Figure A1a shows the very accurate reproduction of tyre forces, as in the case of the braking manoeuvre, while Figure A1b shows the slip ratio of both tyres, with close accordance with the simulated dataset.



(a) Longitudinal forces.

(b) Slip ratios.

Figure A1. Tyre longitudinal force and slip ratio results (second gear acceleration).