Science&Philosophy Volume 2021, pp. 133- 159

Ellipses and ovals: two curves so close and so far

Giuseppe Conti*
Raffaella Paoletti**
Alberto Trotta***

Abstract

In this work we will deal with ellipses and ovals, comparing them both from the geometric point of view and from the one of applications. There is a notable similarity between these curves so often it's not possible to recognize which of the two figures is, unless we consider other elements to distinguish them. We will show the presence of both curves in architectural works and in treatises, motivating their use, when it's possible, with geometric and technological considerations.

Keywords: conics, regular curves, normals, vaults, pointed arches, curvature. [1]

---

* Giuseppe Conti, Dipartimento di matematica DIMAI, Università di Firenze. Italia. gconti@unifi.it

** Raffaella Paoletti, Dipartimento di matematica DIMAI, Università di Firenze, Italia. raffaella.paoletti@unifi.it

***Alberto Trotta, IISS Santa Caterina-Amendola, Salerno, Italia. albertotrotta@virgilio.it

[1] Received on May 6th, 2021. Accepted on June 15th, 2021. Published on June 30th, 2021. doi: 10.23756/sp.v9i1.611. ISSN: 2282-7757. eISSN: 2282-7765.

1. Introduction

Ellipses and ovals are very similar figures; however, they are distant by geometric genesis and even in applications they are used for different reasons. The ellipse is a more "elegant" curve since its curvature varies continuously; moreover, it appears naturally in the construction of cross vaults or pavilion vaults. The meridian curves of the sails of the Dome of Santa Maria del Fiore are arches of ellipses and this fact also contributes to the upward momentum of this construction. The elegance of Santa Trinita Bridge in Florence also depends on the elliptical shape of its arches; moreover, this shape allows a greater quantity of water to flow under the arches without raising their keystones too much. The oval, being formed by arcs of circumferences, inherits some typical advantages of circular figures such as equidistance and convergence towards the centers of the arcs that compose it. It follows from its geometric construction that its curvatures undergo sudden changes in the points of tangency of two adjacent arcs; also, for this reason, sometimes the oval is a little less "graceful" than the ellipse. In this paper we will discuss these questions, showing examples of applications of these curves in art and architecture.

2. Ellipses

The ellipse, together with the circumference, which is a particular case of it, the parabola and the hyperbola form the family of *conics* (not *degenerate*). They have been known since ancient times; the first mentions of them date back to the Hellenistic period; Eudoxus (408 BC-355 BC), Menecmo (380 BC-320 BC), Euclid (IV century BC-III century BC), Archimedes (287 BC-212 BC) and others considered conics as the intersection of a right circular cone with a plane (hence the name of *conic sections*). We note that these authors obtained the three types of conics by varying the angle at the vertex of the cone (acute for the ellipse, obtuse for the hyperbola, right for the parabola), keeping the plane always perpendicular to a generatrix of the cone (see [15] pg. 59).

In the third century BC Apollonius of Perga (262 BC-190 BC) wrote a fundamental treatise on conics, consisting of eight books; the eighth one has been lost and was reconstructed following the writings of Pappus (290 AD-350 AD). We owe to Apollonius the study of conics as the intersection of a fixed right circular cone with a variable plane (see [15] pg.107). Although they were known and deeply studied, the conics did not have practical applications until the beginning of the seventeenth century, especially starting from the works of Giovanni Keplero (1571-1630) and Galileo Galilei (1564-1642).

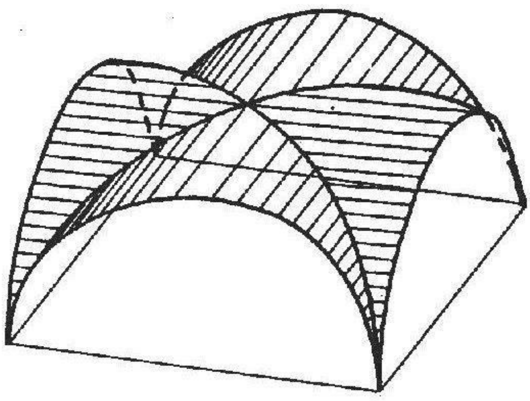

Figure 1. Cross vault

We observe that in the cross vaults of the Roman era there are quite precise arches of ellipse[2], as it was possible to establish by means of tests carried out with laser scanners. In fact, for geometric reasons, the intersection of two right circular half-cylinders, equal and orthogonal to each other, is a semi-ellipse; consequently, the diagonal arches of a cross vault formed by the interpenetration of two equal circular barrel vaults are two semi-ellipses. However, we think that the ancient Romans most likely obtained the ellipses corresponding to the diagonal arches by constructing them by points, starting from the semicircular perimetric arches and without realizing that they were ellipses. Figure 2 shows a possible geometric construction that could have been used by the ancient Romans to determine (by points) the ellipse which defines the profiles of the diagonal arches of a cross vault starting from the semicircular profile of the rounded perimetric arches of the same vault (see [22] pg. 62).

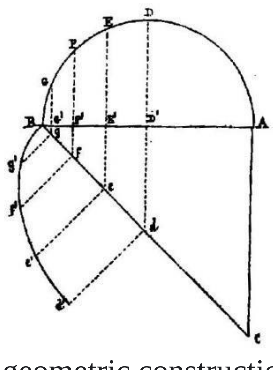

Figure 2. The geometric construction of the ellipse

[2] Recall that an ellipse is the geometric locus of the points of the plane for which the sum of their distances from two fixed points, called foci of the ellipse, is constant.

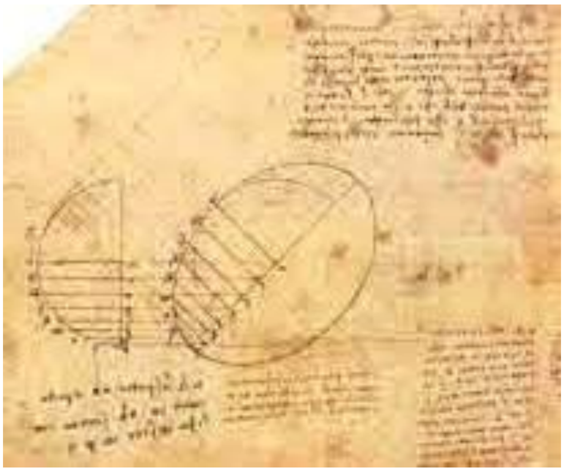

Figure 3. (Codice Atlantico, Vol. IV, t. 318b)

Leonardo da Vinci (1452-1519) did the same construction, as we can see from Figure 3.

Medieval architects were unable to build elliptical arches, both for tracing reasons and for difficulties that arose in cutting the wall ashlars[3]. They built the diagonal arches of the cross vaults in semicircular shape (round arch); in this way, the perimetric arches necessarily had to assume an elliptical profile. Bechmann states (see [4] pg. 169) that the builders tried to replace ellipses with simpler curves and for this reason they introduced the *ogival arches*, which approximated the ellipses with figures based on arcs of circumference.

It is interesting to note that the Gothic *fourth pointed arches* are, among the curves involving circular arcs, the best approximations of the elliptical lateral arches that would be obtained in a cross vault with a square base, starting from the semicircular diagonal arches (maximum error 4%).

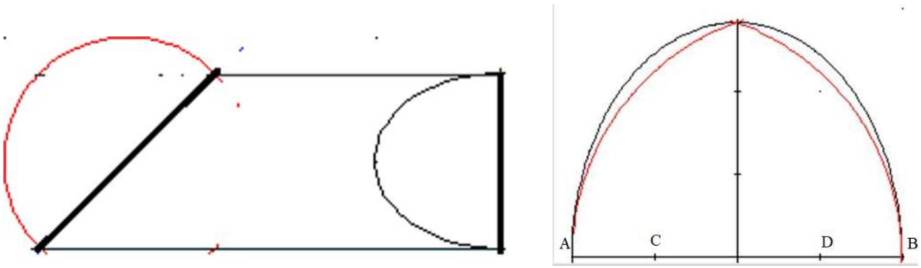

Figure 4. The best approximations of the elliptical lateral arches

[3] Note that in the elliptical arches the wall ashlars have different shapes while in the circular ones they all have the same shape.

Figure 4 highlights the fact that the projection of a semicircle (semicircular diagonal arc) on a plane (perimetral plane), placed at 45° with respect to the plane containing the semicircle, is an ellipse and that the ellipse itself (the black curve on the right in Figure 4) is well approximated by the Gothic *fourth pointed arch* (the red curve on the right in Figure 4).

We recall that a *fourth pointed arch* is formed by two arcs of circumference, with radius 3 units, each having the center (points C and D in Figure 4) placed at a quarter (hence the name "fourth pointed") from the ends of a segment AB measuring 4 units. We can make the same considerations for the pavilion vaults; if their base is a square and they are obtained starting from two straight circular cylinders orthogonal to each other, then the edge curves are two semi-ellipses, while the median curves (those obtained by intersecting the vault with the two planes containing its axis and parallel to one pair of sides of the base square) are two semicircles.

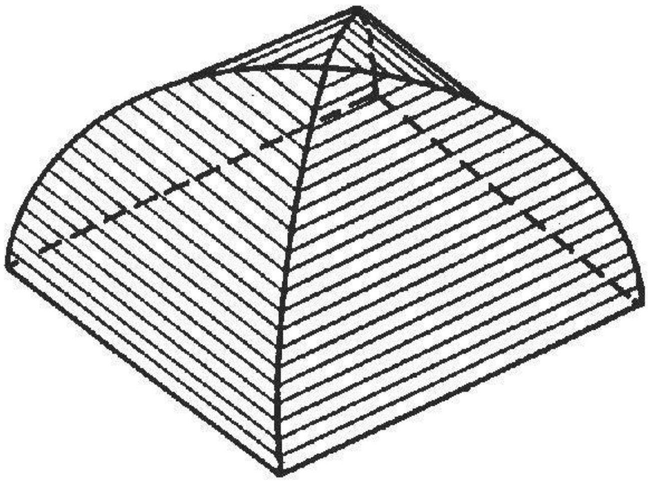

Figure 5. Pavilion vault

It is interesting to note that the Dome of Santa Maria del Fiore in Florence is also a pavilion vault, but with an octagonal base.

However, in this case, the edge ribs are arcs of circumference, while the meridian curves of the sails are elliptical arches (see [10] pg. 194). Therefore, we obtain the surface of the Dome by the interpenetration of four elliptical (and not circular) cylinders which, intersecting, form eight arcs of circumference.

Figure 6. The Dome of Santa Maria del Fiore

Starting from the second half of the 15th century[4], many artists and scholars resumed the study of ellipses, which they applied in their works. We mention, among the many: Piero della Francesca (1416-1492), Leonardo da Vinci, Sebastiano Serlio (1475-1554), Albrecht Dürer (1471-1528).

Serlio also built an elliptical compass. Leonardo was responsible for a graphic construction of the ellipse found in the Codex Atlanticus. Piero della Francesca drew an elliptical halo in *San Giuliano* (fresco detached from the

[4] We note that the first presence in Italy of the book *Conics* by Apollonius, in Greek, dates back to 1427, when the humanist Francesco Filelfo (1398-1481) brought a copy from Greece. In 1501, the Latin translations of some parts of the *Conics* appeared in the book *De expetendis et fugiendis rebus opus* by the humanist Giorgio Valla (1447-1500). Federico Commandino (1509- 1575) wrote the Latin translation of the first four books of the *Conics* in 1566. The first complete edition in Latin is from 1710; the translation was written by the astronomer Edmond Halley (1656-1742), the one after whom was named the comet, and it was made from the Arabic.

former church of Sant'Agostino in Sansepolcro, which is now in the Civic Museum of the same city), obtained, almost certainly, as an axonometric oblique projection of a circular halo.

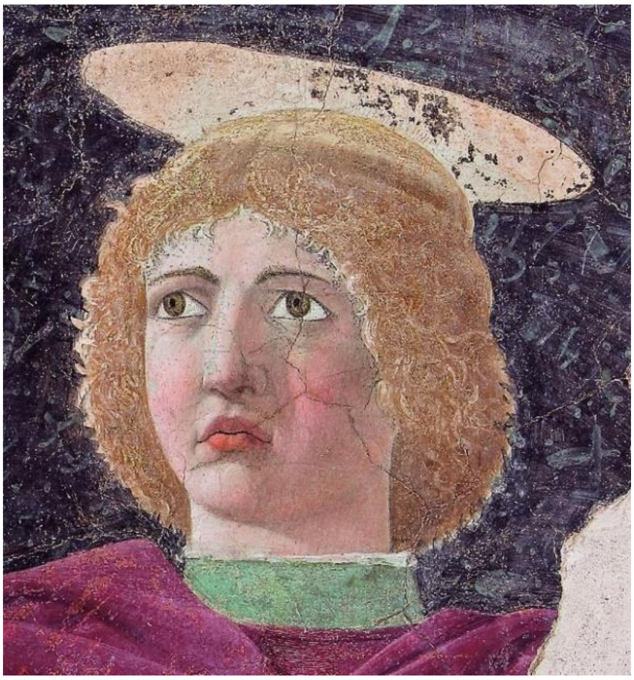

Figure 7. *San Giuliano*, Piero della Francesca[5]

In the tomb of Giuliano de' Medici, Duke of Nemours, located in the New Sacristy in the Medici Chapels in Florence, there are the sculptures made by Michelangelo Buonarroti (1475-1564) named *Day* and *Night*. We can see that the upper profile of the lid of the tomb, located under these figures, is a broken line (ending with two volutes) which is a part of an ellipse.

The red curve in Figure 8 is an ellipse drawn with the computer, starting from its equation: we can note the remarkable coincidence of the theoretical ellipse with the one used by Michelangelo.

As a first approximation, this ellipse has the same the axes) as the ellipses forming the arches of the Santa Trinita Bridge in Florence. Also in this case, we superposed the theoretical ellipse (in red) which, we repeat, is similar to

[5] The drawing in Figure 7 was taken from [21] pg. 253.

that of the Medici Chapels, on the curve of the arches of the bridge, noting an excellent coincidence. We emphasize the fact that the elliptical arch allows for large spans and, therefore, facilitates the flow of water under the bridge, without raising its keystone excessively.

Figure 8. The ellipse in Giuliano de' Medici's tomb[6]

Thus, it is likely that Bartolomeo Ammannati (1511-1592), the architect who designed the Santa Trinita Bridge, followed the suggestions of Michelangelo, with whom he was bound by a deep friendship. Both Enrico Felleni in 1957 and Piero Bargellini in 1964 (see [2] pg.56) stated that there was a remarkable similarity between the curve of the arches of the bridge and that of the covering of the sarcophagi found in the Medici Chapels. Actually, as we have already mentioned, the profiles of the arches of this bridge are very close to elliptical curves but not equal.

The bridge was rebuilt after its destruction by the German army in the Second World War (4 August 1944).

[6] The drawing in Figure 8 was taken from [1] pg. 186.

Figure 9. The ellipse of Santa Trinita Bridge

During its reconstruction (completed on March 16, 1958) a lively debate on the true shape of its arches broke out, involving engineers, architects, art historians and mathematicians[7], including prof. Luigi Campedelli.

3. Ovals

An oval (with four centers[8]) is a curve formed by four arcs of circumference constructed as follows.

Consider a segment AB and its midpoint O (*centre* of the oval). With reference to Figure 10, we take inside the segment AB two points L and M symmetrical with respect to point O and, on the axis of segment AB, two points E and F symmetrical with respect to point O. Consider, on the half-line FL, a point R, external to the segment FL, such that RL is congruent to AL. In the same way, we construct the segments EL and LP. With centre at point L and radius equal to the measurement of the length of AL we trace the arc of circumference PR. With a similar procedure, we construct the points Q and S and we trace the arc of circumference QS with centre at point M and radius equal

[7] We just want to point out that the difference between the elliptical curve and the one, with which the bridge was rebuilt, formed by two semi-chains reaches a maximum of 30 cm in the central arch (about 29 meters long); therefore a trifle compared to the size of the bridge which in all measures about 98 meters. Anyone interested in this question, which also involves compelling mathematical aspects, can refer to [5], [9], [16].

[8] There are also ovals with six (or bigger even numbers) centers, which will not be covered in this paper.

to the measurement of the length of MB (which is, by construction, equal to that of AL). Finally, with radius equal to the measurement of the length of EP, we trace the arcs of circumference PQ with center at point E, and RS, with center at point F. It follows from the construction that the arcs RP and PQ have the same tangent line in P, i.e., the line orthogonal to EP. For similar reasons, also the following arcs have the same tangent line in their intersection points: the arcs PR and RS in R, the arcs PQ and QS in Q and the arcs QS and RS in S.

We call the resulting curve an oval with *major axis* AB. Let C and D be the intersections of the line EF with the oval; we call the segment CD the *minor axis* of the oval.

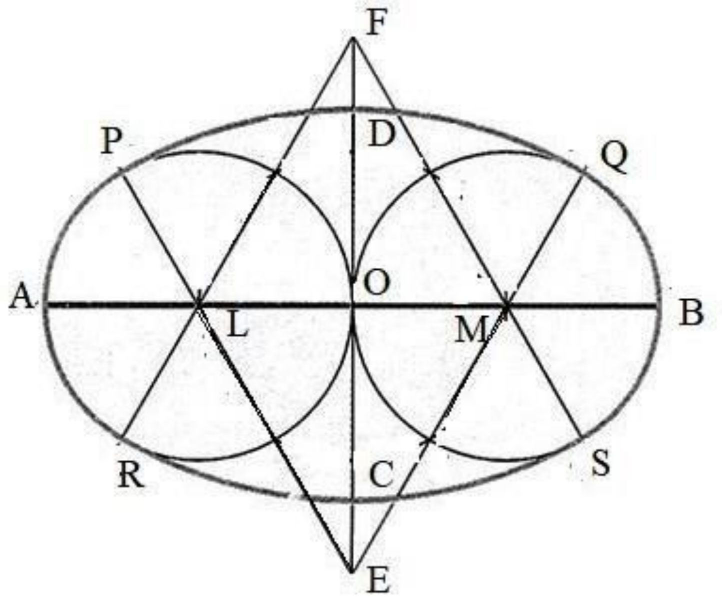

Figure 10. Geometric construction of an oval

Thus, ellipses and ovals are different curves from the point of view of their geometric constructions. We observe that, given two orthogonal segments intersecting at their midpoints, there is only ***one ellipse*** having those two segments as axes of symmetry, but there are ***infinite ovals*** having those two segments as their axes; their shape varies according to the position of the centers of the smaller circles on the major axis or of the centers of the larger circumferences on the minor axis line. We can easily see this fact from the construction of the ovals given by Abraham Bosse (1602-1676) in 1655.

Let us consider the rectangle having as base the semi-axis OB and as height the semi-axis OD of the oval to be built (see Figure 11)[9]; in this way the semi-

[9] The drawing in Figure 11 was taken from [12] pg. 17.

axes of the oval are fixed. We choose the radius *r* of the smallest circumference, then, we take a point M on OB and a point G on OD so that the length of the congruent segments MB and DG measures *r*. Let E be the intersection point between the axis of segment GM and the straight-line OD; let Q be the point of the half-line EM such that the segments ED and EQ are congruent. Then, by construction, also the segments GD and MQ are congruent and their length measures *r*. Now we can trace the arcs of circumference DQ, with center at point E and radius equal to the measurement of the length of EQ, and QB, with center at point M and radius equal to *r*. We observe that the arcs DQ and QB have the same tangent line at the point Q and, therefore, the arc QB is just a quarter of an oval with semi-axes OB and OD.

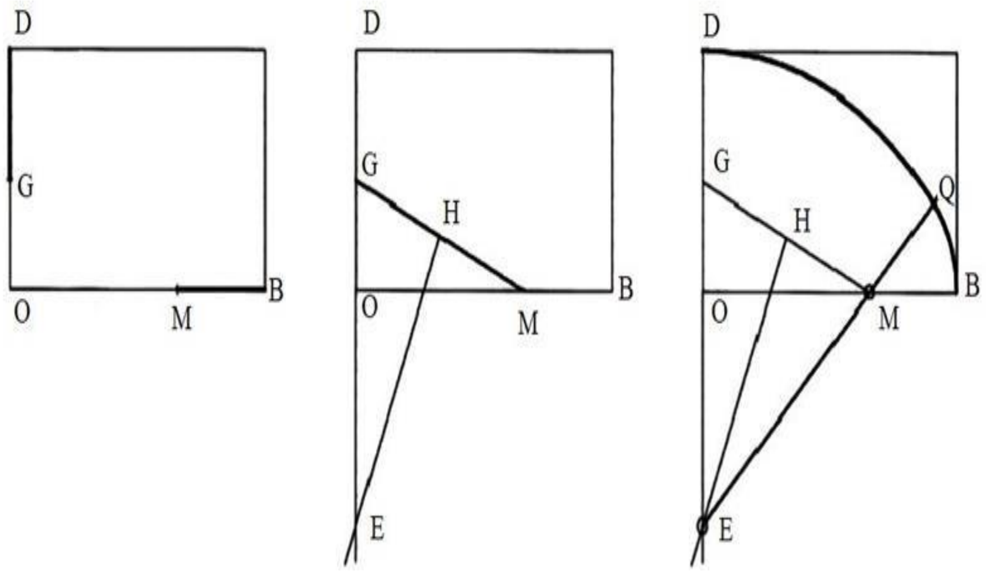

Figure 11. The construction of the ovals given by Abraham Bosse

From this construction we deduce that the ovals with given axes are ∞^1, since they depend on the position of the center of one of the two smaller circles. We can give the same proof also algebraically.

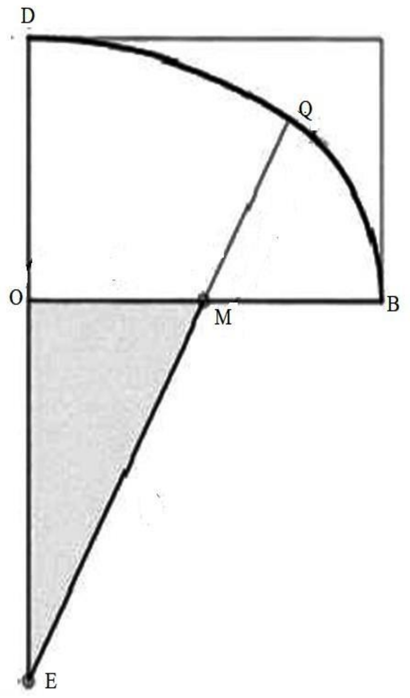

Figure 12. Relationship between the measures of the elements of an oval

With reference to Figure 12[10], let OB = l, OD = m, OM = a, MQ = MB = r, EM = x, EO = b, so that ED = $b + m$.
We have: $a + r = l$, $x + r = b+m$, $a^2 + b^2 = x^2$, from which we obtain

$$x = \frac{(l-r)^2 + (m-r)^2}{2(m-r)}$$

Thus, knowing the measurements of the semi-axes and the radius of the smallest circumference, we obtain the radius of the largest circumference.

We can quite easily distinguish some ovals from ellipses, while others are so similar to ellipses that one is unable to distinguish the two types of curves. Sometimes (but not always) the two curves can be recognized because in some ovals the curvature at the point of contact between the two arcs of circumference changes abruptly. For example, in Figure 13 we can see that the edge of the red region is an ellipse while that of the blue region is an oval.

[9] The drawing in Figure 11 was taken from [12] pg. 17.

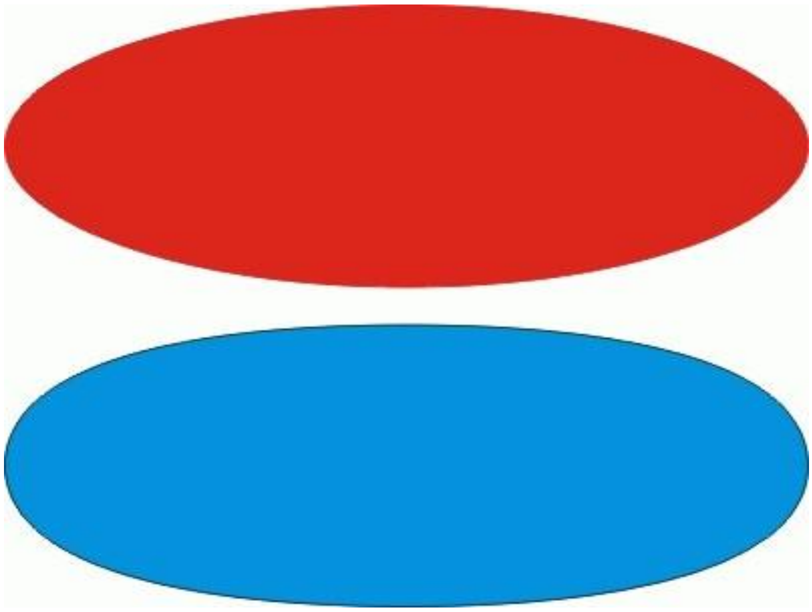

Figure 13. Ellipse (red) and oval (blue)

The ancient Egyptians also used the oval; in fact, as Choisy states, they used a three-center arc, which is half of a four-center oval, as an approximation of an elliptical arc (see [7], pp. 45-46).

It should be borne in mind that numerous studies have proved that, in the construction of amphitheaters, the ancient Romans used ovals instead of ellipses. The reason was essentially technological and not due to the difficulty of tracing these two types of curves (see [11] pp. 13-24).

Balbus Mensor[11] in his land-surveying treatise *Ad Celsum expositio et ratio omnium formarum*, written between 102 and 106 AD, states that the arena of the amphitheaters is formed by 4 circular arcs: "ex pluribus circulis forma sine angulo ut harenae ex quattuor circulis. " (see [14] pp. 181-182).

Now we show the reasons why ovals were preferred to ellipses in the construction of amphitheaters.

First of all, we show that a curve parallel[12] to an ellipse is an eighth-degree curve. In fact, the equation of the curve parallel to the ellipse with equation:

---

[11] Balbus was an engineer, surveyor and land-surveyor who lived in the time of Trajan. He followed the emperor during the military campaign in Dacia.

[12] Given a regular curve α, consider one of its points *P*. On the normal for *P* to the curve α, take a point *P*' having a fixed distance *d* from *P*. As *P* varies over α, the set of points *P*' (always taken from the same side with respect to the chosen direction of the normal) determine a curve β called

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ it is obtained by eliminating the parameters $x_0$and y_0 from the following eighth degree system, where *d* is the distance between the two parallel curves:

$$\begin{cases} y-y_0=\dfrac{a^2y_0}{b^2x_0}\,(x-x_0) \\ \dfrac{x_0^2}{a^2}+\dfrac{y_0^2}{b^2}=1 \\ (x-x_0)^2+(y-y_0)^2=d^2 \end{cases}$$

Now, let an ellipse with semi-axes *a* and *b,* with *a* > *b,* be given; the measure of the focal axis of this ellipse is $\sqrt{a^2-b^2}$. Let us now consider the ellipse, concentric to this, with semi-axes $a+d$ and $b+d$ with $d>0$; its focal axis measures $\sqrt{(a+d)^2-(b+d)^2}>\sqrt{a^2-b^2}$. Therefore, to make a series of concentric ellipses, it would be necessary to determine each time the (variable) position of the respective foci.

On the contrary, the ovals, being formed by arcs of circumferences, maintain the parallelism in the case they have the same centers. Note that the parallelism between curves is very important in the construction of amphitheaters, since the rows of steps must be at a constant distance.

In Figure 14 we have drawn two ellipses: the semi-axes of the inner one measure 4 and 2 and the semi-axes of the outer one measure 6 and 4. They look like two parallel curves but they are not! Moreover, the focal half-axis of the internal ellipse measures $\sqrt{12}$ while the focal half-axis of the external one measures $\sqrt{20}$. So, if we had to build many rows of steps using ellipses, we should draw a different ellipse for each row.

---

a curve parallel to the curve α; *d* is called the *distance* between the curves α and β. We note that the tangent line in *P* to the curve α is parallel to the tangent line in *P*' to the curve β. Furthermore, the distance between these parallel lines is just *d.*

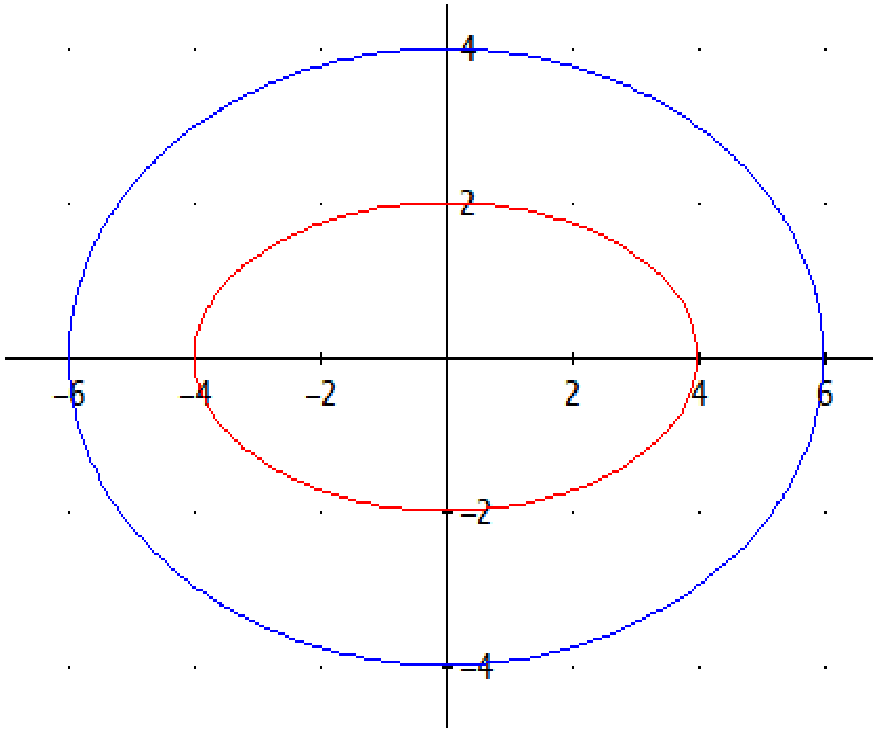

Figure 14. Non-parallel ellipses

For instance, ovals and not ellipses were used in the construction of the Colosseum The arches of these ovals belong to concentric circles; the four centers of all these ovals are the vertices of a rhombus formed by two equilateral triangles (see [3] pg. 106). In the following Figures 15, 16 and 17 we show, by way of example, three of these ovals: we can see that they all have the same centers. The axes of the outermost oval of the Colosseum measure 188 and 156 meters, the axes of the inner arena measure 88 and 54 meters.

We will see later that these ovals are built according to Serlio's first rule; thanks to this construction, the rows determined by the steps are all parallel curves; in fact, two concentric circles are parallel curves whose distance is equal to the absolute value of the difference between their radii.

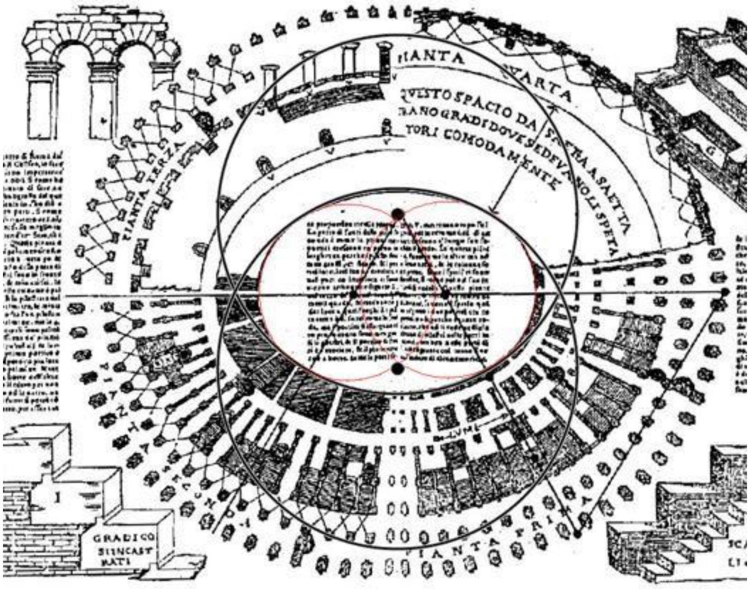

Figure 15. The oval of the Colosseum arena[13]

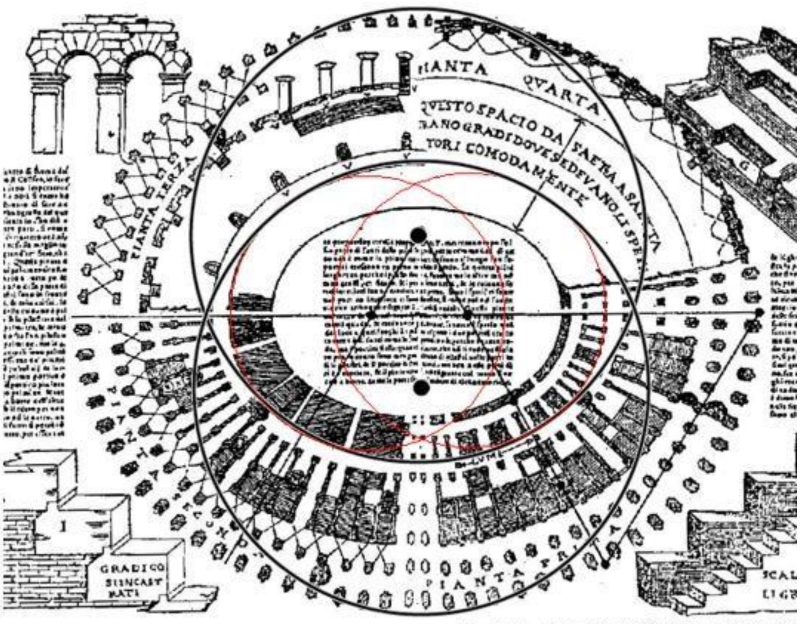

Figure 16. An intermediate oval of the Colosseum

[13] The drawings in Figures 15, 16, 17 and 18 was taken from [3] pg.105.

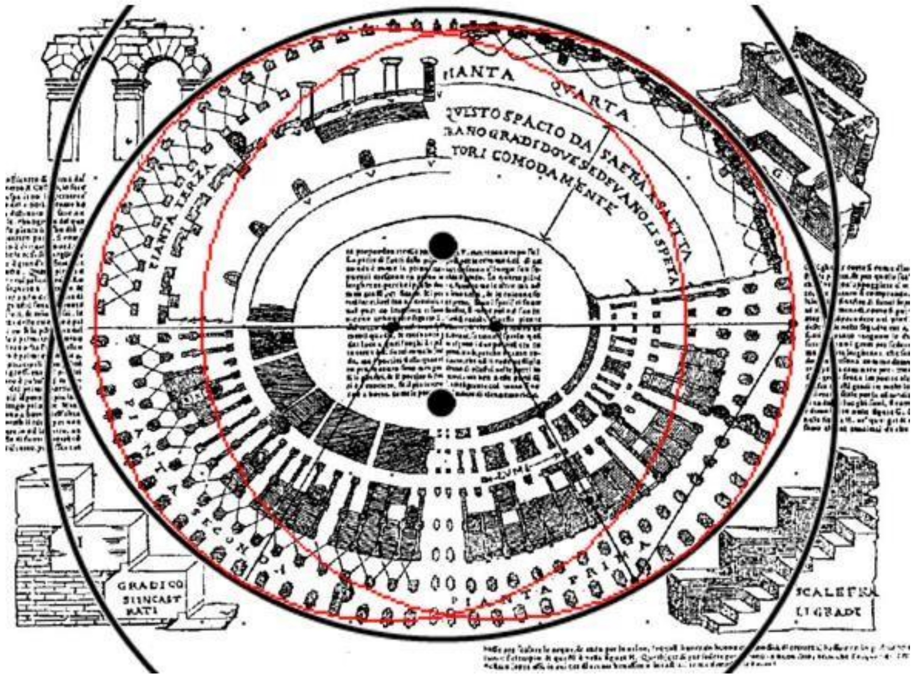

Figure 17. The external oval of the Colosseum

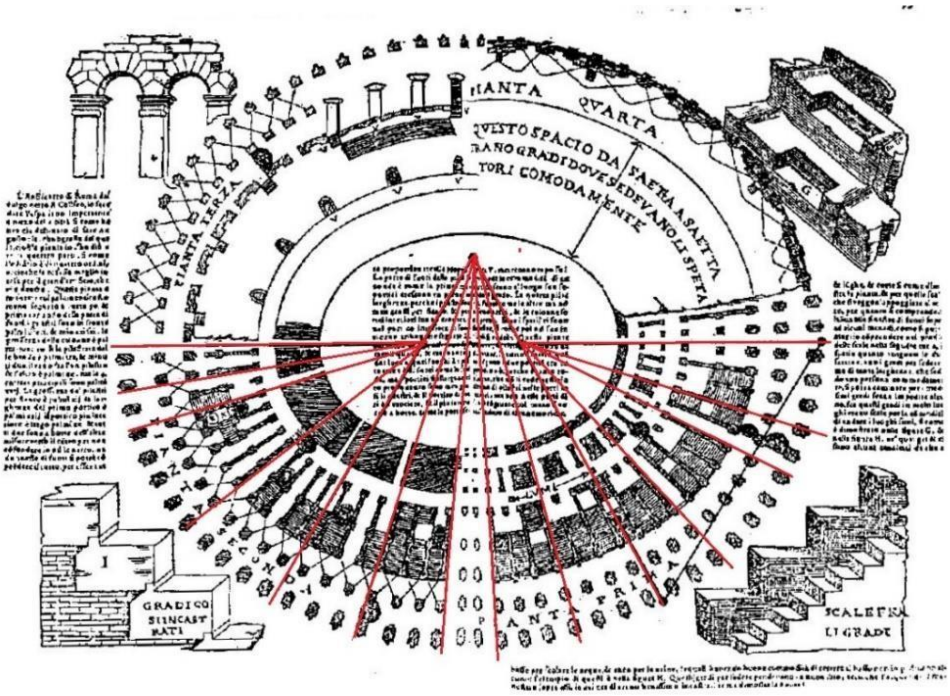

Figure 18. Directions of architectural elements towards the centers of the Colosseum ovals

It is interesting to note that the presence of the oval allows the architectural elements to converge towards the centers of the arcs of circumference that form the oval, as it can be seen from the Figure 18; we also observe that, once again, this is a property that does not apply to ellipses.

The same considerations can be made, for example, for the Roman amphitheaters in Pola and Verona (see [3] pp. 104-107) and for the roofs of some modern soccer stadiums (see Figure 19).

.

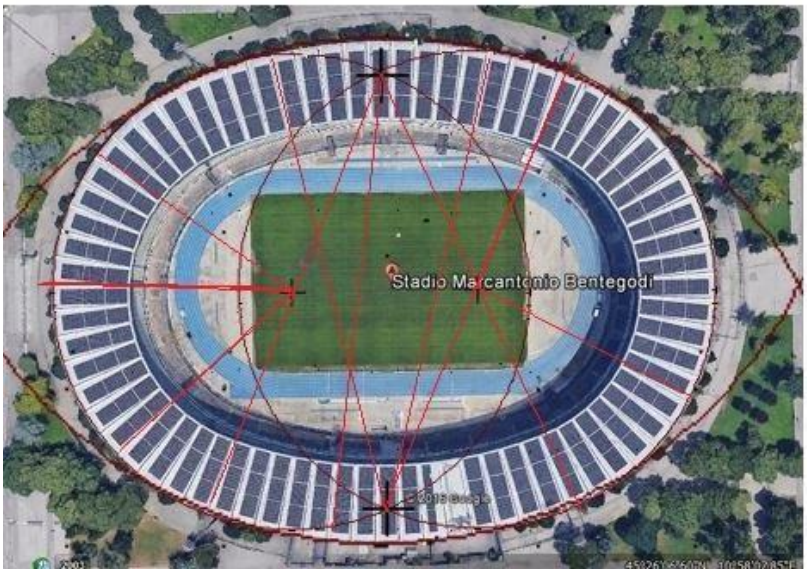

Figure 19. The Bentegodi stadium in Verona

Consider now the wall ashlars that form the arches. In the elliptical arches

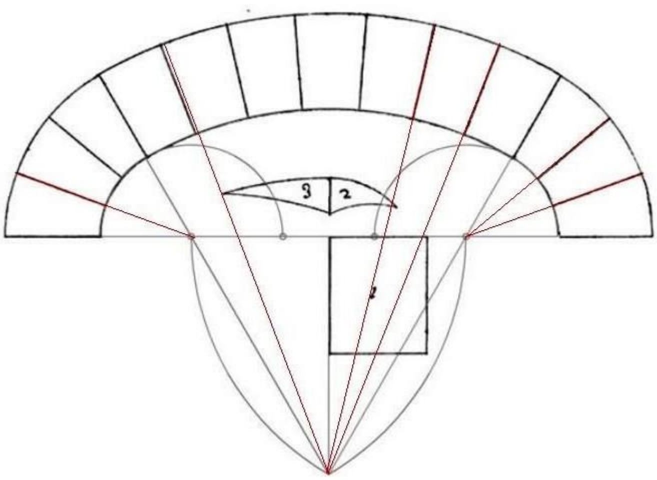

Figure 20. Wall ashlars normal to the profiles of the oval arches

the ashlars have different shapes while in the oval arches with three centers the ashlars have only two different forms. Furthermore, in this case, the lines normal to the circular profiles determined by the oval arch converge to these centers.

Thus, these lines identify the directions of the wall ashlars, as it can be seen from the above Figure 20, elaborated by the authors on the basis of a drawing taken from the text on stereotomy by Joseph Gelabert: *De l'art de Picapedrer* of 1653. This fact makes the arrangement of the aforesaid masonry elements easier.

Starting from the sixteenth century, many artists in their works used ovals, especially those with four centers; among these we mention: Leonardo da Vinci, Baldassare Peruzzi (1481-1536), Michelangelo Buonarroti, Sebastiano Serlio, Andrea Palladio (1508-1580), Jacopo Barozzi known as Vignola (1507-1573).

The *Hostinato rigor* emblem bears Leonardo da Vinci's favorite motto; it is kept in the Royal Library, Windsor Castle, in Windsor. He drew it, along with other emblems, between 1506 and 1510, during the second Milanese period (see Figure 21)[14]. The drawing on the left shows that the edge of this emblem is not an ellipse, while the one on the right shows that it is enclosed by an oval.

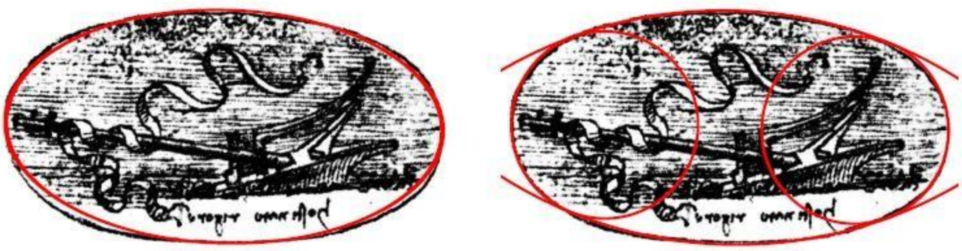

Figure 21. Emblem *Hostinato rigore* by Leonardo da Vinci

Let us consider the steps of the staircase of the Vestibule of the Laurentian Library, designed by Michelangelo; actually, this staircase was built by Bartolomeo Ammannati[15] starting in 1559, after the Master sent a clay model. In fact, starting from 1524, Michelangelo made some drawings and began the construction of the Laurentian Library, commissioned by Pope Clement VII. We know that in the autumn of 1534 Michelangelo left Florence and he never returned (see [13], pp. 259 and following). Afterwards, due to the insistence of Cosimo I de' Medici, he sent from Rome drawings and instructions for the construction of the library. The works were carried out by Vasari, by

[14] The drawing in Figure 21 was taken from [20].

[15] We already said, studying the Ponte Santa Trinita, that Ammannati and Michelangelo were friends.

Ammannati and by Tribolo, who scrupulously followed Michelangelo's instructions.

Looking at the steps, it seems quite evident that their profiles are ovals and not ellipses; this can also be seen from the abrupt change of their curvature towards the vertices of the major axis (a typical configuration of ovals, though not easily observable in all).

Figure 22. Oval steps of the staircase of the Vestibule of the Laurentian Library[16]

[16] The drawing in Figure 22 was taken from [1] pg. 259.
[17] Florence, Casa Buonarroti, Drawings by Michelangelo, Foglio 92 Ar [Corpus Tolnay 525r] and Foglio 92 Aw, [Corpus Tolnay 525v]. See also [6], pp. 53-67.

This fact is also visible in the drawings made by Michelangelo and kept in Casa Buonarroti[17]. In fact, Michelangelo knew the ovals; he also used these curves in the design of the Piazza del Campidoglio in Rome, whose profile fits perfectly into an oval with four centers.

Antonio Munoz in 1940 created the white oval with the internal white design of the pavement in Piazza del Campidoglio; Munoz based his project on an engraving (*ex Michaelis Angeli Bonaroti architectura*) made by Bartolomeo Faleti in 1567, according to the design of Michelangelo (see [17] pp. 73, 74 and [18]). As we can see from the following Figure 23, the profile of this square is not an ellipse.

Figure 23. The outline of Piazza del Campidoglio is not an ellipse

Instead, this profile is an oval that fits perfectly, as we can see in Figure 24, to the oval of Serlio drawn according to the fourth rule; moreover, we will highlight that this oval is of the same type as the one with which St. Peter's Square in Rome is drawn.

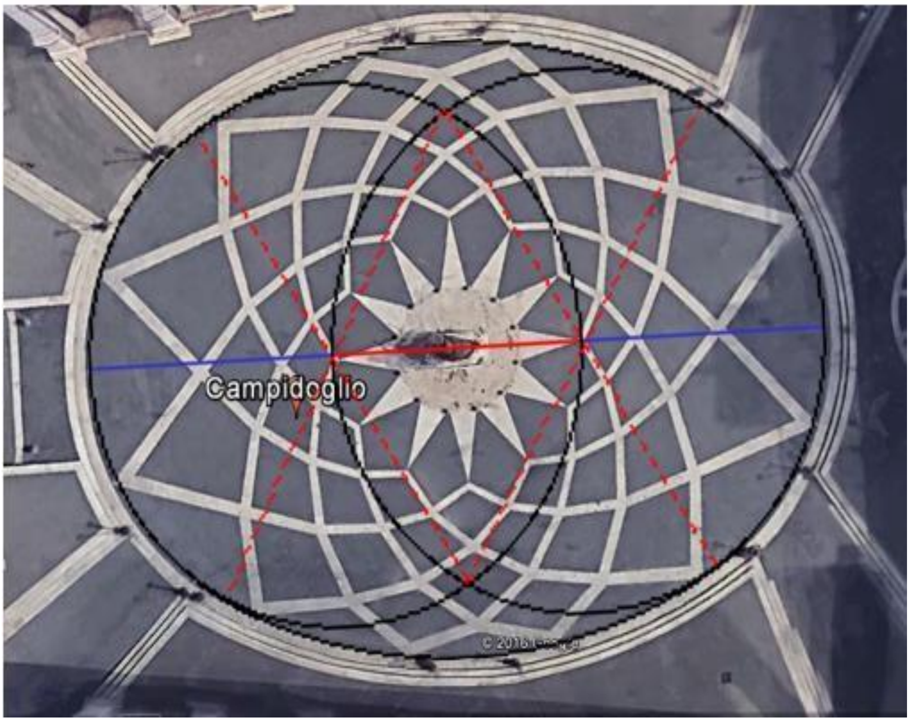

Figure 24. Michelangelo's oval in Piazza del Campidoglio

Sebastiano Serlio in the *First Book* of the 1545 *Treaty of Architecture* (see [19]) drew four types of ovals.

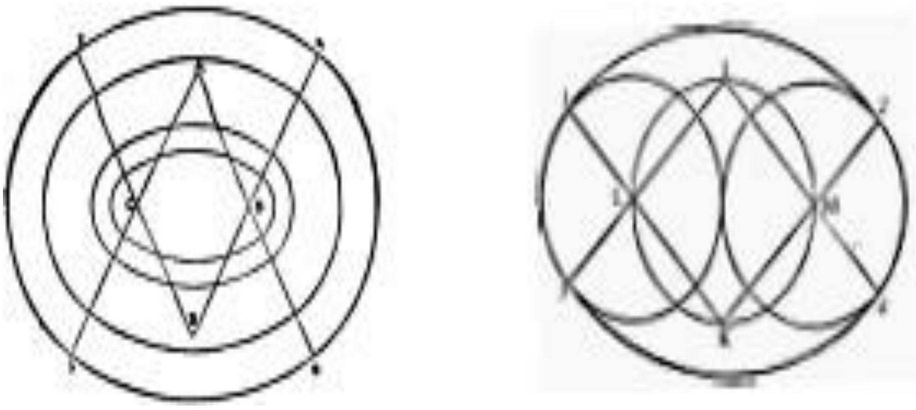

Figure 25. Serlio's ovals drawn according to the first and second rule respectively

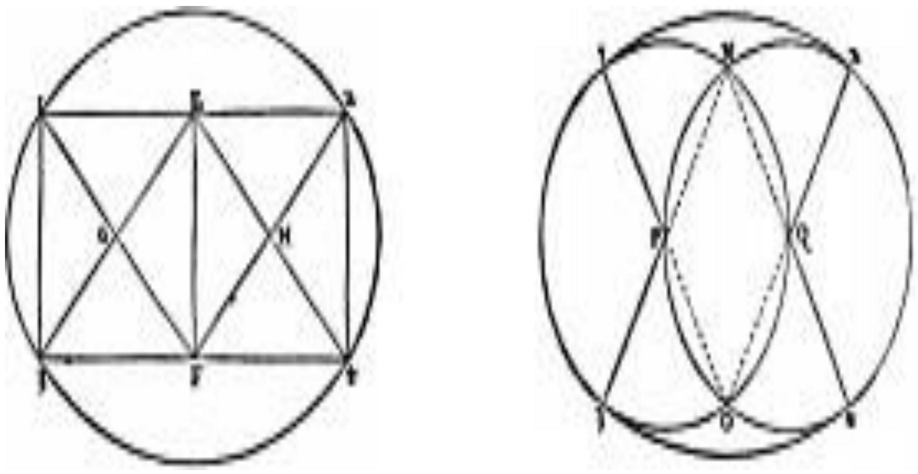

Figure 26. Serlio's ovals drawn according to the third and fourth rules respectively[18]

We note that the equilateral triangle is present in the first and fourth oval, while the square is present in the second and third; these figures were widely used in Renaissance art[19].

Giovanni Keplero, before choosing ellipses to describe the motion of the planets, tried to correct the discrepancies between the circular orbit, hypothesized up to then, and the data obtained from the observations of the motion of the planets, assuming that the orbits were ovals. Probably the reason for the choice of ovals was that these curves are formed by arcs of circumference, which was considered the perfect curve. With this hypothesis the situation improved but the data were still conflicting; only by adopting the elliptic curves there was a perfect coincidence with the astronomical observations (see [8]).

In the Baroque period, numerous artists used the ovals as, for example, Gian Lorenzo Bernini (1598-1680) and Francesco Borromini (1599-1667). In fact, many scholars hypothesize that even the elliptical structures of the Baroque are actually ovals; however, the question is still much debated (see, for example, [8]).

[18] The drawings in Figures 25 and 26 was taken from [3] pg.102.
[19] In his treatise Serlio states that there are many ways to draw an oval but he would have given the rules for only four of these.

Let's consider now St. Peter's Square in Rome: we can say with certainty that its profile is an oval built according to the fourth rule of Serlio, as it can be seen from the geometric construction shown in Figure 27.

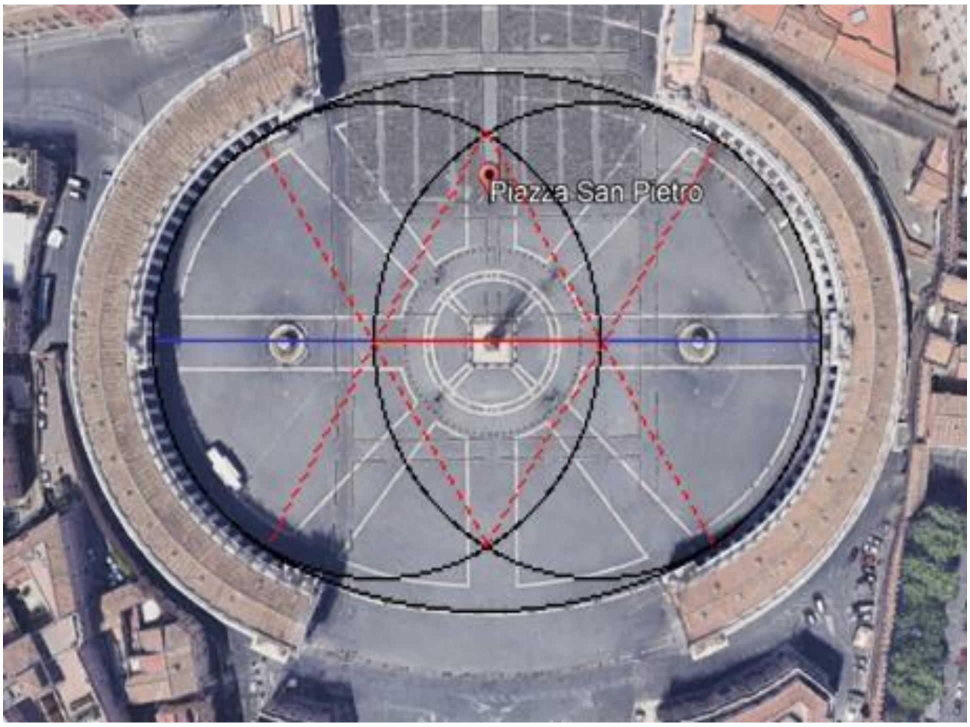

Figure 27. The oval of St. Peter's Square

Bernini himself makes clear which curve he used to draw the square: he inserts two stones in the pavement of the square with the inscription *"CENTRO DEL COLONNATO"* as we can see from Figure 28.

Indeed, these stones are located exactly in the centers of the two smaller circumferences that form the oval with four centers, which determines the internal profile of the square. Exactly from these points, one can see aligned the columns of the colonnades that surround the square.

If we approximated this oval with an ellipse, this would have the focuses moved from the aforesaid centers of the oval by about 25 meters towards the colonnades. Thus, it is completely wrong to state, as many says, that from the foci of the ellipse (assuming it is an ellipse) of the square one can see the aligned

Figure 28. Inscription of Bernini in St. Peter's Square

columns; in addition, it should be borne in mind that the foci of an ellipse do not satisfy this property.

Approximating the internal profile of the square with an ellipse, its foci would be in the fountain centers, about 59 meters from the center of the square.By the way, note that the axes of this oval measure about 196 and 156 meters, almost the same as those of the Colosseum.

Approximating the external profile of the square with an ellipse, the axes of which measure 240 and 180 meters, this would have the foci displaced by about 20 meters towards the two colonnades with respect to the foci of the internal ellipse, so Bernini should have used two ellipses with four different foci to determine the internal and external profile. Furthermore, as we have already said, the two ellipses would not have been two parallel curves. Instead, the oval that determines the external profile of the square has the same centers as the one that corresponds to the internal profile. This fact, as we have already observed, makes it easier to trace the curves that determine the structure, even if the drawing of an ellipse is quite simple to perform: just use the elliptical compass or the so-called "gardener's construction of an ellipse".

4. Conclusion

We think we have given an idea of the difference between ellipses and ovals, both. Regarding their geometric genesis and their properties that make these figures useful in applications. However, it is not always easy to distinguish these curves; sometimes, as we have seen, it is from their use that we can determine which curve it is, as we have highlighted in the case of the oval in St. Peter's Square or the ovals with which the Roman amphitheaters were traced.

Bibliography

[1] U. BARDINI, B. NARDINI (a cura di). *San Lorenzo, la Basilica, la Sagrestia, le Cappelle, la Ciblioteca.* Firenze: Nardini Editore. 1984.

[2] P. BARGELLINI. *I ponti di Firenze*. Firenze: Edizioni dell'Istituto Professionale "Leonardo da Vinci". 1964

[3] M. T. BARTOLI. *Le Ragioni Geometriche del Disegno Architettonico.* Firenze: Alonea Editrice. 1997

[4] R. BECHMANN. *Le radici delle Cattedrali.* Milano: Arnoldo Mondadori Editore. 1989.

[5] A. BELLUZZI, G. BELLI. *Il ponte a Santa Trinita.* Firenze: Edizioni Polistampa. 2003.

[6] S. CATITTI. *Michelangelo e il disegno architettonico come strumento progettuale ed esecutivo: il caso della Biblioteca Laurenziana.* Atti del Convegno "Michelangelo e il linguaggio dei disegni di Architettura", Firenze 30 gennaio - 1 febbraio 2009 (a cura di G. Maurer e A. Nova), Collana del Kunsthistorisches Institute in Florenz, nr.16-17. 2009.

[7] A. CHOISY. *L'art de bâtir chez les égyptiens.* Paris: É. Rouveyre. 1904.

[8] F. COLONNESE. *Kepler, Galileo, Bernini e Gaspari. Note sulla controversa associazione tra Ellisse e Barocco.* Proceedings of European Network for Baroque Cultural Heritage General Conference (Roma, 27-29 marzo 2014). Parigi: Open Edition. 2014.

[9] G. CONTI, G. LITTERA, S. MARRAGHINI. *Costruzioni geometriche in alcuni monumenti fiorentini del cinquecento.* Bollettino Studi Storici fiorentini, 28-29. 2019-2020. pp. 94-102.

[10] R. CORAZZI, G. CONTI. *Il segreto della Cupola del Brunelleschi a Firenze.* Firenze: Pontecorboli Editore. 2011.

[11] M. DOCCI, R. MIGLIARI. *Architettura e geometria nel Colosseo di Roma*, in *Matematica e Architettura, Metodi analitici, metodi geometrici e rappresentazione in Architettura* (a cura di O. Arena). Firenze: Alinea Editrice. 2001. pp.13-24.

[12] E. DOTTO. *Il disegno degli ovali armonici*. Catania: Le Nove Muse Editrice. 2002.

[13] A. FORCELLINO. *Michelangelo. Una vita inquieta.* Roma-Bari: Editori Laterza. 2005.

[14] C. INGLESE , A. PIZZO, *L'anfiteatro di Augusta Emerita: dallo studio formale, geometrico e proporzionale al progetto.* In Carlo Bianchini, La documentazione dei teatri antichi del Mediterraneo. Le attività del progetto Athena a Merida. Roma: Cangemi Editore. 2013. pp. 181-188.

[15] M. KLINE. *Storia del pensiero matematico.* Vol. I. Torino: Giulio Einaudi Editore. 1991.

[16] P. PAOLETTI. *Il ponte a Santa Trinita. Com'era e dov'era.* Firenze: Becocci Editore. 1987.

[17] G. M. PILO. *Come Michelangelo concepì la sistemazione della piazza del Campidoglio.* Arte Documento, vol. 9. 1995.

[18] A. M. RACHELI. *Restauro a Roma. 1870-1990 architettura in città.* Venezia: Marsilio. 2000.

[19] P. L. ROSIN. *On Serlio's construction of ovals.* The Mathematical Intelligencer, 23. 2001. pp. 58-69.

[20] P. L. ROSIN. *On the construction of ovals*. www.researchgate.net/publication/2904488

[21] V. VALERIO. *La Forma dell'ellisse.* Arte e matematica. Un sorprendente binomio (a cura di E. Ambrisi, L. Basile, L. D'Apuzzo, A. Pellegrini). Istituto Italiano per gli Studi Filosofici. Napoli: Arte Tipografica Editrice. 2006. pp. 241-262.

[22] E. VIOLLET-LE- DUC. *L'Architettura ragionata.* Milano: Jaca Book. 1982.