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Author: Kolmakov, Evgeny A.

Title: On Shavrukov's non-isomorphism theorem for diagonalizable algebras.

MR Number: MR4722267

Primary classification:

Secondary classification(s):

Review text:

This very rich paper presents several improvements in the theory of *diagonalizable algebras* (also known as Magari algebras after [Magari, R., Boll. Un. Mat. Ital., 4(1975); MR0460109]), which feature a modal operator \Box_T representing a provability predicate of a formal theory T extending elementary arithmetic. The paper is centered on some results by V. Yu. Shavrukov on the existence of isomorphisms between diagonalizable algebras of (Σ_1 -sound) formal theories. In particular, Shavrukov proved a theorem (see, e.g., Theor. 2.11 of [J. Symb. Log., 62 (1976); MR1450515]) identifying a sufficient condition for the diagonalizable algebras of two theories to be not elementarily equivalent, hence not isomorphic. This result was later refined by G. Adamsson in [Diagonalizable algebras and the length of proofs, Master's thesis, University of Göteborg], who proved in particular that the diagonalizable algebra \mathfrak{D}_T of T is not isomorphic to the one whose modal operator is $\Box_T^{(6)}$, i.e. the sixth iteration of the operator \Box_T of \mathfrak{D}_T .

Kolmakov's main result (Theor. 4.24) further improves Adamsson's theorem by identifying a non-isomorphisms condition implying that \mathfrak{D}_T is already not isomorphic to the algebra having $\Box_T^{(2)}$, i.e. $\Box_T\Box_T$, as its operator. This theorem exploits the properties of a class of speed-up functions, the Σ_1 -*reflection function* of a theory, which is investigated at length in the paper, along with other classes of functions which generalize the class of functions that are provably total in a theory T and are implicitly referred to by Adamsson. The relations between these classes of functions are also illustrated in details.

Among the many results connected with the paper's main theorem that the author discusses, Theor. 5.33 identifies a proof-theoretic condition for the non-isomorphism result to apply. It consists in T proving the local Σ_1 -reflection principle of S , which improves L. Beklemishev's observation from [Russian Math. Surveys, 60(2005); MR2152943] about Shavrukov's condition following from the provability in T of the *uniform* Σ_1 -reflection principle of S . In addition, Theor. 5.36 identifies a family of theories that are Π_1 -equivalent provably in elementary arithmetic, but have non-isomorphic algebras.

Finally, hinging upon work done by L. Beklemishev since [Lecture Notes in Pure and Appl. Math., 180(1996);MR1404946], the author considers diagonalizable algebras of theories with a signature extended by a second modality and finds examples of pairs of theories having isomorphic diagonalizable algebras, but non-isomorphic algebras of this bi-modal kind.