THE MOVING ARROWS ENVIRONMENT: A DIGITAL ARTIFACT FOR MEDIATING THE MEANINGS OF VARIABLE AND UNKNOWN

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This study, part of a greater research project, describes a digital interactive artifact designed for mediating the mathematical meanings of variable, constant, functional dependency, structure and value of a linear literal expression, and relationship between two expressions depending on the same variable. We present the a-priori analysis of the semiotic potential of this artifact and the a-posetriori analysis of its actual unfolding in the context of a pre-experimentation with low achieving students (aged 15-16). In particular, we focus on the signs produced by these students during the accomplishment of exploratory tasks involving the artifact.

Keywords: variable, linear literal expression, semiotic potential, low achievement in mathematics

INTRODUCTION

Wide research has documented students' difficulties in the domain of school algebra, that include giving meaning to algebraic symbols, unknowns, variables, equations and inequalities (e.g., Arcavi et al., 2017; Kieran, 1992). In particular, algebra can be extremely challenging for low achieving students (Xin et al., 2022). Yet, studies investigating the implementation of appropriately designed digital artifacts in the teaching-learning of algebra suggest that they can be helpful for students with a history of low achievement in mathematics (e.g., Baccaglini-Frank, 2021; Mariotti & Cerulli, 2001). We engage in this line of research, since further studies are still needed, to shed some light on what aspects of such artifacts seem to help students the most, in order to guide the design of appropriate instructional interventions. This study is part of a greater funded research project (DynaMat) that, through a design-based methodology, is conducting case studies of second year high school students with a history of low achievement in mathematics. These students, volunteering from different Italian high schools, participate to an intervention conducted by researchers during which they engage in a set of newly designed digital activities in the context of algebra. In this study we describe a specific digital interactive artifact designed for mediating the mathematical meanings of variable, constant, functional dependency, structure and value of a linear literal expression, relationship between two expressions depending on the same variable; and we explore its semiotic and didactic potential.

THEORETICAL FRAMEWORK

We adopt the lens provided by the Theory of Semiotic Mediation (TSM) (Bartolini Bussi & Mariotti, 2008) in order to explore the affordances of a specific digital artifact. The TSM finds its roots in the socio-cultural frame for learning developed by Vygotsky, according to which signs play a crucial role for learning. According to Vygotsky (1978) signs (symbols, texts, formulae, graphics, tables, and most fundamentally, language) are *psychological tools*, that mediate the development of cognitive processes such as thinking, concepts formation, voluntary attention, and so on, in ways appropriate to our cultures. Within such a general frame, the TSM conceives teaching-learning as a semiotic process which originates from the activity with an artifact and develops through a continuous interplay of signs. Three main categories of signs can be distinguished in this process: *artifact signs*, which refer to the context of the use of the artifact, *mathematical signs*, which refer to mathematical context, and *pivot signs*, which act as bridges between the artifact signs and the mathematical signs. The potential of an artifact to trigger such a complex process rests on its so-called *semiotic potential*: the use of an artifact for accomplishing a task can foster the development of personal signs which, from the expert's point of view, can be put in relation to and evolve towards specific mathematical signs. It is a double semiotic link that can be established by an expert, as a mathematics teacher, between the artifact and the task, and between the artifact and a piece of mathematical knowledge, as culturally and historically established; this double semiotic link is the semiotic potential of the artifact with respect to an activity and a specific mathematical knowledge. As emphasized by Bartolini Bussi and Mariotti (2008), the semiotic potential of an artifact is not naturally activated and exploited but the teacher must take advantage of it to foster the transition of signs produced by the students towards the mathematical signs which are objectives of the didactical intervention.

The research objective of this paper is to study the semiotic potential of the "Moving Arrows Environment" (MAE), a digital artifact that we designed for mediating specific mathematical knowledge in the context of school algebra. We would contribute to the stream of research about the implementation of digital resources, which other studies has shown to be promising for algebra and calculus teaching-learning processes (e.g., Falcade et al., 2007). More specifically, we analyze the semiotic potential of the artifact MAE and its actual unfolding in the context of a pre-experimentation conducted with pairs of students in a laboratory setting. In order to do so, we focus on the artifacts signs produced by students during the accomplishment of exploratory tasks involving the artifact.

THE ARTIFACT DESIGNED FOR THIS STUDY

We designed a GeoGebra applet that we call MAE (Figure 1). The applet presents a red arrow, labeled *a*, on a horizontal oriented number line; the origin of the arrow is the point 0, the endpoint is marked with a red tick mark and is directly draggable bound to the oriented number line. Then, there are two black arrows labelled *a* and *-a* whose endpoints move indirectly, one opposite to the other, as the endpoint of the red arrow is dragged, maintaining the same length of the red arrow, and two other black arrows representing *1* and *-1* that are fixed.

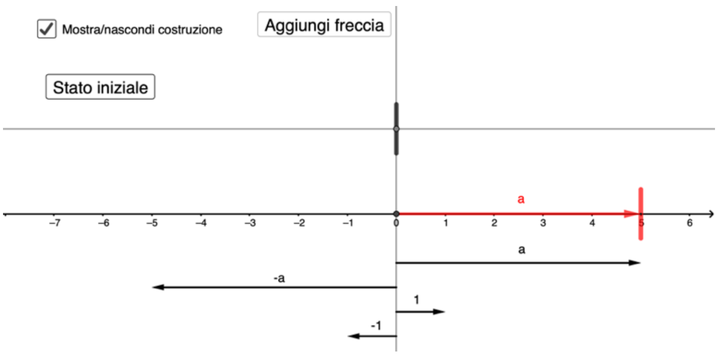

Figure 1. Screenshot of the moving arrows environment

New arrows can be constructed on a horizontal non-numbered line through the "Add arrow" button (in Italian "Aggiungi freccia"). This button allows the user to create different consecutive copies of the black arrows (that is arrows with the same length and same orientation); the origin of the first copied arrow is at the intersection of the horizontal line with a vertical line passing through the origin of the oriented number line, the origin of each other arrow is the endpoint of the previous one (Figure 2). The endpoint of the last copied arrow is marked with a black tick mark. The endpoint of the copied arrows cannot be directly dragged but moves accordingly with the dragging of the red arrow. The user can choose to visualize either the entire construction, that is, all the arrows copied on the horizontal non-numbered line, or the black tick mark alone, that is, only the endpoint of the last arrow, by clicking on "Show/hide construction" (in Italian "Mostra/nascondi costruzione"). The

“Reset” button (in Italian “Stato iniziale”) restores the initial configuration by deleting any construction made. Figure 2 shows the result of the construction of two consecutive copies of the black arrow *a* and a copy of the black arrow *1*. In Figure 2a the red tick mark is on number 3, in Figure 2b it is on -2. In this latter case, the last copied arrow (a copy of *1*) overlaps a copy of the arrow *a*; however, the black tick mark still denotes the endpoint of the last arrow.

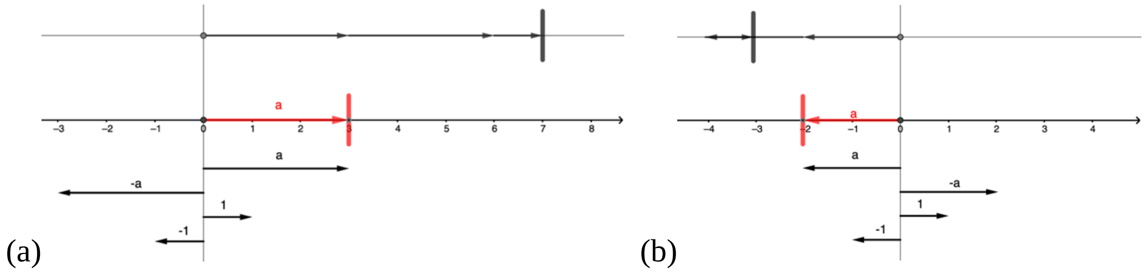

Figure 2. Two screenshots of the moving arrows environment in which two consecutive copies of *a* and a copy of *1* have been constructed, and the red tick mark is on the number 3 (a) and -2 (b)

Moreover, we created another version of the same applet, with two horizontal lines parallel to the line containing the red arrow, so that two expressions dependent on the same variable *a* can be represented simultaneously. In this version, there are two “Add arrow” buttons that allow the user to choose on which line to create a copy of the selected arrow. The endpoints of the last copied arrow on the two lines are marked respectively with a black and a blue tick mark. As in the previous version, the constructed sequence of arrows moves accordingly how the endpoint of the red arrow is dragged.

Analysis of the semiotic potential of the artifact

The MAE artifact is designed to mediate the mathematical signs of variable, functional dependence between variables, value and structure of a linear literal expression, and reciprocal relations between two expressions depending on the same variable. In this section we analyze the semiotic potential of the artifact with respect to particular tasks, seeking to highlight the semiotic link that can be established firstly between the artifact and the mathematical knowledge, and secondly between the artifact and a task. Specifically, in Table 1 we report the representations provided within the MAE, the possible actions on them or feedback of them, and in the third column we relate these to the mathematical knowledge they might be exploited to mediate.

In GeoGebra, it is possible to distinguish two types of motion: direct, when the user acts directly on a base object, as the red tick mark in our applet; and indirect if the observed movement is obtained as a consequence of dragging another object (Falcade et al., 2007). We exploited this functionality for designing the MAE, where the dependence relation can be experienced, thanks to the use of dragging, in terms of these two different types of motion. Moreover, a characterizing feature of our artifact is that it provides a twofold representation of algebraic expressions. In functional terms, the tick marks represent two variables, one dependent and the other independent, and their movement represents the functional relationship between them. Moreover, the construction of the arrows obtained with the “Add arrow” button provides a representation of the structure of the expression. For instance 2*a*+1 can be obtained adding two copies of *a* and a copy of *1*. The order in which the user copies the black arrows determines the structure of the expression, but it does not affect its functional aspect.

Representations in the MAE	**Related possible actions / feedback**	**Related mathematical signs**

Red arrow, red tick mark	Can be directly dragged on the oriented number line	Independent variable
	Can be dragged to specific numerical values or positions on the number line	Assignment of a value to a variable
Black arrows *a* and -*a*, and their copies	Change length as an effect of the dragging of the red tick mark	Dependent variable
Black arrows *1* and -*1*, and their copies	Fixed constant length	Constant numerical value
Sequence of arrows on the non-numbered line and its corresponding black tick marks	Can be constructed through the "Add arrow" button	Construction of a linear literal expression/Definition of a linear function in one variable
	Change length as an effect of the dragging of the endpoint of the red arrow	Dependent variable
		Evaluation of an expression or function for specific instatiation of the independent variable
	Show/hide construction	Consider the explicit analytical definition of a function / consider function as a set of ordered pairs
Red and black arrows, red and black tick marks	Simultaneous change of the length of the arrows / deplacement of the tick marks as an effect of the dragging of the endpoint of the red arrow	Functional relationship between variables
Two sequences of arrows on two horizontal lines and their corresponding blue and black tick marks	Can be both constructed through the "Add arrow" button on the two lines	Construction of two linear literal expressions/Definition of two linear function in one variable
	Simultaneous deplacement of the blue and black tick marks, as an effect of the dragging of the red tick mark	Dependency of both the two expressions/functions on the same variable
	Reciprocal position of the blue and black tick marks, at a given position of the red tick mark	Equal-/greater than-/less than-, relationship between the values of the two expressions/functions for a given value of the variable
		Values of the unknown that verify the equality/inequality of the two expressions

Table 1. Overview of the semiotic link between the artifact and the mathematical knowledge

METHODOLOGY

We conducted a pre-experimentation in October and November 2022 that took place in extra-school hours, in a learning center outside of school. There were involved 12 students (aged 15-16) from different schools, volunteering, with a history of low achievement in mathematics. They worked in pairs, under the guidance of a researcher, for five sessions of 2 hours each. They had at disposal two touch-screen tablets: one to interact with artifacts and one for writing their own notes. Data collected consist of video-recordings of the sessions, tablets' screen-recordings, and students' writings.

The activities we designed within the MAE consist of two tasks, simultaneously given to students, written on a single page on the screen of a tablet. In the first task one construction (or two, in the case of the applet in the second version) is required, with the "Add arrow" button, representing a certain expression in the variable *a*, given in words. For example: *We want to construct the arrow representing the double of a, minus 1. To do this, use the "Add arrow" button.* In the second task, a description of the observed movement is required, after dragging the red tick mark: *How does this new arrow move as a varies?* Moreover, during the activities the researcher guided students' explorations through some questions, orally posed, asking for predictions, comments or synthesis. Preliminarily, the first time that students were introduced to the MAE, the researcher asked them to explore it by dragging and to describe what happens, before doing any construction.

In this paper, we focus on the preliminary request (episode 1) and on two activities (episodes 2 and 3). Both these activities consist of the two mentioned tasks but we focus on the second one, which we expected may support a rich semiotic production by students. In particular, the task is designed to foster the production of signs, related to the activity with the artifact, that have potentials consistent with the embedded mathematical knowledge and the mathematical signs that an expert can use to express that knowledge. In our a-posteriori analysis, to gain insights onto the actual unfolding of the hypothesized semiotic potential of the artifact, we investigate the possible emergence of signs:

- Referring to the distinction between direct dragging of the red arrow and indirect movement of the constructed sequence of arrows
- Referring to specific positions of the tick marks or to the length of the arrows
- Indicating the impossibility to stretch the *1* and *-1* arrows
- Expressing that the relationship considered in the construction of the sequence of arrows is invariant under dragging of "*a*"
- Relating the movements of the arrows constructed on the two lines or of the relative tick marks

DATA ANALYSIS

In the following sections we present three short episodes in which three pairs of students (names are pseudonyms) interact with the MAE. We chose these episodes since in a few turns they allow to have an overview of students' rich semiotic production.

Episode 1 – Andrea and Hugo

This episode comes from the students' preliminary exploration of the artifact. The students are requested to drag the red tick mark, without having yet used the "Add arrow" button.

Researcher:	What does it happen to the arrows one and minus one?
Hugo:	They remain there
Researcher:	Why?
Hugo:	Because these lines, let's say, tell us that up to here its value is one or minus one
Researcher:	Okay, try to tell me this again
Hugo:	These remain still because, let's say, they tell us that this arrow makes, is… the length of this arrow is one or minus one
[…]	
Researcher:	Why are these ones called *a* and minus *a*?

Hugo:	Because these one, let's say, they have no measure, in fact *a* and minus *a* change from the position of where the red line goes, indeed for example if I put it on two *a* and minus *a* go always onto the same position
Researcher:	Okay, so you said that *a* and minus *a* are called like that because they have no measure, you said? Okay what do you mean by that?
Hugo:	That they have no value. Because their value depends on where the red line goes

The request gives rise to an intense production of signs referring to the activity with the artifact. Among these we point out the artifact signs "these remain still" and "they have no measure, in fact *a* and minus *a* changes from the position of where the red line goes" with which Hugo describes the difference between the arrows *1* and *-1*, and those *a* and *-a*. In these turns, the different dependence relationships on the red tick mark are made explicit in terms of no movement ("these remain still") or dependent movement ("their value depends on where the red line goes"). These signs could be exploited to mediate the difference between constant numerical value and variable.

Episode 2 – Grazia and Lucia

This episode follows the students' construction corresponding to the "double of *a*, plus one". The red tick mark is now on the value 2 and the researcher asks what they expect will happen if it is moved.

Lucia:	Unless also the arrow that we constructed moves, this arrow will no longer be the double of *a* plus one, that is, because we constructed it considering *a* as two, but if we move this onto four this one will no longer be the double of *a* plus one
Researcher:	Okay do you agree Grazia?
Grazia:	It makes sense what she said, however if I move, for example, I point *a*, the red arrow, for example on three, in turn, *a* also moves on three and then, in turn, also
Lucia:	You didn't listen to me well… I said unless it also moves, that is, unless the arrow above will also update, so, this *a*, that eventually is this one, will change in size along with the guide
Grazia:	But this is what she asked!
Lucia:	No… I mean, unless it keeps moving according to *a*
Researcher:	And do we expect it will move according to *a*, or not? For how we constructed it
Lucia:	One didn't move even before, so it should stay still and, instead, *a* in principle should move according to how the guide moves, and so yes, it would remain the arrow corresponding to the double of *a* plus one

This episode is full of artifact signs used by Grazia and Lucia to express their prediction about the behavior of the sequence of arrows representing the double of *a* plus 1 as *a* moves. In the opening part, the students describe in detail two possible behaviors of their construction under the dragging of *a*: either the constructed arrows will not move and therefore "this will no longer be the double of *a* plus one", or "the arrow above will also update so [...] this will change in size along with the guide". The students' description of these two possibilities involves many artifact signs. For example, they refer to the red tick mark as the "guide". This sign, with the mediation of the teacher, has the potential to evolve towards the mathematical signs of independent variable and functional relationship. In the concluding part, the students observe that since each arrow *a* in the construction should move under dragging of the red tick mark, the construction made "would remain the arrow corresponding to the double of *a* plus one". Therefore, they seem to recognize the functional relation between *a* and their construction and also the invariance under the dragging of *a* (it keeps being the double of *a* plus 1).

Episode 3 – Cora and Letizia

This episode follows the two students' constructions corresponding to the "double of *a*, minus one" and "*a* plus three" with the "Add arrow" button on two parallel lines (Figure 3).

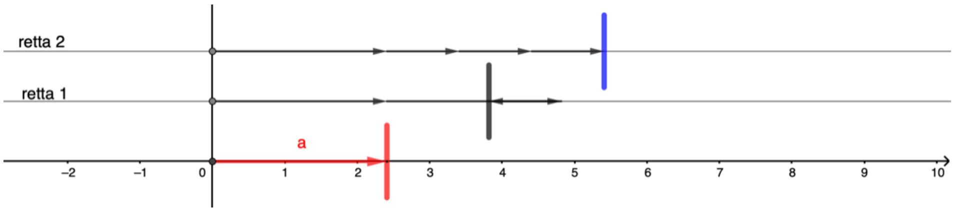

Figure 3. Screenshot of the "moving arrows" representing 2*a*-1 (line 1, below) and *a*+3 (line 2, above)

Initially, as asked by the task, the students describe the observed movements by dragging *a*.

Cora: The first part of the arrow that indicates *a*, which is present in all three, remains the same, so it goes together with the red arrow. Then also three always remains three and doesn't get… I mean, while this arrow gets longer, the three remains, that is, the part of the arrow that indicates three… these ones here [pointing to the three *1* arrows] remain the same [...] The black one, practically, both the first part of arrow that is up to here and the second part of arrow that is up to here get longer with *a*. Then, instead, the one stays still, that is, it is always the segment and goes back

There are many artifacts signs similar to those highlighted in the episode 1 referring to the dependency on *a*. The signs produced by Cora seem to be related to the structure of the expressions, since she not only describes how the blue and black tick marks move in dependence on the red one, but also how the respective arrows involved in the construction vary ("while this arrow let's say gets longer (...) these ones here remain the same"). Concerning the black tick mark, Cora's last words can be read as signs about the behavior of the *a* and *-1* arrows following the same direction and opposite orientation.

Then the researcher suggests hiding the construction with the "Hide construction" button and, focusing on the tick marks, describing their reciprocal position when dragging the red one.

Letizia: It seems that here at four they are one above the other, while as they move or it increases, I mean that one, they are always like consecutive

Cora and Letizia focus on positive *a*–values and Letizia finds a value for which the tick marks are "one above the other". An expert can recognize that it is possible to establish a link between this artifact sign and what can be expressed through the mathematical sign "the equality is verified for the value 4 of the unknown". Finally, Letizia, referring to *a*–values greater than 4, describes the reciprocal position of the two tick marks as being "always like consecutive". An expert can recognize a possible link with the inequality that is verified in that interval *a*>4. In a classroom context, the teacher can exploit this potential link to promote the evolution of these signs towards target mathematical ones.

CONCLUSION

In this paper we presented the a-priori analysis of the semiotic potential of the MAE (Table 1) and the a-posteriori analysis of its actual unfolding in the context of a pre-experimentation with low achieving students. In particular, driven by our research objective we focused on the artifacts signs produced by these students during the accomplishment of an exploratory task involving the artifact.

The episodes analyzed show how an expert could recognize these signs and, through purposefully designed actions (Bartolini Bussi & Mariotti, 2008), promote their evolution towards the mathematical signs of variable, functional dependency, linear literal expression, and reciprocal relations between two expressions depending on the same variable. This is particularly relevant considering that the students participating in the DynaMat project are students with a history of low achievement and difficulties in mathematics (e.g., Xin et al., 2022).

Although our results come from a preliminary study, they provide grounds for hope with respect to the didactical implementation of the MAE for the teaching and learning of school algebra. Indeed, the activity with the artifact fostered a rich production of signs by the students, that could become effective teaching resources for the mediation of mathematical knowledge. This study opens the door for further research investigating the possible evolution from artifact signs to mathematical signs, which in tune with TMS, will bring us to deeply focus on the role and actions of the teacher.

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