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Editorial

Preface for "Combinatorics, Riordan Matrices and Umbral Calculus—In Memory of Professor Emanuele Munarini"

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Abstract

This editorial introduces the Special Issue of *Mathematics* "Combinatorics, Riordan Matrices and Umbral Calculus—in Memory of Professor Emanuele Munarini," which collects ten peer-reviewed papers on theoretical and applied aspects of enumerative combinatorics, combinatorial matrices, and polynomial sequences. The contributions cover a variety of topics and methods, including generating functions, Riordan arrays, recurrence relations, and related combinatorial structures. The Special Issue's scope and individual contributions are briefly discussed.

MSC: 05A15; 05A10; 05A19; 11B65

Combinatorial analysis is one of the branches of modern mathematics that has experienced particularly rapid development in recent decades. Its methods and results have applications in a wide range of areas, including enumerative and algebraic combinatorics, combinatorial number theory, graph theory, discrete mathematics, physics, computer science, and biology.

Among the fundamental tools of combinatorial analysis are generating functions, infinite matrices, and polynomial sequences. Deep connections exist between finite structures and generating functions, as illustrated by the theory of combinatorial species, as well as between enumeration, infinite matrices, formal power series, and polynomial sequences, notably in the theories of Riordan matrices and umbral calculus.

This Special Issue was originally conceived by Professor Emanuele Munarini, who served as its first Guest Editor and sadly passed away in October 2024. His research scope was broad, encompassing formal power series and symbolic methods; Riordan and Sheffer matrices, Sheffer polynomial sequences, and umbral calculus; enumerative and bijective combinatorics; combinatorial identities and special polynomial sequences; partially ordered sets and lattices of paths; and graph theory, with contributions to the study of Fibonacci and Lucas cubes, Pell graphs, antiregular graphs, double graphs, and graph coloring. He was highly appreciated as both a researcher and a teacher at the Politecnico di Milano, Italy, and is warmly remembered by colleagues and students alike for his kindness.

After his passing, I was pleased to accept the invitation to continue the project he had initiated. Given the close connections between the themes of this Special Issue and many of his research interests, it is particularly fitting that the volume is dedicated to his memory. The ten contributions collected in this Special Issue reflect the breadth of combinatorial analysis and, at the same time, highlight several recurring themes, including generating functions, Riordan arrays, polynomial sequences, recurrence relations, lattice paths, and combinatorial identities.

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Generating functions and recurrence relations play a central role in Contribution 1, where multiple sums involving circular products of binomial coefficients are investigated through a recursive construction, leading to rational generating functions involving Fibonacci and Lucas numbers. In Contribution 2, C-finite sequences, including several classical homogeneous linear recurrence relations with constant coefficients, are studied from the perspective of Riordan arrays, providing a general representation in terms of recurrence coefficients and initial conditions.

Riordan arrays are also at the heart of Contributions 4, 7, and 8. Contribution 4 introduces two extensions of the notion of Riordan arrays, reverse symmetrization, and interleaved Riordan arrays and explores applications ranging from integer sequences associated with integrable lattice models to lattice-path enumeration. Contribution 7 develops a promotion framework for lattice paths based on Riordan arrays and the Binomial and Chebyshev transforms, connecting Dyck, generalized Motzkin, Schröder, and grand paths. Contribution 8 studies the Janjic–Petkovic counting function as a generalization of binomial coefficients, showing how it can be represented as a Riordan array and applying the resulting framework to a constrained many-body physical system.

Polynomial sequences and symbolic methods provide another connecting thread. Contribution 6 develops symbolic methods for identities involving s-Appell polynomials, Stirling numbers, and binomial coefficients and investigates connections with Riordan arrays, Sheffer matrices, and their q-analogues. Contribution 10 focuses on Bernoulli polynomials of the second kind, deriving new connection, derivative, linearization, and integral formulas with possible applications to polynomial approximation and differential equations.

The other contributions illustrate the diversity of structures and techniques encompassed by modern combinatorics. Contribution 5 studies families of underdiagonal paths arising from permutations avoiding vincular patterns, combining bijections, geometric characterizations, and generating trees. Contribution 9 investigates terminating Appell and Gauss functions and derives a corrected reduction identity whose structure reveals connections with binomial and Pascal coefficients.

Finally, Contribution 3 brings a probabilistic perspective to the volume by studying clade-size distributions in random labeled tree topologies under Ford's alpha model, including the probability that a randomly selected set of leaves is monophyletic.

Taken together, these contributions illustrate both the diversity of contemporary combinatorial analysis and the strong connections among its different approaches. Generating functions, matrix methods, recurrence relations, polynomial sequences, and bijective techniques provide common tools for problems ranging from lattice paths and permutation patterns to special functions, random trees, and mathematical physics.

I hope that this Special Issue will provide readers with an interesting perspective on current research in combinatorial analysis and, above all, that it will serve as a fitting tribute to the memory of Professor Emanuele Munarini and his lasting scientific contribution to the field.

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List of Contributions

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