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Bayesian model updating of historic masonry towers through dynamic experimental data

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Abstract

The numerical model of existing masonry buildings, especially in case of monumental constructions, must consider the unavoidable lack of knowledge and the consequent effects of the uncertain parameters (material properties, geometry, boundary conditions, etc.). In this work, a Bayesian approach is proposed to update the finite element model of masonry towers by using experimental data. The towers of San Gimignano (Italy) were considered as an effective case study to test this approach thanks to the availability both of geometric data and dynamic measurements. The possibility to obtain a reliable numerical model is relevant also from the point of view of the seismic risk assessment, which is a crucial issue to ensure the conservation of heritage over the centuries. In fact, both seismic capacity and demand are strictly dependent on the dynamic characteristics of the structure, and a reliable updating of the modal properties of numerical models plays a primary role in the assessment of the seismic risk. In this respect, Bayes' theorem is herein employed to convert the prior distribution of the elastic modulus E, into the posterior distribution by using the experimental data (first and second natural period). The measurement errors were accounted for by means of a Gaussian distribution centred on the measured values of the natural periods. In addition, modelling uncertainties were defined to incorporate the lack of knowledge on the restraint effect caused by the neighbouring buildings. The oscillating height of the tower was modelled according to a lognormal distribution whose interval starts from the height of the confined buildings to the tower top. Particular attention was devoted to the parameters involved in the Bayesian procedure to define their effect on the obtained posterior distributions. The achieved results encourage the spread this approach, already employed in many engineering fields, to the safeguard of cultural heritage. In view of further challenges, the methodology will be extended to seismic analyses, aiming to obtain the assessment of the seismic risk of the particular structural typology herein considered: the historic masonry towers.

Keywords: Built heritage, Masonry towers, Uncertainty parameters, FE model updating, Bayesian approach

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1. Introduction

The challenging issue of obtaining a reliable structural model has gained increasing relevance for historical buildings since, in recent decades, the conservation and the safety assessment of cultural heritage have become a growing concern. Moreover, the development of sophisticated tools of analysis has made essential the tuning of proper numerical models. However, the significant lack of knowledge on historical structures (*e.g.*, material properties, geometry, construction technique, boundary conditions), makes still difficult their numerical modeling, as some of the authors reported in previous works [1-3]. The availability of experimental data can be used to update the finite element (FE) models. For historical structures, in many cases, only data from non-destructive techniques are available for historic buildings. Several works adopt operational modal analysis (OMA) for estimating the dynamic characteristics of historic masonry towers with the aim to improve the knowledge on materials and structural schemes [4-9]. Because of their high sensitivity to dynamic actions, ambient vibration tests are commonly used for slender structures. The Bayesian statistical framework has gained great interest in civil engineering research field, especially for the structural model updating based on experimental dynamic data [10, 11]. Recent works have used the Bayesian methodology to update the uncertainty of the mechanical parameters of reinforced concrete [12, 13] and masonry structures [14, 15]. Indeed, the process of construction, the material properties and the theoretical model are affected by errors. Due to these errors, the model updating can be tackled as a structural inference problem, and the uncertain parameters can be represented by random variables with a given distribution function.

This paper proposes a methodology based on the Bayesian approach for the FE model updating of historic masonry towers starting by dynamic experimental data. As shown in [16] the dynamic behavior of these structures is strongly dependent of the elastic and mass parameters and of the boundary conditions. Therefore, the measurements of the first two natural periods of the analysed tower were herein used as new information to update posterior probability density function (PDF) of the elastic modulus. This updating was carried out by taking into account both the uncertainties due to the lateral restraints imposed by adjacent buildings on the tower (previously defined as boundary conditions), and the uncertainties due to measurement errors. Sensitivity analyses were conducted to show the effects of some assumptions concerning the prior PDF and the modeling of both epistemic and measurement errors. Section 2 describes the methodology proposed and successively applied to the case study explained in section 3. After, the main results are shown, and some concluding remarks are reported at the end of the paper.

2. Bayesian Methodology

According to the Bayesian approach, the random variable was assumed as the elastic modulus of the masonry E, whose prior PDF, based on expert judgment and/or previous information reported in the literature, was updated by new information gained from measurements of the natural periods $\bar{T}$ of the tower. The posterior PDF of E, given the measurement $\bar{T}$, is obtained by applying the Bayes theorem, *i.e.*

$$P(E|\bar{T}) = \frac{P(\bar{T}|E)P_0(E)}{\int P(\bar{T}|E)P(E)dE} \tag{1}$$

where $P_0(E)$ is the prior PDF; $P(\bar{T}|E)$ is the likelihood function, which takes into account both the modelling and measurement uncertainties. If the two sources of uncertainties are estimated separately, the likelihood function can be obtained by:

$$P(\bar{T}|E) = \int P(\bar{T}|E,T)P(T|E)dT \tag{2}$$

in which $P(\bar{T}|E,T)$ is the PDF that describes the measurement error, while $P(T|E)$ denotes the PDF used to characterize the modelling uncertainties. This latter function was defined starting from the natural periods of the tower, evaluated for several restraint conditions and for several values of E. The natural period was assumed as information data of the Bayesian approach, since it is an experimental parameter capable of synthetizing material properties, boundary conditions and stiffness of the investigated structure.

3. Case Study

The historic towers of San Gimignano (Tuscany, Italy) were recently analysed, as an illustrative case study, within the research project RiSEM (Italian acronym of Seismic Risk of Monumental Buildings). The project was aimed at developing and testing expeditious and innovative methodologies (*i.e.* without direct contact with the masonry construction) to assess the structural data needed for the subsequent evaluation of the seismic risk.

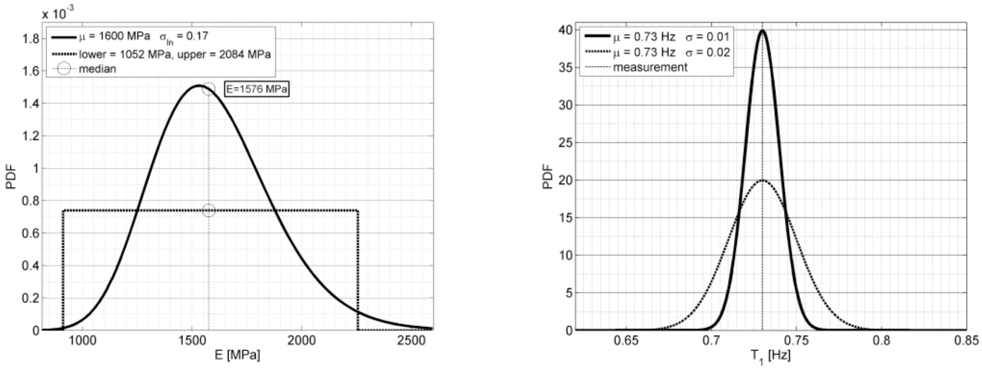
The methodology herein proposed was applied to the Becci tower (Fig. 1), one of the biggest towers in the city centre. Becci tower is characterized by a cross-section length of 6.6 m, width of 6.8 m and a height of 39.4 m. The masonry walls are 1.5 m thick and are constituted by a multi-leaf stone masonry with the internal and the external faces made with the same typology of material (and presumably the same thickness). The internal core of the multi-leaf walls is unknown but, likely, is composed of heterogeneous stone blocks tied by a good mortar.

(a) (b)

Fig. 1. Becci tower (a), FE model (b)

The FE model was built by using the commercial code ANSYS; three-dimensional elements were employed to reproduce the geometry and the model was used to perform linear modal dynamic analyses. Maximum dimension of the mesh was about 70 cm (Fig 1b).

Different prior distributions (Fig. 2a) were considered to evaluate the sensitivity of the posterior distribution with respect to the initial assumptions [17]. A lognormal distribution was employed, starting from the mechanical properties proposed in [18] for different typologies of masonry. The selected distribution is characterized by a mean value of 1600 MPa. Moreover, a uniform distribution was defined starting from 1052 MPa to 2084 MPa, and assuming the same median value of the previous curve.

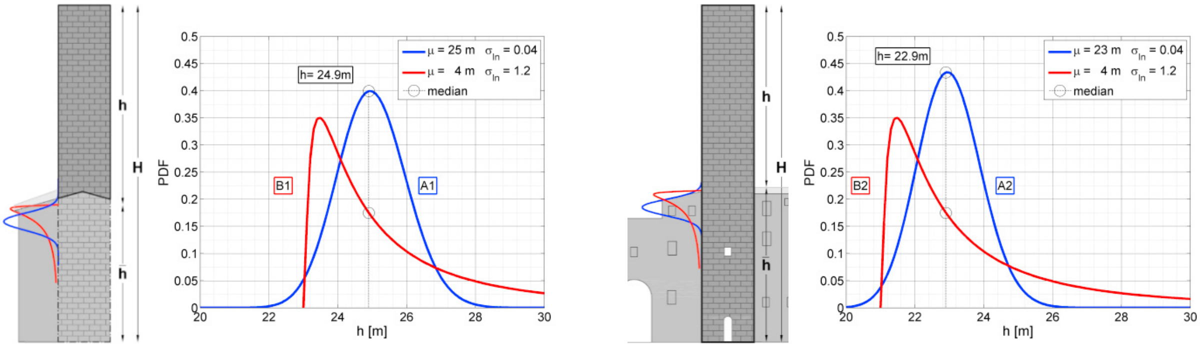


(a) (b)

Fig. 2. PDFs of (a) two prior distributions of E and of (b) two measurement errors for the first natural period

As shown in Fig. 1, at the lower level the tower is incorporated into the neighbouring buildings and hence the lower sections present several openings to allow communication with the confining buildings. The definition of the restraint conditions imposed by the neighbouring buildings to the tower was clearly a hard task. This led to include in the FEM fixed horizontal constraints up to a certain height (considered as a random variable).

In order to represent the height of the restrained part of the tower, two types of lognormal distributions were considered for the main directions of the tower (N-S and E-W), as shown in Fig. 3. In particular, A1 and A2 are characterized by a standard deviation of 0.04 and a mean value equal to 25 m and 23 m, respectively for N-S and E-W directions. Additional lognormal distributions, denoted as B1 and B2, were built with standard deviation equal to 1.2 and a mean value of 4 m shifted of 23 m and 21 m, respectively. These two groups of distributions, for each side of the tower, are characterized by the same median value. The samples of *h* were obtained by the Latin Hypercube Sampling. In particular, 200 values of height were selected for each lognormal distribution. Moreover, 200 values of elastic modulus (from 800 MPa to 3000 MPa) and two values of density were considered. The results of 40,000 modal analyses were used to quantify the uncertainty of the model.



(a) (b)

Fig. 3. PDFs of the height of the unrestrained tower (a) N-S direction; (b) W-E direction.

Within the RiSEM project, experimental tests were performed and dynamic parameters were acquired [19, 20]. The main periods of the Becci towers are shown in Table 1. A Gaussian distribution was employed to take into account the measurement errors in the estimation of the natural periods. The distributions are centred on the experimental values and their standard deviations were evaluated according to [19, 20]. As shown in Fig. 2b, different values of the standard deviation were considered to assess the effect of non-accurate measurements.

Table 1. Experimental measurements of Becci tower.

Tower	Direction	$\bar{T}$ [s]	μ [s]	σ [s]
Becci	N-S	0.73	0.73	0.01-0.02
	E-W	0.60	0.60	0.01-0.02

4. Results and Discussion

The posterior distributions *E* are herein presented as the result of two successive updates, realized using the measurements of the first two natural periods. The introduction of the second natural period produces, in each case, a relevant reduction of the uncertainty (in terms of standard deviation) and a minimum variation in terms of the mean value. The sensitivity of the posterior distribution was evaluated for different assumptions of prior distributions, material density and modelling error. Note that, the figures reported below correspond to the same measurement error (standard deviation equal to 0.01). Different measurement errors were considered; but this effect was neglected because of its less relevance compared to the other assumptions. Fig. 4 shows the effect of different assumptions on the prior PDF for the modeling uncertainties A. It is evident that, in the case of a uniform prior distribution, the probability in the posterior distributions are more concentrated to the higher values of *E*.

Fig. 4a and Fig. 5a highlight the effect of the modeling uncertainties, which cause significant variations both in terms of the mean value and standard deviation. Another relevant role is played by the material density, which causes a shift of the posterior curves, as shown in Fig. 5 for the model uncertainty B. This case seems to suggest a correlation between the elastic modulus and the material density. The last comparison, shown in fig. 6, underlines which the modeling uncertainty B is almost insensitive to variation of prior distribution, also in terms of the mean value. Moreover, the introduction of the second period, in the Bayesian procedure, causes a relevant reduction of the uncertainty, maintaining approximately the same median value.

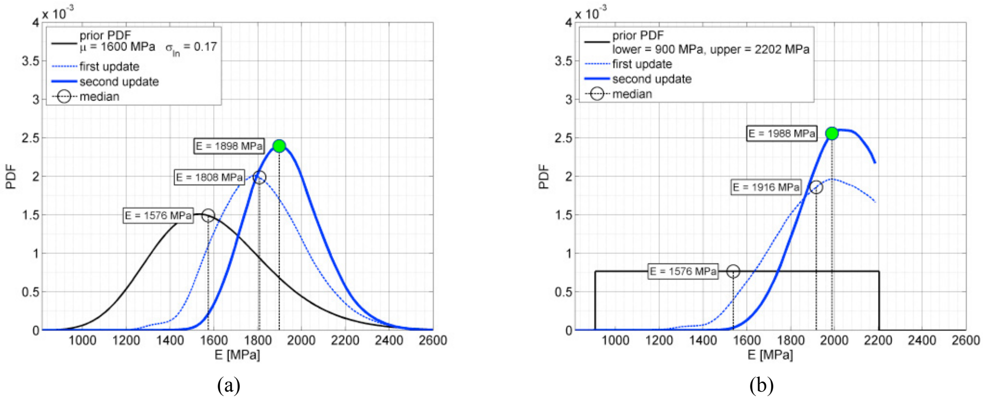


Fig. 4. Posterior distributions obtained updating a (a) lognormal prior distribution and a (b) uniform prior distribution.

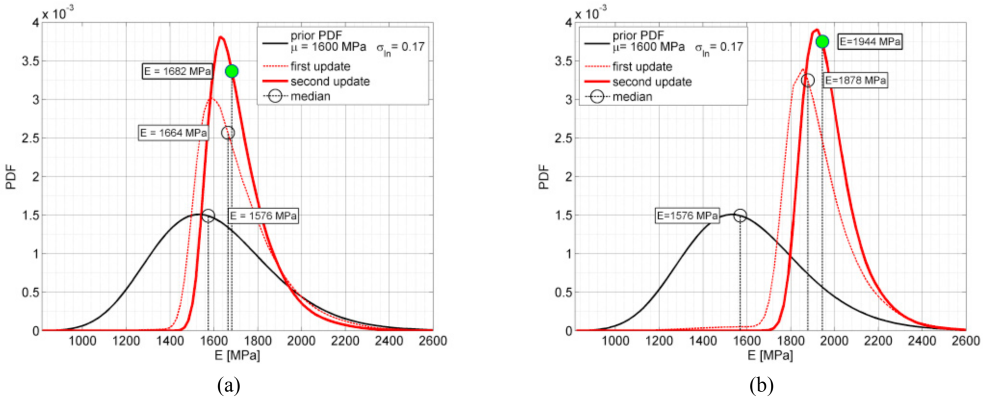


Fig. 5. Posterior distributions with a material density of (a) $16 kN/m^3$ and (b) $19 kN/m^3$.

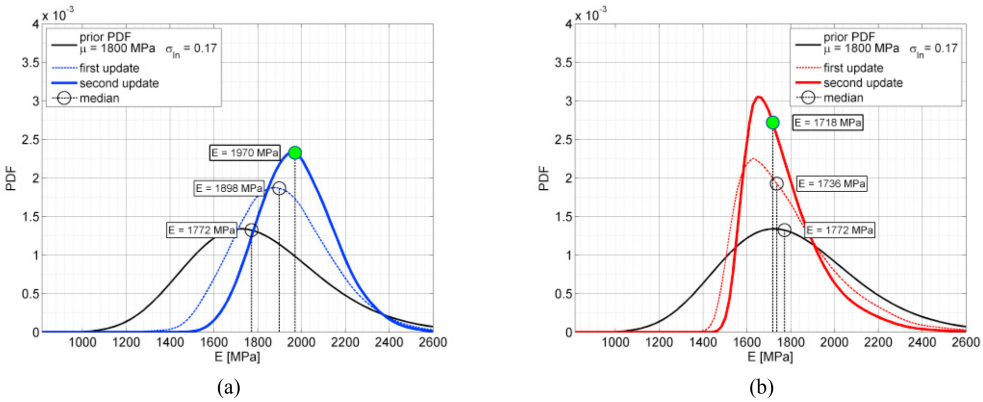


Fig. 6. Posterior distribution with material density equal to 16 kN/m^3 and modelling error: (a) A and (b) B.

Table 2 and Table 3 synthesize the posterior distributions of E in terms of percentiles. The results underline the uncertainty reduction compared to the initial assumptions on the elastic modulus but also the importance of the choice of restraint conditions and material density.

Table 2. Percentiles of the posterior distributions (MPa), related to the material density 16 kN/m^3.

Percentiles	25th	50th	75th	25th	50th	75th
Prior PDF	1396	1576	1756	1576	1772	1976
A	1790	1898	2024	1844	1970	2078
B	1610	1662	1772	1628	1718	1844

Table 3. Percentiles of the posterior distributions (MPa), related to the material density 19 kN/m^3.

Percentiles	25th	50th	75th	25th	50th	75th
Prior PDF	1396	1576	1756	1576	1772	1976
A	2010	2142	2274	2076	2208	2340
B	1878	1944	2032	1900	1966	2076

5. Conclusions

The procedure herein presented gives an opportunity to take advantage of experimental dynamic data for updating the FE model of masonry towers through Bayesian approach, is easily applicable to other similar buildings and gives an alternative way to define the mechanical characteristics of the masonry, considering different sources of uncertainties. Particular attention was devoted to the sensitivity of the posterior distribution toward the different terms involved in the Bayesian methodology. These comparisons show that the modeling uncertainties play a relevant role in the sensitivity analysis herein presented. This procedure can be considered the starting point for successive nonlinear models necessary for the seismic risk assessment of these structures. Take into account this aspect could be the crucial point to ensure the conservation of historical structures over the time.

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