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*Original Citation:*

Heat and mass transfer coefficients of falling-film absorption on a partially wetted horizontal tube / Giannetti, Niccolò; Rocchetti, Andrea; Yamaguchi, Seiichi; Saito, Kiyoshi. - In: INTERNATIONAL JOURNAL OF THERMAL SCIENCES. - ISSN 1290-0729. - ELETTRONICO. - 126:(2018), pp. 56-66. [10.1016/j.ijthermalsci.2017.12.020]

*Availability:*

The webpage <https://hdl.handle.net/2158/1124113> of the repository was last updated on 2018-04-05T22:24:47Z

*Published version:*

DOI: 10.1016/j.ijthermalsci.2017.12.020

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(Article begins on next page)

# Heat and mass transfer coefficients of falling-film absorption on a partially wetted horizontal tube

Niccolò Giannetti<sup>1\*</sup>, Andrea Rocchetti<sup>2</sup>, Seiichi Yamaguchi<sup>1</sup>, Kiyoshi Saito<sup>1</sup>

<sup>1</sup> Department of Applied Mechanics and Aerospace Engineering, Waseda University  
3-4-1 Okubo, Shinjuku-ku, Tokyo 169-8555, Japan

<sup>2</sup> Department of Industrial Engineering of Florence, University of Florence  
Via Santa Marta 3, Firenze, 50139, Italy

\* Corresponding author: niccolo@aoni.waseda.jp

**Abstract-** Detailed, reliable, and time-saving methods to predict the transfer characteristics of horizontal-tube falling-film absorbers are critical to control system operability, such that it is closer to its technical limitations, and to optimise increasingly complex configurations. In this context, analytical approaches continue to hold their fundamental importance. This study presents an analytical solution of the governing transport equations of film absorption around a partially wetted tube. A film stability criterion and a wettability model extend the validity range of the resulting solution and increase its accuracy. Temperature and mass fraction fields are analytically expressed as functions of Prandtl, Schmidt, and Reynolds numbers as well as tube dimensionless diameter and wetting ratio of the exchange surface. Inlet conditions are arbitrary. The Lewis number and a dimensionless heat of absorption affect the characteristic equation and the corresponding eigenvalues. Consequently, local and average transfer coefficients are estimated and discussed with reference to the main geometrical and operative parameters. Finally, a first comparison with the numerical solution of the problem and experimental data from previous literature is presented to support the simplifying assumptions, which are introduced and as a first model validation.

**34 Nomenclature**

35	A, B	Eigenfunction coefficients
36	a, b	Power series coefficients
37	$c_p$	Isobaric specific heat, $J \cdot kg^{-1}K^{-1}$
38	D	Mass diffusivity, $m^2 \cdot s^{-1}$
39	d	Diameter, m
40	E, H	Single variable exponential functions
41	F, G	Eigenfunctions
42	g	Gravity, $m \cdot s^{-2}$
43	h	Specific enthalpy, $kJ \cdot kg^{-1}$
44	htc	Heat transfer coefficient, $kW \cdot m^{-2}K^{-1}$
45	k	Thermal conductivity, $W \cdot m^{-1}K^{-1}$
46	l	Reference axial length, m
47	$L_c$	Characteristic length, m [ $L_c = v^{2/3} \cdot g^{-1/3}$ ]
48	Le	Lewis number [ $Le = \alpha \cdot D^{-1}$ ]
49	mtc	Mass transfer coefficient, $m \cdot s^{-1}$
50	Nu	Nusselt number [ $Nu = htc \cdot L_c \cdot k^{-1}$ ]
51	P	Pressure, kPa
52	Pr	Prandtl number [ $Pr = \nu \cdot \alpha^{-1}$ ]
53	Q	Heat flux, W
54	r	Outer tube radius, m
55	Re	Reynolds Number [ $Re = 4\Gamma \cdot \mu^{-1}$ ]
56	S	Area, $m^2$
57	Sc	Schmidt Number [ $Sc = \mu \cdot \rho^{-1}D^{-1}$ ]
58	Sh	Sherwood Number [ $Sh = mtc \cdot L_c \cdot D^{-1}$ ]
59	t	Tube wall thickness, m
60	T	Temperature, K
61	u	Streamwise Velocity, $m \cdot s^{-1}$
62	v	Normal Velocity, $m \cdot s^{-1}$
63	W	Transversal extension of the wet part, m
64	WR	Wetting Ratio
65	x	Local tangential position, m
66	y	Local normal position, m
67		

**68 Greek symbols**

69	$\alpha$	Thermal diffusivity, $m^2 \cdot s^{-1}$
70	$\beta$	Contact angle
71	$\epsilon$	Dimensionless tangential position
72	$\gamma$	Dimensionless LiBr mass fraction
73		distribution
74	$\eta$	Dimensionless normal position
75	$\Lambda$	Normalised heat of absorption
76		[ $\Lambda = h_{abs}(\omega_e - \omega_{in}) \cdot (T_e - T_{in})^{-1} \omega_e^{-1} c_p^{-1}$ ]
77	$\lambda, \phi$	Eigenvalues
78	$\theta$	Dimensionless temperature distribution
79	$\Gamma$	Mass flow rate per unit length, $kg \cdot s^{-1}m^{-1}$
80	$\delta$	Film thickness, m
81	$\mu$	Dynamic viscosity, $Pa \cdot s$
82	$\rho$	Density, $kg \cdot m^{-3}$
83	$\omega$	LiBr mass fraction
84		

**85 Subscripts**

86	0	Film breaking condition
87	abs	Absorption
88	av	Average
89	b	Bulk value
90	c	Cooling water side
91	e	Equilibrium
92	g	Global
93	i	Power series index
94	if	Interface
95	in	Inlet
96	max	Maximum
97	n, m	Eigenvalue/Eigenfunction indexes
98	o	Outlet
99	sat	Phases equilibrium
100	T	Temperature
101	v	Vapour
102	W	Wall

104 **Superscripts**

106

107 **1. Introduction**

108 It is not possible to consider heat transfer and mass transfer  
109 separately in several technical circumstances and physical  
110 processes. Absorption systems, such as chillers, heat amplifiers,  
111 and heat transformers, belong to the aforementioned category and  
112 represent an opportunity for clean and efficient energy  
113 conversion systems (1). The main advantages of these systems  
114 include low-grade heat as the main energy source, higher  
115 reliability, and environmentally friendly refrigerants. This is  
116 accompanied by the possibility of realising the refrigerant  
117 pressure jump in a liquid phase. Accordingly, the compressor of  
118 a conventional system is substituted with a set of components,  
119 such as a solution pump, a generator, an absorber, and a  
120 solution heat exchanger, termed as a "thermal compressor". As a  
121 downside, this requires a significantly larger exchange surface.  
122 In addition, extant studies indicated that the highest amount of  
123 irreversibility occurs in an absorber (2) and that global  
124 capacity and first law efficiency are limited by the amount of  
125 refrigerant that is absorbed in this component (3-4). Therefore,  
126 the intensification of the absorption process and proper design  
127 of an absorber are the critical factors that should be addressed.  
128 Conversely, the recent technical development of absorption  
129 chillers, heat pumps, and heat transformers corresponds to  
130 increasingly complex plant configurations (5-6), and  
131 specifically constitutes a step forward with respect to the  
132 theoretical background required for an accurate performance  
133 prediction, optimisation, and control. In general, the systems  
134 design approach continues to rely on empirical rules, heuristic  
135 correlations, or trial and error procedures on a global and  
136 component scale. The correlations rely on large sets of data, in  
137 which each set depends on experimental equipment as well as the

138 specific boundary conditions of these measurements. Furthermore,  
139 devices that are designed to achieve high performance under  
140 nominal conditions may not exhibit a sufficient performance over  
141 most of the actual operative range. Similarly, in practice,  
142 conditions are transient and change continuously, because they  
143 are affected by interrelations with the external environment.  
144 Consequently, instantaneous conditions significantly differ from  
145 the design point. The construction of reliable and widely  
146 applicable theoretical models enables the design, optimisation,  
147 and definition of an effective control method without depending  
148 on trial and error procedures or empirical rules.  
149 More specifically, horizontal-tube falling-film absorbers can  
150 realise high heat and mass transfer rates with compact size and  
151 negligible pressure losses. Nevertheless, prior experimental  
152 studies on falling film absorption (7-12) report a limited  
153 amount of results with high uncertainties and within a  
154 relatively narrow range of operative conditions.  
155 Reference (13) numerically discusses a model for film absorption  
156 and desorption of a laminar liquid film with constant thickness  
157 that flows over a vertical isothermal plate. A similar model was  
158 applied by (14) to a horizontal tube heat exchanger. References  
159 (15-18) introduce the effects of thickness and velocity  
160 distributions around a tube surface via numerical analyses.  
161 Finally, references (19-25) use the Volume of Fluid technique to  
162 examine and extract detailed descriptions of the wavy film  
163 dynamics, inter-tube droplets formation, detachment, and impact.  
164 Numerical analysis and computational fluid dynamics (CFD) are  
165 powerful tools that could be very precise when the problem is  
166 properly formulated. However, it is necessary to adequately  
167 consider the time required to reach an accurate solution and the  
168 fact that its validity is restricted to the specific case and  
169 the selected operative condition. Generalisable design  
170 guidelines are not directly provided by specific results as well  
171 as heuristic methods. Given this viewpoint, analytical  
172 approaches continue to maintain their fundamental importance to

173 capture the physics of the problem and generalise the validity  
174 of the solution. The main limitations of extant analytical  
175 models include the geometry of the solid surface, assumptions of  
176 complete wetting, equilibrium of an inlet solution with the  
177 refrigerant vapour, uniform velocity profile, and film thickness  
178 (26-29). Reference (28) indicated that uniform velocity profile  
179 and film thickness are responsible for approximately 20%  
180 deviations in the heat and mass transfer coefficients, and they  
181 under-predict approximately 40% of the distance required for the  
182 development of the thermal boundary layer. Therefore, this study  
183 successfully achieves an accurate and widely applicable  
184 analytical solution of the governing equations of falling film  
185 absorption over a horizontal tube including the effects of  
186 thickness variations, incomplete wetting, and the corresponding  
187 reduction in transfer interfaces.

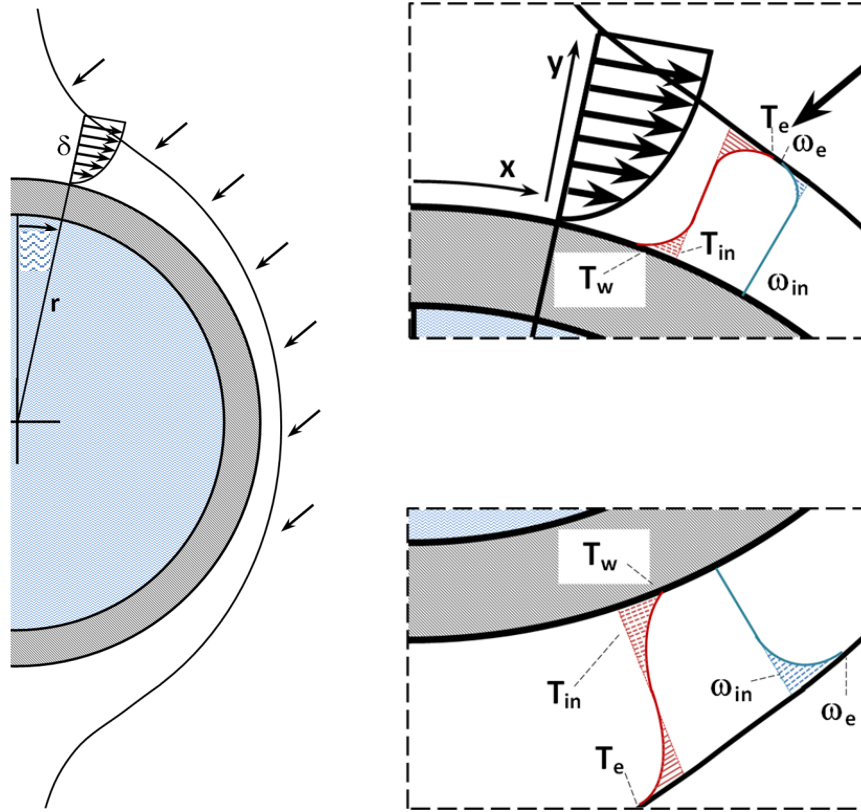
188

## 189 **2. Physical model**

190 The present analysis focuses on an absorptive liquid film flowing  
191 over a vertical row of horizontal smooth tubes. Droplet impact  
192 and hydrodynamic boundary layer development (19-25, 30) are not  
193 discussed herein. Figure 1 schematically illustrates the system  
194 under consideration. A single tube at uniform wall-temperature,  
195  $T_w$ , is considered. A thin film of LiBr-H<sub>2</sub>O solution impinges at  
196 the top ( $x=0$ ) and flows viscously down the tube due to gravity as  
197 a laminar incompressible liquid. Additionally, absorption can  
198 occur at the free-interface of the film based on the thermo-  
199 physical relation between the solution and the vapour. The  
200 enthalpy of vapour condensation that is released in the lithium-  
201 bromide/water mixture is rejected to the cooling water flowing  
202 inside the tube. Following the development of the thermal  
203 boundary layer, the temperature gradient related to the cooling  
204 process at the wall also influences the temperature at the  
205 interface, and this in turn establishes the equilibrium mass

206 fraction at the vapour pressure within the heat exchanger and  
207 consequently controls mass transfer.

208



209

210

**Figure 1. Local coordinate system**

211

212 In order to reach a closed analytical solution of the governing  
213 transport equations, heat and mass transfer processes are  
214 considered under the following main assumptions:

- 215 - The zone of impingement is assumed as a small fraction of the  
216 total periphery, and it is assumed that the thermal boundary  
217 layer starts its growth from the upper stagnation point ( $x \approx 0$ );
- 218 - It is assumed that both the tube circumference and length are  
219 large when compared to the film thickness and that the  
220 disturbances at the edges of the system can be neglected;
- 221 - The flow is laminar;
- 222 - Neither interfacial shear forces with the vapour nor  
223 interfacial waves exist;

224 - Thermodynamic equilibrium occurs at the film inlet-interface  
225 with the vapour at the heat exchanger pressure, and thus mass  
226 transfer occurs without any resistance;

227 - Thermo-physical solution properties are similar to those of an  
228 ideal mixture and remain constant along the film thickness and  
229 around the tube. As a corollary, natural and Marangoni convection  
230 are not considered;

231 - Heat transfer to the vapour environment is neglected;

232 - The variation of the mass flowrate due to the absorbed vapour  
233 is negligible;

234 - According to the thin film approximation introduced by (27),  
235 body fitted coordinates ( $x$  along the tube surface and  $y$  normal to  
236 it at any point) are used because the film thickness is low when  
237 compared to the tube diameter.

238 A curvilinear coordinate transformation is adopted to map the  
239 flow domain of the physical space to a simple rectangular domain  
240 (16). The dimensionless variables considered in the  
241 circumferential and radial directions correspond to  $\varepsilon=x/\pi r$  and  $\eta=y/\delta$ ,  
242 respectively. Tangential (eq. 1) and normal (eq. 2) velocity  
243 components based on the Nusselt integral solution of the boundary  
244 layer momentum and continuity equations with constant properties  
245 form (see, for instance, references 13-18) are employed under the  
246 assumption that the momentum transfer of the fluid is dominated  
247 by viscous forces in the absence of inertia and pressure forces.

248

$$249 \quad u = \frac{\rho g \delta^2}{2\mu} \sin \pi \varepsilon (2\eta - \eta^2) \quad (1)$$

$$250 \quad v = -\frac{\rho g \delta^2 \eta^2}{2\mu r} \left[ \frac{1}{\pi} \frac{d\delta}{dx} \sin \pi \varepsilon + \delta \left( 1 - \frac{\eta}{3} \right) \cos \pi \varepsilon \right] \quad (2)$$

251

252 Accordingly, once the film mass flowrate per unit length of the  
253 tube is known, the corresponding film thickness is given by eq.

254 3.

255 
$$\delta = \left( \frac{3\mu\Gamma}{\rho^2 g \sin \pi\epsilon} \right)^{1/3} \quad (3)$$

256 A small thermal resistance is associated with a thinner film at  
257 low specific mass flowrates, and thus moving the operability of  
258 falling film absorbers to a low Reynolds number is attractive in  
259 increasing the performance of absorption systems and reducing  
260 their overall size. However, it is necessary to consider the  
261 reduction in the contact area due to partial wetting as a  
262 critical related issue. In these operative conditions,  
263 specifically at a low film Reynolds number ( $Re=4\Gamma/\mu$ ) and while  
264 employing liquids with high surface tension (i.e., low Weber  
265 numbers), it is not possible to consider the assumptions of a  
266 film with uniform thickness and complete wetting of the transfer  
267 surface as even approximately rigorous. This leads to an  
268 unacceptable inaccuracy of simulation results (i.e., the  
269 obtained trend of the predicted heat transfer coefficient itself  
270 disagrees with measurements (31)). Furthermore, it is recognised  
271 that partial wetting occurs even at typical operative conditions.  
272 Among the previously proposed models, the effect of the amount  
273 of wetted surface is not assessed or is merely assumed as a  
274 fixed value imposed on the calculation (15, 32) albeit with a  
275 few exceptions (9, 33-35). Moreover, related experimental data  
276 and visual descriptions by digital image processing are also  
277 extremely limited in terms of the number of studies that report  
278 the same as well as in the range of conditions that is covered  
279 (36-39). Nevertheless, the role of wettability is recognised as  
280 a dominant factor in determining the efficiency of the  
281 absorption process. Therefore, both a criterion of stability of  
282 the uniform film to identify the minimum flow rate to ensure a  
283 complete wetting of the surface and a method to estimate the  
284 wetted area after the film breakage should be included to  
285 enhance the model capability to predict the performance of these  
286 devices.

287 To consider the effect of partial wetting, after the thermo-  
 288 physical properties of the solution are given, the extension of  
 289 the range affected by the phenomenon is identified by the  
 290 critical condition for a uniform film in terms of minimum wetting  
 291 rate  $\Gamma_0$  that corresponds to a critical Reynolds number  $Re_0$ . The  
 292 latter can be experimentally measured (37-41) or analytically  
 293 estimated for a surface with generic inclination (42-43) once the  
 294 characteristic contact angle that is representative of the  
 295 affinity of the solid-liquid interaction is known. Among the  
 296 various available methods (44-47), the principle of minimising  
 297 the energy contained in a given stream wise length of the  
 298 falling film is hereby used to assess the stability of the  
 299 uniform configuration (eq. 4) and to provide an estimate (eq. 5)  
 300 of the rivulet wetting ability (42-43) given the assumption of a  
 301 rivulet cross-section geometry. The value of the dimensionless  
 302 group  $(Re_0 \cdot We_0^3)^{1/15}$  in (43) is directly proportional to the  
 303 dimensionless critical thickness  $\delta_0^*$  that is defined in (42) (eq.  
 304 4). Therefore, equation 4 represents the flow regime transition  
 305 between a uniform film and a rivulet flow configuration with  
 306 circular cross-section shape and contact angle  $\beta$ , which  
 307 partially wets the solid surface. This is obtained from the  
 308 condition of equivalent kinetic plus surface tension energy, and  
 309 flowrate of the two regimes, when the stable condition of the  
 310 rivulet is identified through the principle of minimum energy.  
 311

$$312 \quad \delta_0^{*5} + (1 - \cos \beta) - G(\beta) \delta_0^{*3} = 0 \quad , \quad \delta_0^* = \left( \frac{\rho^3 g^2}{15 \mu^2 \sigma} \right)^{1/5} \delta_0 \quad (4)$$

313 Equation 5 corresponds to the minimisation of the energy  
 314 contents of a given stream-wise length of the rivulet, with  
 315 respect to the geometrical parameter that defines its wetting  
 316 ability  $WR$  (the ratio of the base of the rivulet  $w$  to the total  
 317 axial length  $l$  taken as a reference).  
 318

319 
$$WR = \left\{ \frac{1}{15} \frac{\rho g}{\sigma} \frac{\psi(\beta)}{\sin \beta} \left[ \frac{\beta}{\sin \beta} - \cos \beta \right]^{-1} \right\}^{\frac{3}{5}} \frac{\sin \beta}{\gamma(\beta)} Re \quad (5)$$

320

321 where  $G(\beta)$ ,  $\psi(\beta)$ , and  $\gamma(\beta)$  denote geometrical functions of the contact  
 322 angle  $\beta$  between the liquid-gas interface of the rivulet (further  
 323 details are given in reference 43). When  $WR$  is used to estimate  
 324 the wetting ability of the film along the absorber tube, its  
 325 value corresponds to the ratio of the wetted portion  $w$  to the  
 326 tube unit length  $l$  (Figure 2).

327 For lower solution flowrates, methods based on the principle of  
 328 minimum energy (eq. 5) as well as experiments (37-41, 43) are in  
 329 agreement with a linearised wettability model (eq. 6) relative to  
 330 the film Reynolds number, which gives zero wetting when Reynolds  
 331 number is zero, and complete wetting at  $Re=Re_0$ .

332

333 
$$WR = \frac{Re}{Re_0} \quad (6)$$

334

335 Therefore,  $\delta_0^*$  can be evaluated from eq. 4, once the value of the  
 336 characteristic contact angle of the liquid-solid pair is known.  
 337 Afterwards, using the Nusselt velocity profile for a vertical  
 338 falling film, the film thickness can be directly related to the  
 339 film Reynolds number ( $Re=4\Gamma/\mu$ ) and the critical Reynolds number  
 340 at which the film breaking occurs  $Re_0$  is calculated as in eq. 7.

341 
$$Re_0 = \left( \frac{3^5 g \mu^4}{4^5 15^3 \rho \sigma^3} \right)^{-\frac{1}{5}} \delta_0^{*3} \quad (7)$$

342 The approach aims at estimating the wetted exchange area on an  
 343 average basis while not targeting a local description of the  
 344 complex film hydrodynamics. Furthermore, a closed solution  
 345 requires considering  $WR$  as an independent function of the angular  
 346 position on the tube surface. Accordingly, the film thickness  
 347 distribution (eq. 9) is adjusted to assure the consistency

348 between uniform and partial wetting configurations (eq. 8) by  
 349 using a modified form of the Nusselt equation (as in (32)).

350

351

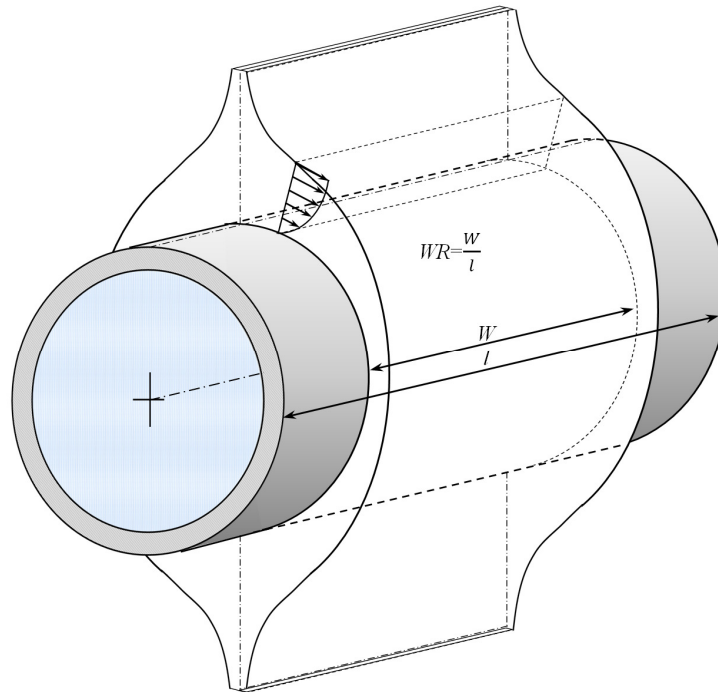
$$352 \quad \frac{\Gamma}{2WR} = \int_0^{\delta} \rho u(y) dy = \frac{1}{3} \frac{\rho^2 g \sin \beta}{\mu} \delta^3 \quad (8)$$

$$353 \quad \delta = \left( \frac{3\mu\Gamma}{WR\rho^2 g \sin \beta} \right)^{1/3} \quad (9)$$

354

355 To the authors' knowledge, a direct validation of eq. 9 has not  
 356 been achieved in previous literature and further research efforts  
 357 in this regard are needed.

358



359

360

**Figure 2. A physical model of film partial wetting**

361

362 Heat and mass transfer characteristics of the system under  
 363 analysis are studied with reference to eq.s 10 and 11. This two-  
 364 dimensional form of the energy and species transport equations is  
 365 written for a steady flow with constant properties without  
 366 internal heat generation and viscous dissipation and neglecting

367 diffusion terms in the flowing direction (see, for instance, 15-  
368 16, 27).

369

$$370 \quad \frac{\partial T}{\partial \varepsilon} = \frac{\pi r \alpha}{u \delta^2} \frac{\partial^2 T}{\partial \eta^2} + \left( \frac{\eta}{\delta} \frac{d\delta}{d\varepsilon} - \frac{\pi r v}{u \delta} \right) \frac{\partial T}{\partial \eta} \quad (10)$$

$$371 \quad \frac{\partial \omega}{\partial \varepsilon} = \frac{\pi r D}{u \delta^2} \frac{\partial^2 \omega}{\partial \eta^2} + \left( \frac{\eta}{\delta} \frac{d\delta}{d\varepsilon} - \frac{\pi r v}{u \delta} \right) \frac{\partial \omega}{\partial \eta} \quad (11)$$

372

373 Where,

374

$$375 \quad \frac{d\delta}{d\varepsilon} = - \left( \frac{\mu \Gamma \pi^3}{9WR\rho^2 g} \right)^{1/3} \frac{1}{\sin^{1/3} \pi \varepsilon} \frac{1}{\tan \pi \varepsilon} \quad (12)$$

376

377 It is shown that eq. 13 is generally applicable for the velocity  
378 distribution expressed in eq. 1 and eq. 2.

379

$$380 \quad \left( \frac{\eta}{\delta} \frac{d\delta}{d\varepsilon} - \frac{\pi r v}{u \delta} \right) = 0 \quad (13)$$

381

382 As a result, the simplified expression is obtained as follows:

383

$$384 \quad \frac{\partial T}{\partial \varepsilon} = \frac{\pi r \alpha}{u \delta^2} \frac{\partial^2 T}{\partial \eta^2} \quad (14)$$

$$385 \quad \frac{\partial \omega}{\partial \varepsilon} = \frac{\pi r D}{u \delta^2} \frac{\partial^2 \omega}{\partial \eta^2} \quad (15)$$

386

387 An analytical solution of the coupled set of equations is  
388 approached with the final aim of obtaining Nusselt and Sherwood  
389 number expressions in terms of the operative parameters,  
390 geometrical features, and boundary conditions.

391 It is advantageous for the solution of the problem to use a  
392 dimensionless form of the variables  $T$  and  $\omega$  as defined by eqs. 16-  
393 17 where  $T_e$  and  $\omega_e$  are defined in (28). These values are,

394 respectively, the equilibrium temperature of the solution at LiBr  
 395 mass fraction  $\omega_{in}$  and the equilibrium LiBr mass fraction of the  
 396 solution at temperature  $T_{in}$ , namely, the temperature and the mass  
 397 fraction reached if thermodynamic equilibrium is obtained without  
 398 changes in mass fraction and temperature.

399

$$400 \quad \theta(\varepsilon, \eta) = \frac{T(\varepsilon, \eta) - T_w}{T_e - T_w} \quad (16)$$

$$401 \quad \gamma(\varepsilon, \eta) = \frac{\omega(\varepsilon, \eta) - \omega_{in}}{\omega_e - \omega_{in}} \quad (17)$$

402

403 Accordingly,  $T_e - T_w$  represents the level of sub-cooling of the wall  
 404 while  $\omega_e - \omega_{in}$  embodies the driving force for vapour diffusion at the  
 405 inlet of the calculation domain. The dimensionless tube diameter  
 406  $d^* = 2\pi r / L_c$  is defined as the ratio of the tube circumference to the  
 407 characteristic length  $L_c$ , which is expressed in eq. 18 as follows  
 408 (17):

409

$$410 \quad L_c = \left( \frac{\mu^2}{\rho^2 g} \right)^{1/3} \quad (18)$$

411

412 Finally, non-constant terms of eq.s 14 and 15 are developed and  
 413 dimensionless variables and parameters are used to express energy  
 414 and species transport equations in eq. 19 and eq. 20,  
 415 respectively, in which the independent variables are separated  
 416 between the sides of the equations as follows:

417

$$418 \quad \frac{1}{d^* \sin^{1/3} \pi \varepsilon} \left( \frac{3 \text{Re}}{4WR} \right)^{4/3} \frac{\partial \theta}{\partial \varepsilon} = \frac{1}{\text{Pr} (2\eta - \eta^2)} \frac{\partial^2 \theta}{\partial \eta^2} \quad (19)$$

$$419 \quad \frac{1}{d^* \sin^{1/3} \pi \varepsilon} \left( \frac{3 \text{Re}}{4WR} \right)^{4/3} \frac{\partial \gamma}{\partial \varepsilon} = \frac{1}{\text{Sc} (2\eta - \eta^2)} \frac{\partial^2 \gamma}{\partial \eta^2} \quad (20)$$

420

421 The solution is approached with the following boundary and inlet  
 422 conditions: solution temperature and mass fraction at the  
 423 distributor or, by assuming that complete mixing occurs, the bulk  
 424 values of the solution coming from the previous tube ( $x \approx 0$  and  
 425  $0 < y < \delta$ ;  $T = T_{in}$ ,  $\theta(0, \eta) = \theta_{in}$ ;  $\omega = \omega_{in}$ ,  $\gamma(0, \eta) = 0$ ), at the tube wall constant  
 426 temperature and non-permeability to species are assured ( $y = 0$ ;  
 427  $T = T_w$ ,  $\theta(\varepsilon, 0) = 0$ ;  $\partial \alpha / \partial y = 0$ ,  $\partial \gamma / \partial \eta|_w = 0$ ), and at the phase interface  
 428 ( $y = \delta$ ;  $T = T_{sat}(\omega_{if}, P)$ ,  $\omega = \omega_{if}$ ) phase equilibrium is established.  
 429

$$430 \quad \left. \frac{\partial \theta}{\partial \eta} \right|_{if} = \frac{\Lambda}{Le} \left. \frac{\partial \gamma}{\partial \eta} \right|_{if} \quad (21)$$

431  
 432 Equation 21 constitutes a rearrangement of Fick's law of  
 433 diffusion and Fourier law that assures that the heat produced by  
 434 absorption at the film interface is conducted through the film  
 435 towards the tube surface. Where, the following expression holds  
 436 and defines the normalised heat of absorption (28):  
 437

$$438 \quad \Lambda = - \frac{h_{abs} (\omega_e - \omega_w)}{\omega_e c_p (T_e - T_w)} \quad (22)$$

439  
 440 Additionally, with respect to the vapour pressure equilibrium at  
 441 the interface, a linear relation (as in (28)) between temperature  
 442 and mass fraction at the film interface is employed. Accordingly,  
 443 in terms of the dimensionless variables at a constant pressure,  
 444 the relation expressed by eq. 23 is obtained.  
 445

$$446 \quad \gamma_{if} = 1 - \theta_{if} \quad (23)$$

447 Equation 23 was found in good agreement for a wide range of  
 448 operative conditions of LiBr-H<sub>2</sub>O solution and a thermodynamic  
 449 justification (although it limited to electrolytic solutions) was  
 450 presented in reference (48).  
 451

### 452 3. Solution method

453 The dependent functions (eq.s 24-25) are assumed as a infinite  
 454 series of products of a number of eigenfunctions in which each is  
 455 dependent on a single variable as shown in (12) and (13).  
 456

$$457 \quad \theta(\varepsilon, \eta) = \sum_{n=1}^{\infty} A_n F_n(\eta) E_n(\varepsilon) \quad (24)$$

$$458 \quad \gamma(\varepsilon, \eta) = 1 - \sum_{n=1}^{\infty} B_n G_n(\eta) H_n(\varepsilon) \quad (25)$$

459  
 460 The application of this method results in four ordinary  
 461 differential equations as follows:  
 462

$$463 \quad \frac{1}{d^* \sin^{1/3} \pi \varepsilon} \left( \frac{3 \text{Re}}{4WR} \right)^{4/3} \frac{E_n'}{E_n} = \frac{1}{\text{Pr}(2\eta - \eta^2)} \frac{F_n''}{F_n} = -\lambda_n^2 \quad (25)$$

$$464 \quad \frac{1}{d^* \sin^{1/3} \pi \varepsilon} \left( \frac{3 \text{Re}}{4WR} \right)^{4/3} \frac{H_n'}{H_n} = \frac{1}{\text{Sc}(2\eta - \eta^2)} \frac{G_n''}{G_n} = -\phi_n^2 \quad (27)$$

465  
 466 The general solutions of the left members of both eq. 26 and eq.  
 467 27 are as follows:  
 468

$$469 \quad E_n(\varepsilon) = e^{-\lambda_n^2 d^* \left( \frac{4WR}{3\text{Re}} \right)^{4/3} \int_0^\varepsilon \sin^{1/3} \pi \varepsilon d\varepsilon} \quad (28)$$

$$470 \quad H_n(\varepsilon) = e^{-\phi_n^2 d^* \left( \frac{4WR}{3\text{Re}} \right)^{4/3} \int_0^\varepsilon \sin^{1/3} \pi \varepsilon d\varepsilon} \quad (29)$$

471  
 472 Where  $\lambda_n$  and  $\phi_n$  denote the eigenvalues corresponding to the  
 473 eigenfunctions  $F_n$  and  $G_n$ , respectively. Additionally, for the  
 474 linear equilibrium condition at the interface (eq. 23) that  
 475 should be satisfied for every  $\varepsilon$ , it is necessary for every  $n$  that  
 476  $\lambda_n = \phi_n$ . The boundary conditions at the wall require  $F_n(0) = 0$  and  
 477  $G_n'(0) = 0$ , while eq. 30 and eq. 31 are obtained at the interface.  
 478

479  $A_n F_n(1) = B_n G_n(1)$  (30)

480  $A_n F_n'(1) = -\frac{\Lambda}{Le} B_n G_n'(1)$  (31)

481

482 Equation 30 and eq. 31 represent two homogeneous equations for  $A_n$   
 483 and  $B_n$ , and thus a non-null solution is reached given the  
 484 condition that the determinant equals zero.

485

486  $\frac{F_n'(1)}{F_n(1)} = -\frac{\Lambda}{Le} \frac{G_n'(1)}{G_n(1)}$  (32)

487

488 Equation 32 represents the characteristic equation to determine  
 489 the eigenvalues  $\lambda_n$  when the solution for  $F_n$  and  $G_n$  is determined.  
 490 The power series solutions for the right-side members of eq. 26  
 491 and eq. 27 are expressed as follows:

492

493  $F_n(\eta) = \sum_{i=0}^{\infty} a_{n,i} \eta^i$  (33)

494  $G_n(\eta) = \sum_{i=0}^{\infty} b_{n,i} \eta^i$  (34)

495

496 The boundary conditions at the wall  $F_n(0)=0$  and  $G_n'(0)=0$ , namely  
 497 constant temperature and non-permeability to species , are used  
 498 to calculate the coefficients  $a_{n,i}$  and  $b_{n,i}$  by the recursive  
 499 relations represented by eq. 35 and eq. 36, respectively.

500

501  $a_{n,0} = 0, a_{n,1} = 1, a_{n,2} = 0, a_{n,3} = 0, a_{n,i} = \frac{\lambda_n^2 \text{Pr}(a_{n,i-4} - 2a_{n,i-3})}{i(i-1)}, i \geq 4$  (35)

502  $b_{n,0} = 1, b_{n,1} = 0, b_{n,2} = 0, b_{n,3} = -\lambda_n^2 / 3,$

503  $b_{n,i} = \frac{\lambda_n^2 \text{Sc}(b_{n,i-4} - 2b_{n,i-3})}{i(i-1)}, i \geq 4$  (36)

504

505 The coefficients  $A_n$  and  $B_n$  are determined by using a Sturm-  
506 Liouville orthogonality condition at the inlet and the boundary  
507 conditions at the interface. The solution method follows the  
508 procedure presented in (28) although the inlet temperature value  
509 in this case is different from the constant value at the wall.  
510 Equations 37 and 38 are expressed by multiplying the right-side  
511 members of eq. 26 and eq. 27 by the eigenfunctions  $F_m$  and  $G_m$ ,  
512 respectively, in the specified order and integrating with respect  
513 to  $\eta$ . This is expressed as follows:

514

$$515 \quad \lambda_n^2 \text{Pr} \int_0^1 (2\eta - \eta^2) F_m F_n d\eta = - \int_0^1 F_m F_n'' d\eta = F_m(0)F_n'(0) - F_m(1)F_n'(1) + \int_0^1 F_m' F_n' d\eta \quad (37)$$

$$516 \quad \lambda_n^2 \text{Sc} \int_0^1 (2\eta - \eta^2) G_m G_n d\eta = - \int_0^1 G_m G_n'' d\eta = G_m(0)G_n'(0) - G_m(1)G_n'(1) + \int_0^1 G_m' G_n' d\eta \quad (38)$$

517

518 The corresponding equations (obtained by proceeding in the same  
519 way for eigenvalues and eigenfunctions with index  $m$ ) are  
520 subtracted and the boundary conditions expressed in eq. 30 and eq.  
521 31 are used to yield eq. 39 and eq. 40 as follows:

522

$$523 \quad \text{Pr} (\lambda_n^2 - \lambda_m^2) \int_0^1 (2\eta - \eta^2) F_n F_m d\eta = F_n(1)F_m'(1) - F_m(1)F_n'(1) \quad (39)$$

$$524 \quad \text{Sc} (\lambda_n^2 - \lambda_m^2) \int_0^1 (2\eta - \eta^2) G_n G_m d\eta = G_n(1)G_m'(1) - G_m(1)G_n'(1) \quad (40)$$

525

526 The coupling between the previous two conditions is established  
527 by using eq. 30 and eq. 31 as follows:

528

$$529 \quad F_n(1)F_m'(1) - F_m(1)F_n'(1) = - \frac{\Lambda}{Le} \frac{B_n B_m}{A_n A_m} [G_n(1)G_m'(1) - G_m(1)G_n'(1)] \quad (41)$$

530

531 Equation 41 enables the combination of eq. 39 and eq. 40 as  
532 follows:

533

$$534 \quad Sc(\lambda_n^2 - \lambda_m^2) \int_0^1 (2\eta - \eta^2) (\text{Pr} Le A_n A_m F_n F_m + Sc \Lambda B_n B_m G_n G_m) d\eta = 0 \quad (42)$$

535

536 This directly implies,

537

$$538 \quad \int_0^1 (2\eta - \eta^2) (\text{Pr} Le A_n A_m F_n F_m + Sc \Lambda B_n B_m G_n G_m) d\eta \begin{cases} = 0, n \neq m \\ \neq 0, n = m \end{cases} \quad (43)$$

539

540 The boundary conditions of constant temperature and mass fraction  
541 are used over the entire film thickness at the inlet of the  
542 calculation domain as follows:

543

$$544 \quad \sum_{n=1}^{\infty} A_n F_n(\eta) = \theta_m \quad (44)$$

$$545 \quad \sum_{n=1}^{\infty} B_n G_n(\eta) = 1 \quad (45)$$

546

547 The summation of the integrals is simplified as follows:

548

$$549 \quad \sum_{n=1}^{\infty} \int_0^1 (2\eta - \eta^2) (\text{Pr} Le A_n A_m F_n F_m + Sc \Lambda B_n B_m G_n G_m) d\eta = \int_0^1 (2\eta - \eta^2) (\text{Pr} Le \theta_m A_m F_m + Sc \Lambda B_m G_m) d\eta$$

550

(46)

551 According to eq. 43, the first relation between  $A_n$  and  $B_n$  can be  
552 obtained in eq. 44, while the second relation is expressed by  
553 either eq. 30 or eq. 31.

554

$$555 \quad \int_0^1 (2\eta - \eta^2) (\text{Pr} Le A_n^2 F_n^2 + Sc \Lambda B_n^2 G_n^2) d\eta = \int_0^1 (2\eta - \eta^2) (\text{Pr} Le \theta_m A_n F_n + Sc \Lambda B_n G_n) d\eta \quad (47)$$

556

557 Finally,  $A_n$  and  $B_n$  are solved for as follows:

558

559  $A_n = B_n \frac{G_n(1)}{F_n(1)}$  (48)

560  $B_n = \frac{\int_0^1 (2\eta - \eta^2) \left( \text{Pr} Le \theta_{in} \frac{G_n(1)}{F_n(1)} F_n(\eta) + Sc \Lambda G_n(\eta) \right) d\eta}{\int_0^1 (2\eta - \eta^2) \left( \text{Pr} Le \frac{G_n^2(1)}{F_n^2(1)} F_n^2(\eta) + Sc \Lambda G_n^2(\eta) \right) d\eta}$  (49)

561

562 As a result, temperature and mass fraction fields are expressed  
563 in eq.s 50 and 51.

564

565  $T(\varepsilon, \eta) = T_w + (T_e - T_w) \sum_{n=1}^{\infty} \left[ A_n \sum_{i=0}^{\infty} (a_{n,i} \eta^i) e^{-\lambda_n^2 d^* \left( \frac{4WR}{3Re} \right)^{4/3} \int_0^\varepsilon \sin^{1/3} \pi e d\varepsilon} \right]$  (50)

566  $\omega(\varepsilon, \eta) = \omega_e + (\omega_{in} - \omega_e) \sum_{n=1}^{\infty} \left[ B_n \sum_{i=0}^{\infty} (b_{n,i} \eta^i) e^{-\lambda_n^2 d^* \left( \frac{4WR}{3Re} \right)^{4/3} \int_0^\varepsilon \sin^{1/3} \pi e d\varepsilon} \right]$  (51)

567

#### 568 4. Results

569 The following analysis is performed for a set of representative  
570 operative conditions of the absorber in a cooling system (Table  
571 1) and LiBr-H<sub>2</sub>O solution properties (49) are calculated for the  
572 values of temperature, pressure, and mass fraction. Subsequently,  
573 the main influential dimensionless parameters are calculated and  
574 listed in Table 2.

575

576

**Table 1. Operative conditions**

T <sub>in</sub> (°C)	T <sub>w</sub> (°C)	ω <sub>in</sub> (%)	P (kPa)	r (m)	β Ref. (39)
40	32	60	1.0	0.0090	32°

577

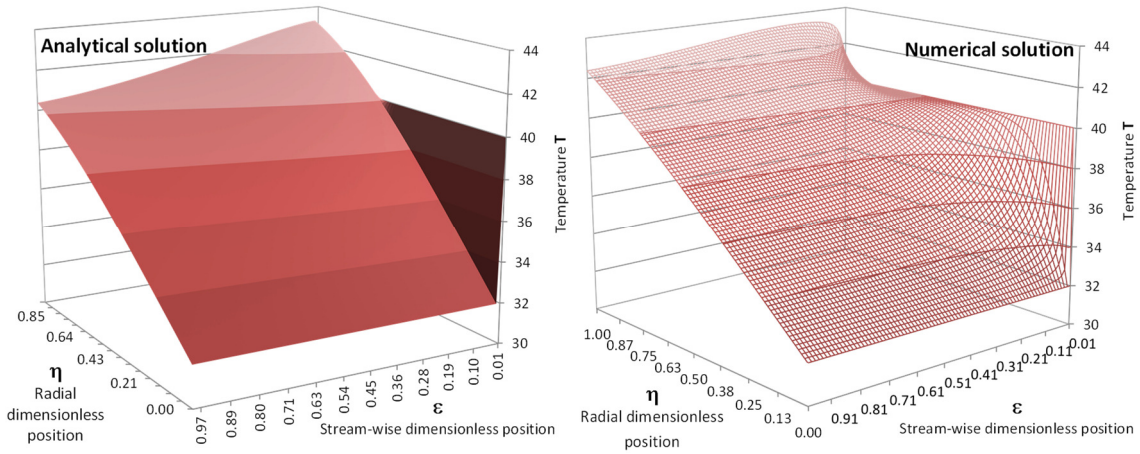
578

**Table 2. Operative dimensionless parameters**

Le	Λ	Sc	Pr	d*	Re	Re <sub>0</sub>
110.8	5.515	2567	23.17	568.4	42.95	95.00

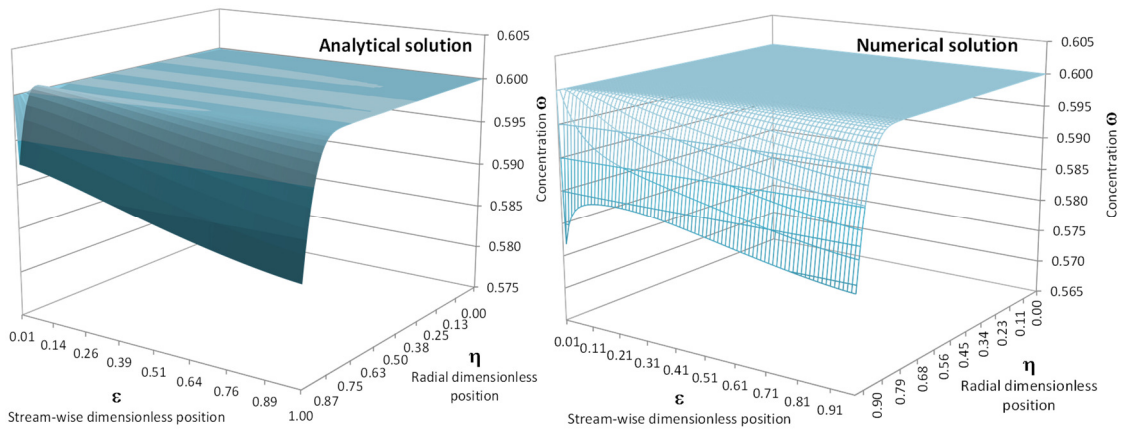
579

580 Figures 3(a) and 3(b) compare temperature and mass fraction  
 581 fields, respectively, as obtained with the first 9  
 582 eigenvalues/eigenfunctions of the present analytical solution  
 583 (Table 3) to the corresponding numerical solutions of energy and  
 584 species transport equations. Both fields indicate good agreement.  
 585 However, the temperature distribution specifically appears as a  
 586 rough approximation at the entrance region in proximity to the  
 587 wall ( $\epsilon \sim 0$ ), where the highest deviations with respect to the  
 588 numerical results are observed.  
 589



590  
 591

(a)



592  
 593

(b)

594 **Figure 3. Film temperature (a) and mass fraction (b) fields in the operative conditions listed in**  
 595 **Table 1**

596

597 It is noteworthy to highlight the agreement between the two  
 598 solution methods at the film interface, where two different

599 equilibrium relations are employed. Equation 23 is used for  
 600 writing the analytical solution, whereas, the thermo-physical  
 601 properties from (49) are used when numerically solving eq.s 10  
 602 and 11. A larger number of eigenvalues and terms representing the  
 603 eigenfunctions  $F_n$  and  $G_n$  are considered, and it is possible to  
 604 model the entrance region with increased accuracy. However, in  
 605 the case of a subcooled or superheated inlet solution, given the  
 606 very small values of the coefficient  $B_n$  for eigenvalues higher  
 607 than 9 (Table 3), which goes beyond the number of significant  
 608 figures available on the calculation platform, this creates  
 609 instability of the analytical solution away from the wall and  
 610 specifically close to the film interface ( $\eta=1$ ).  
 611 The temperature field close to the tube surface obtained with the  
 612 first 14 eigenvalues (Table 4) is compared to the corresponding  
 613 numerical solution in Figure 4. It is observed that this enables  
 614 the analytical solution to model the gradual transition of the  
 615 temperature distribution at the entrance region in proximity to  
 616 the wall. Hence, the heat transfer at the tube surface is  
 617 estimated by considering 14 eigenvalues as listed in Table 3.

618

619

**Table 3. Eigenvalues and eigenfunction coefficients**

n	$\lambda_n$	$A_n$	$B_n$
1	0.0418	0.129	1.34
2	0.116	0.133	-0.551
3	0.189	0.154	0.369
4	0.259	0.176	-0.275
5	0.326	0.168	0.196
6	0.392	0.113	-0.121
7	0.462	0.0536	0.0610
8	0.533	0.0194	-0.0243
9	0.607	0.00328	0.00440
10	2.26	1.28	-9.00E-45
11	3.06	-0.368	-1.00E-70
12	3.91	1.26	-3.00E-45
13	4.72	-0.504	-1.00E-107
14	5.53	1.27	-8.00E-121

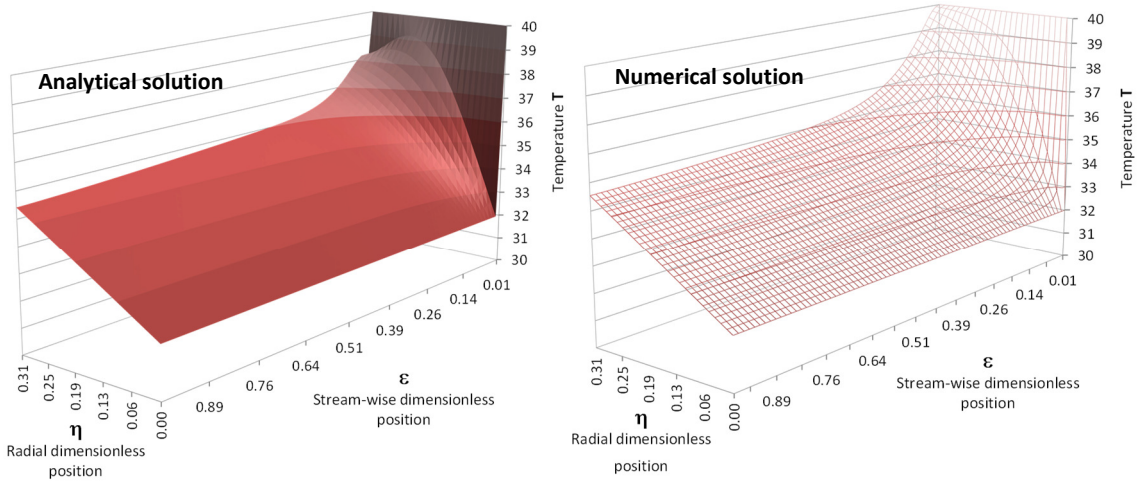


Figure 4. Film temperature field in proximity to the tube wall

621

622

623

### 624 5. Heat and mass transfer coefficients

625 It is assumed that the reduction of the surface in the vapour  
 626 absorption is represented by the values of  $WR$ , and thus the local  
 627 heat and mass transfer coefficient ( $htc$  and  $mtc$ ) are defined by  
 628 eq. 52 and eq. 53, respectively, and by eq. 54 and eq. 55,  
 629 respectively, with respect to the dimensionless parameters (i.e.,  
 630 Nusselt and Sherwood Numbers).

631

$$632 \quad htc = WR \frac{k \left. \frac{\partial T}{\partial y} \right|_w}{T_{av} - T_w} \quad (52)$$

$$633 \quad mtc = -WR \frac{D \left. \frac{\partial \omega}{\partial y} \right|_{if}}{\omega_w - \omega_{if}} \quad (53)$$

$$634 \quad Nu(\varepsilon) = \left( \frac{4 WR^4 \sin \pi \varepsilon}{3 Re} \right)^{1/3} \frac{\sum_{n=1}^{\infty} \left[ \frac{G_n(1)}{F_n(1)} B_n a_{n,1} e^{-\lambda_n^2 d^* \left( \frac{4WR}{3Re} \right)^{4/3} \int_0^\varepsilon \sin^{1/3} \pi \varepsilon d\varepsilon} \right]}{\sum_{n=1}^{\infty} \left[ \frac{G_n(1)}{F_n(1)} B_n \sum_{i=0}^{\infty} \left( \frac{a_{n,i}}{i+1} \right) e^{-\lambda_n^2 d^* \left( \frac{4WR}{3Re} \right)^{4/3} \int_0^\varepsilon \sin^{1/3} \pi \varepsilon d\varepsilon} \right]} \quad (54)$$

$Sh(\varepsilon) =$

$$\begin{aligned}
 & \left( \frac{4WR^4 \sin \pi \varepsilon}{3 \text{Re}} \right)^{1/3} \sum_{n=1}^{\infty} \left[ B_n \sum_{i=1}^{\infty} (ib_{n,i}) e^{-\lambda_n^2 d^* \left( \frac{4WR}{3\text{Re}} \right)^{4/3} \int_0^{\varepsilon} \sin^{1/3} \pi \varepsilon d\varepsilon} \right] \\
 & \left\{ \omega_e + (\omega_m - \omega_e) \sum_{n=1}^{\infty} \left[ B_n \sum_{i=0}^{\infty} (b_{n,i}) e^{-\lambda_n^2 d^* \left( \frac{4WR}{3\text{Re}} \right)^{4/3} \int_0^{\varepsilon} \sin^{1/3} \pi \varepsilon d\varepsilon} \right] \right\} \sum_{n=1}^{\infty} \left[ B_n \sum_{i=1}^{\infty} (b_{n,i}) e^{-\lambda_n^2 d^* \left( \frac{4WR}{3\text{Re}} \right)^{4/3} \int_0^{\varepsilon} \sin^{1/3} \pi \varepsilon d\varepsilon} \right] \\
 & \hspace{20em} (55)
 \end{aligned}$$

636

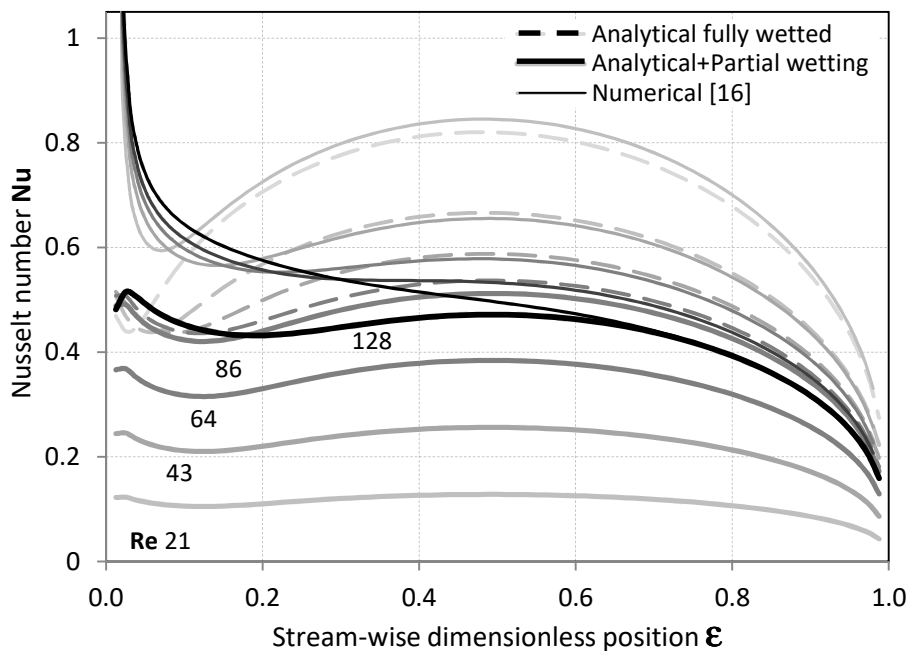
637

638 The denominators of these last two expressions represent the  
 639 driving potentials for heat transfer and that for mass transfer,  
 640 respectively; in the analytical formulation of the Nusselt  
 641 number, corresponding to the temperature difference between the  
 642 bulk value of the liquid film and the solid wall; in the  
 643 expression of the Sherwood number, the difference between the  
 644 mass fraction at the interface and at the tube wall. On the  
 645 right-side of the expressions, the numerators include terms  
 646 corresponding to the temperature gradient at the tube wall and  
 647 the mass fraction gradient at the film interface. Hence, the  
 648 factors on the extreme left-side embody the products of the  
 649 active extension of the film interface and the inverse of the  
 650 variation of the film thickness while normalised with respect to  
 651 the characteristic length  $L_c$ .

652 First, the inferences of the main parameters are locally  
 653 examined for the reference conditions of the absorber as listed  
 654 in Table 1, and the results obtained are compared while  
 655 considering the effect of partial wetting (continuous lines)  
 656 with the solution obtained when the effect is ignored (dashed  
 657 lines). Figure 5 describes the local Nusselt number distribution  
 658 along the tube surface. The large temperature difference between  
 659 the tube wall and the impinging solution at the entrance region  
 660 is responsible for a local peak in the Nusselt number.  
 661 Additionally, a local maximum that is positioned in proximity of  
 662 the vertical part of the tube ( $\varepsilon \sim 0.5$ ) is ascribed to the minimum  
 663 film thickness. Conversely, in the second half of the tube, the

664 thickening of the film is associated to a decreasing trend of  
 665 the local Nusselt number. It is also stated that higher  
 666 flowrates extend the region affected by the development of the  
 667 thermal boundary-layer and are responsible for moving the first  
 668 local minimum of the heat transfer coefficient to higher stream-  
 669 wise positions. This trend matches the trend presented in extant  
 670 studies when the governing equations of horizontal tube falling  
 671 film absorption are numerically solved (16), and the highest  
 672 deviation occurs in proximity of the inlet of the calculation  
 673 domain in which the temperature gradient is steeper due to the  
 674 boundary condition of constant tube wall temperature. The  
 675 discrepancy between the analytical solution and the numerical  
 676 solution of the governing equations (eqs. 10-11) increases when  
 677 the solution flowrate increases. The remaining deviations are  
 678 related to the assumption of a linear equilibrium-relationship  
 679 at the interface.

680



681

682 **Figure 5. Local Nusselt number corresponding to the first 14 eigenfunctions for different solution**  
 683 **mass flowrates at the reference conditions of a refrigerating machine**

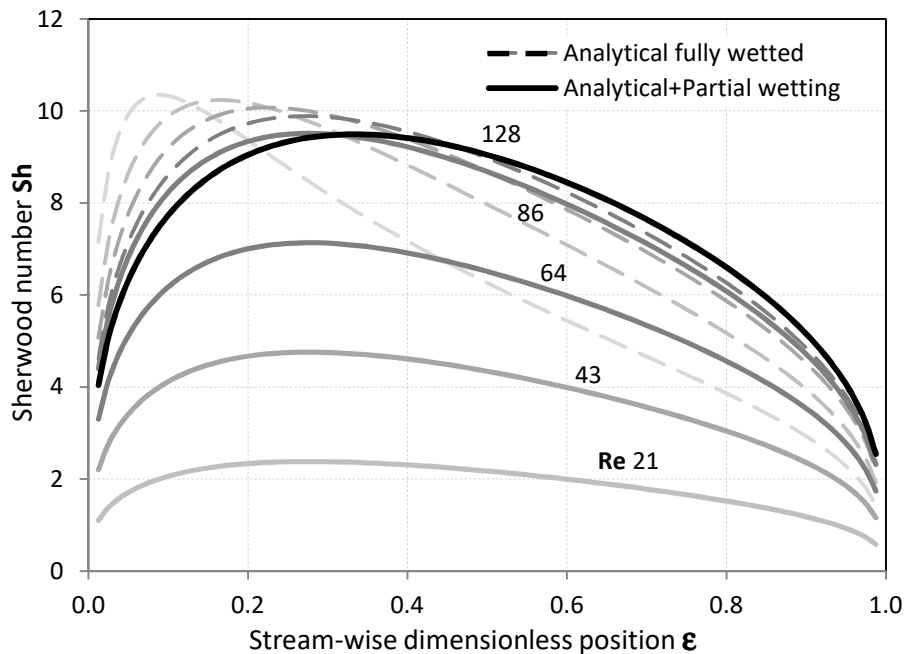
684

685 Figure 5 shows a comparison of continuous and dashed lines of  
 686 corresponding colours and highlights that low Reynolds

687 conditions are associated with a globally higher heat transfer  
 688 rate if partial wetting is overlooked while a gradual reduction  
 689 in the heat transfer coefficient that is mainly related to the  
 690 decreasing wetting ability of the solution is experimentally  
 691 observed (7-11).

692 In figure 6, the mass transfer at the film interface is locally  
 693 considered in terms of Sherwood number and indicates a maximum  
 694 value that grows and moves forward when the solution flowrate  
 695 increases in the partial wetting region (as shown by the  
 696 continuous lines).

697



698

699 **Figure 6. Local Sherwood number for different solution mass flowrates at the reference conditions**  
 700 **of a refrigerating machine**

701

702 Table 4 displays the eigenvalues and their respective  
 703 eigenfunctions coefficients for two different temperatures at  
 704 the tube wall of the absorber. A change in this parameter causes  
 705 the eigenvalues from the characteristic equation (eq. 32) and  
 706 eigenfunctions coefficients to assume different values.

707

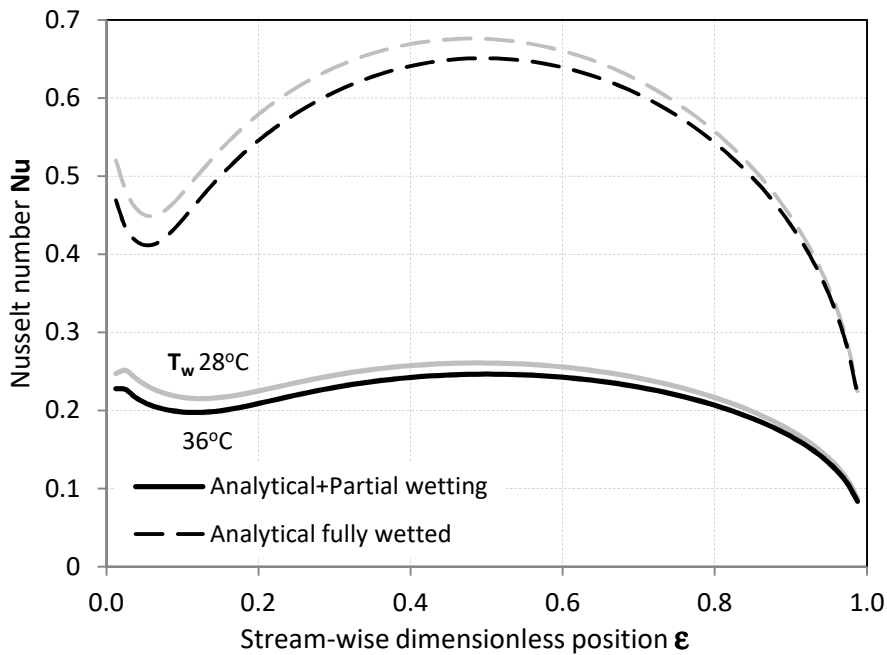
**Table 4. Eigenvalues and coefficients with wall temperatures corresponding to 28°C and 36°C**

n		$\lambda_n$	$A_n$	$B_n$		$\lambda_n$	$A_n$	$B_n$
1	$T_w 28^\circ\text{C}$	0.0424	0.103	1.35	$T_w 36^\circ\text{C}$	0.0409	0.171	1.33
2		0.118	0.112	-0.571		0.114	0.162	-0.517
3		0.191	0.144	0.407		0.186	0.156	0.310
4		0.262	0.199	-0.337		0.256	0.137	-0.196
5		0.327	0.231	0.271		0.325	0.0971	0.113
6		0.391	0.168	-0.185		0.395	0.0477	-0.0497
7		0.459	0.0819	0.105		0.465	0.00537	0.00543
8		0.531	0.0368	-0.0554		0.537	-0.0205	0.0207
9		0.605	0.0177	0.0296		0.609	-0.0340	-0.0345
10		2.23	1.40	-1.E-43		2.30	0.991	-3.E-46
11		3.03	-0.282	-9.E-70		3.10	-0.419	-1.E-71
12		3.83	1.24	-4.E-89		3.90	1.06	-8.E-91
13		4.63	-0.316	-4.E-106		4.70	-0.597	-4.E-107
14		5.45	1.20	-2.E-119		5.51332	1.18	-2.E-120

709

710 As a rule, a lower wall temperature enhances heat and mass  
 711 transfer both locally (Figures 7-8) and globally.

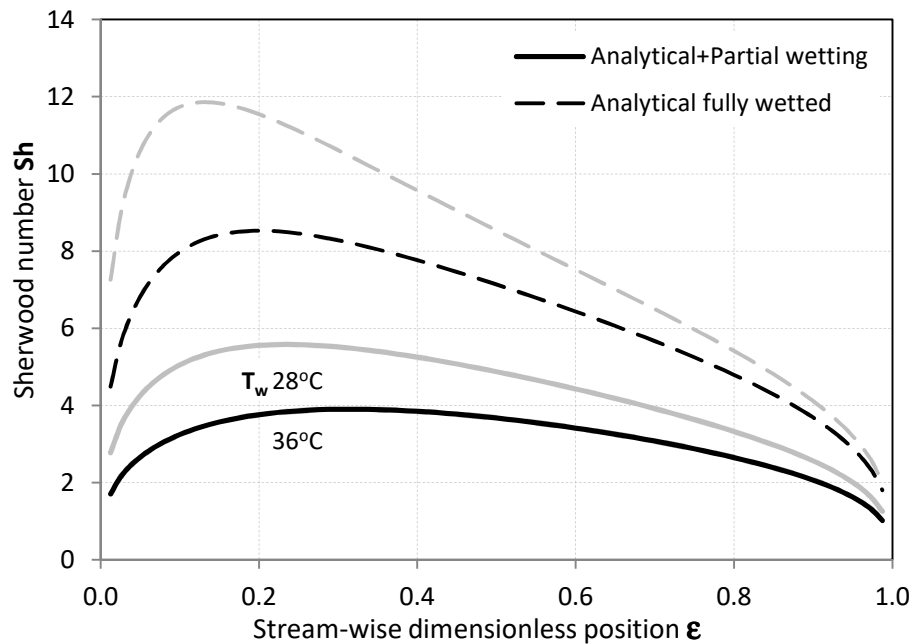
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713

714 **Figure 7. Local Nusselt number for different  $T_w$  at reference conditions of a refrigerating machine**

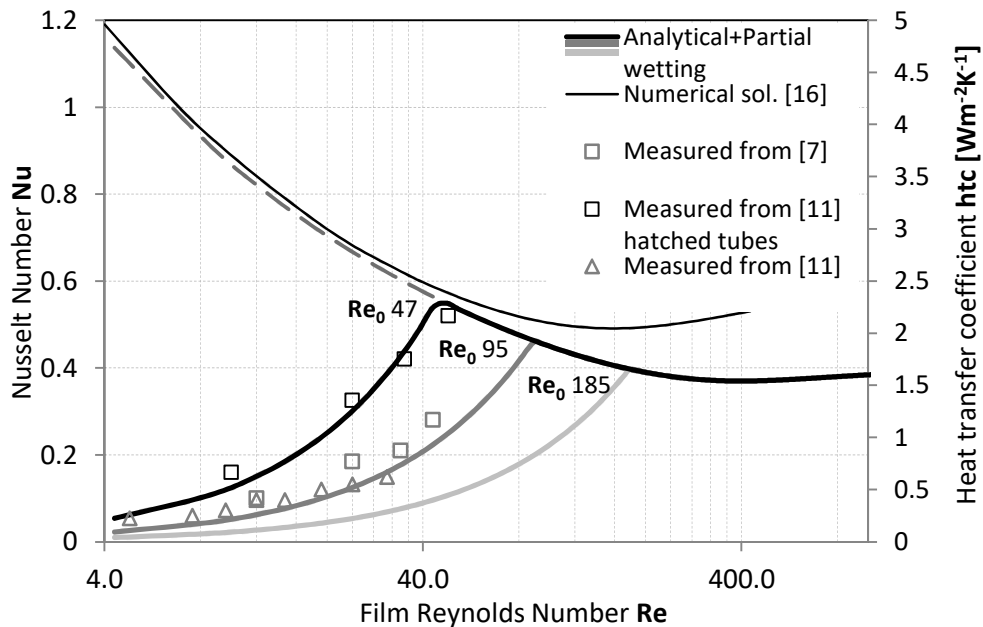
715



716  
 717 **Figure 8. Local Sherwood number for different tube  $T_w$  at reference conditions of a refrigerating**  
 718 **machine**

719  
 720 The wall temperature affects the Sherwood number through the  
 721 interfacial temperature and consequently changes the interface  
 722 mass fraction due to the equilibrium hypothesis. Therefore, a  
 723 lower heat sink temperature can significantly enhance the system  
 724 capacity by increasing the amount of refrigerant that steadily  
 725 circulates within the system for a specific solution flowrate.  
 726 A local analysis further suggests (50) that a lower tube radius  
 727 globally increases heat and mass transfer coefficients although  
 728 it reduces the heat flux per unit length due to a lower heat  
 729 transfer surface. Accordingly, the best selection of the tube  
 730 size results from a compromise between the conflicting effects.  
 731 The local values of  $htc$  and  $mtc$  around the tube are averaged to  
 732 perform a global analysis for the absorber tube in a wide range  
 733 of flowrates. Figures 9 and 10 show that heat and mass transfer  
 734 coefficients are maximised at a certain solution mass flowrate  
 735 based on the extension of the region affected by partial wetting.

736



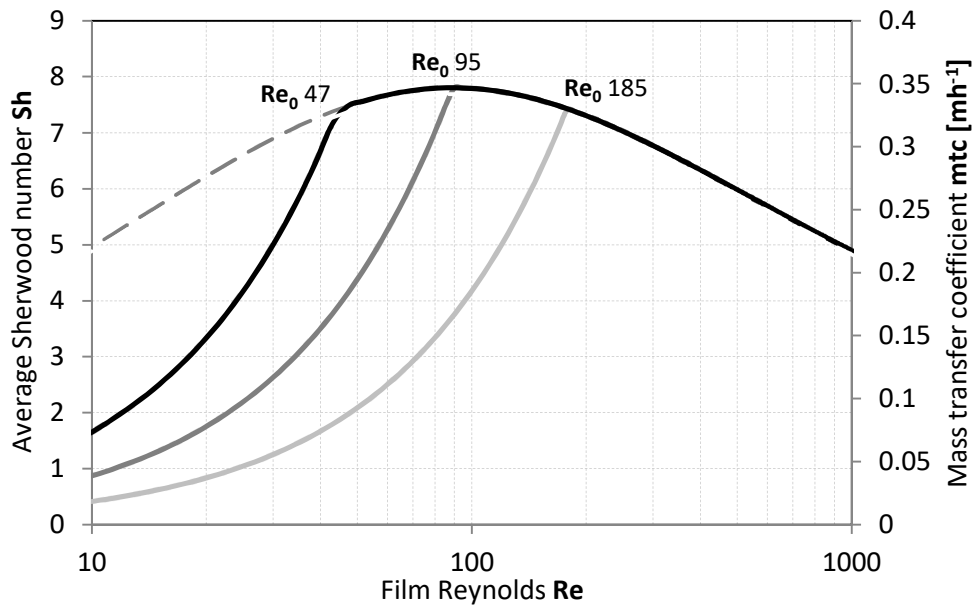
737

738 **Figure 9. Global Nusselt Number for different wetting behaviours at the reference conditions of a**  
 739 **refrigerating machine**

740

741 The wettability of LiBr-H<sub>2</sub>O solution (eq. 5) is increased if  
 742 tensioactive substances are added to the mixture to decrease the  
 743 surface tension  $\sigma$  at the vapour-liquid interface or if the solid  
 744 surface is properly treated (11) to lower the contact angle  $\beta$  at  
 745 the solid liquid interface. This stabilises thinner uniform  
 746 films (eq. 4) and moves the occurrence of the film breaking at a  
 747 lower Reynolds number  $Re_0$ . In contrast, if the affinity between  
 748 the tube surface and the solution worsens, dry patches also  
 749 appear at higher Reynolds numbers due to impurities or surface  
 750 roughness. These two cases are qualitatively represented by the  
 751 lines labelled as  $Re_0$  47 (the simulations are performed by  
 752 considering  $\beta'=\beta/2$ ) and  $Re_0$  185 ( $\beta''=2\beta$ ) in figures 9 and 10,  
 753 respectively, while  $Re_0$  95 represents the case of smooth tubes  
 754 at reference conditions for a Lithium-Bromide refrigeration  
 755 machine (Table 1). The dashed line and thin continuous line  
 756 represent the analytical solution and the numerical results  
 757 obtained, respectively, when partial wetting ( $WR=1$ ) over the  
 758 entire range of operative conditions is neglected.

759 Generally, it is highlighted that both heat and mass transfer  
 760 are critically improved by improving solution wettability. In  
 761 the case in which a partial wetting model is not included, the  
 762 simulated heat transfer coefficients follow an increasing trend  
 763 to decrease the solution mass flowrates. However, this behaviour  
 764 is in disagreement with all the experimental results indicated in  
 765 previous studies (5-11). This indicates the necessity to consider  
 766 partial wetting phenomena in the standard operative range of  
 767 absorbers operating in real plants.  
 768



769  
 770 **Figure 10. Global Sherwood number for different wetting behaviours at reference conditions of a**  
 771 **refrigerating machine**

772  
 773 **7. Conclusions**

774 The presented model for laminar falling film absorption over a  
 775 horizontal cooled tube considers the cylindrical shape of the  
 776 tube, the effect of partial wetting, thickness variation of the  
 777 film flowing around the tube, and arbitrarily selected inlet  
 778 conditions. A simplified linear model for partial wetting is  
 779 included to extend the validity of the obtained expressions when  
 780 complete wetting is not considered as a valid assumption. The  
 781 model provides detailed information to locally and globally

782 characterise heat and mass transfer of falling film absorbers.  
783 The effects related to partial wetting and the main geometrical  
784 and operative parameters are investigated to extract general  
785 guidelines to optimise the aforementioned devices.  
786 Low Reynolds conditions are associated with a globally higher  
787 heat transfer rate when partial wetting is overlooked.  
788 Conversely, a gradual reduction in the heat transfer coefficient  
789 that was mainly related to the decreasing wetting ability of the  
790 solution was experimentally observed in previous studies. In  
791 general, the results highlight that both heat and mass transfer  
792 are critically improved by improving solution wettability.  
793 The study indicates the possibility of an optimal tube radius  
794 from a compromise between lower heat flux per unit length and  
795 higher heat and mass transfer coefficients.  
796 Average heat and mass transfer coefficients around the tube are  
797 analysed in a wide range of flowrates and show that heat and  
798 mass transfer coefficients are maximised at a certain solution  
799 mass flowrate based on the extension of the region affected by  
800 partial wetting.  
801 Given the observed qualitative and quantitative agreements, it is  
802 possible to employ the model as a computationally light and  
803 accurate module in component and system simulations to design and  
804 control actual systems.

805

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