Narrative dynamics in social groups: a discrete choice model

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(Dated: May 2, 2018)

ABSTRACT

Individuals follow different rules for action: they react swiftly, grasping the short-term advantages in sight; or they waste cognitive resources to complete otherwise easy tasks, but are also able to plan ahead future complex decisions. Scholars from different disciplines studied the conditions under which either decision rule may enhance the fitness of its adopters, with a focus on the environmental features. However, we here propose that a crucial feature of the evolution of populations and their decision rules is rather inter-group interactions. Indeed, we study what happens when two groups support different decision rules, encapsulated in narratives, and their populations interact with each other. In particular, we assume that the payoff of each rule depends on the share of both social groups which adopts such rule. We then describe the most salient dynamics scenarios and identify the conditions which lead to chaotic dynamics and multistability regimes.

LEAD PARAGRAPH

The aim of this work is to investigate how the interaction of two social groups whose members carry different worldviews may affect their adoption. In particular, we find that even if the members of the two social groups have institutions and norms favouring either one of the two worldviews, here called narratives, they might adopt the other one due to the interaction with members from the other group. We also study some cyclic and chaotic dynamics, that is situations in which the narratives in the social groups change at every new period or are otherwise very sensitive to exogenous noise and initial conditions.

I. INTRODUCTION

In recent debates of economic psychology and cognitive neuroscience, a well-known position is the one called dualism [10]. This theory postulates the existence of two parallel modes of cognitive functioning. On the one hand, there is the "experiential", or "emotional" cognitive mode, otherwise simply labelled as "System 1". It represents many non-rational decisional aspects, e.g. affective, intuitive, associative, non-verbal, metaphorical, narrative, and so on. By contrast, there is the "analytical-rational" side of cognition, also referred to as "System 2". It is based on conscious, slow, effortful, reflective, deliberative processes based on a formal reasoning [13]. A recurring thesis in the dualist literature suggests that these two types of problem solving rules should not be misinterpreted as hierarchically ordered. In other words, one is not better than the other and analytical cognition does not outperform the experiential one, as the human behaviour is more than the result of the accumulation of rules. Indeed, humans often rely on heuristics, which are simple decision rules that discard part of the available information in order to limit the time employed to take decisions and thus saving on cognitive resources [11, 12]. In this perspective, System 1 heuristics are developed in order to increase the fitness with the surrounding environment and may thus be ecologically rational. This evolutionary argument is supported by Slovic and Peters [19], who attribute the alternate luck of the two cognitive modes to the historical and ecological setting of the human population. In a prehistoric context, in which humans had to discern dangers from safe elements in their environment, System 1 heuristics proved fundamental in providing quick responses to basic needs. In later times, as humans started to live in ever more complex and dense societies and their artefacts allowed them to exercise increased control of the resources, System 2 thinking enhanced human potential. It is relevant to underline that we employ narratives as the main conceptual tool for our theoretical framework because we consider narratives to function at a deeper level of the human mind, identifying the meaningful causal links on which System 1 or System 2 thinking may be applied. Whether an individual privileges one type of thinking with respect to the other, she would still apply it to the same problem in order to achieve the same goal. If System 1 and System 2 represent two different ways to choose a Strategy in Risk! [1], a narrative is the available set of moves and their expected consequences which altogether define the game to our eyes. Beside this illustrative analogy, a narrative also carries with it the potential for creating a social bond: seeing each other facing the same problem with the same set of possible solutions lays the basis for our identification as a team [8].

The environment-bound evolution of cognition modes is also studied by Toupo et al. [20], who model the evolution of two sub-populations: one composed of Emotional minds (automatic agents, in their terminology) and the other by Cognitive minds (controlled agents). In this article, the authors characterise the two types of agents attributing short term advantages to the former and long term advantages to the latter. They then proceed to study how the environmental conditions influence the final composition of the overall population and classify the possible outcomes as follows: convergence to a single strategy; co-existence represented by internal fixed points; oscillation between one strategy and the other; bistability. In our work, similarly to the work by Toupo et al. [20], cognitive and emotive minds may affect each other's payoff. However, we also allow for two distinct social groups to exist, each with a decision rule which is relatively advantageous to adopt. Moreover, the analytical framework we provide could be used to more generally study the juxtaposition of two personal features or behavioural rules. Here we focus on narratives due to the role that such concise messages have in delivering suggestive representations of

[1] For anyone who is not familiar with Risk!, it is one of the most widespread board game, in which players aim to reach indvidually specified objectives.

the world which can effectively alter the behaviour of humans on the one hand and construct a sense of shared identity on the other hand [7, 8]. Nonetheless, we do not exclude that the dynamics of other conceptual couplets might be investigated with the present model. In particular, our model could prove to be useful in studying binary choices conditioned by negative externalities from other agents. Indeed, there exist several contributions building on the seminal paper of Schelling [18], some of whom presenting adoption processes that may give rise to chaotic dynamics (for instance, see [3, 6]). However, these works assume that social interactions take place among individuals belonging to the same population (that is, all individuals have the same payoff structure). Our paper considers a context in which individuals are heterogeneous in terms of payoffs -two groups of individuals exist-, although they have at disposal the same set of choices (two alternative narratives). In such a context, we aim to show how the interplay between the adoption processes of choices in the two groups may give rise to complex dynamics.

For the sake of simplicity, we identify the two groups as *North* and *South*, with no reference intended to any real context. We thus assume that northerners are emotive minds, whereas southerners are cognitive minds. In the terms of the works mentioned so far, northerners are innate System 1 thinkers, whereas southerners mainly employ System 2 processes. In the light of [7], we then assume that the individuals in our model may also adopt different narratives, which are culturally dense representations of the world, i.e. socially constructed and maintained beliefs on the causal links of the world. We name the two alternative narratives cognitive and emotive as well, as each of them resonates with its corresponding type of mind. We might think of the Enlightenment, the Logical positivism, or the Rational choice as some of the most important cognitive narratives in human history, as they all supported a rational representation of the world. By contrast, the Romanticism, the Decadent movement or the Intuitionism, may be seen as instances of emotive narratives. In recent times, an example of opposed emotive and cognitive narratives is provided by the clash of post-truth and technocratic narratives, respectively. Indeed, we are witnesses of the rise of so-called alternative facts and anti-establishment narratives on the one hand and the scientification and de-politicisation of political issues on the other hand [15, 21]. They both offer clear, overly simplistic explanations of the world that can be synthesized and proposed in concise messages: "The system robs us" vs. "There is no alternative". Furthermore, this example also highlights that the causal links embedded in a narrative do not need to be truthful in order to be adopted, as the social and cultural component of a narrative could prove even more relevant to its adopters, to the point that undermining the causal links of a narrative might prove ineffective or even counterproductive [1]. This exemplification of opposed narratives is not the subject of our study, but serves to show the relevance of the topic in the public discourse. We shall thus concern with two generic emotive and cognitive narrative, with no further characterisation to allow for the generalisation of our results.

Since we are interested in studying the diffusion of narratives beyond the boundaries of the groups in which they are mostly accepted, we give to both cognitive and emotive individuals the possibility to adopt either narrative, independently of what type of mind they are. We find that even if the members of the two social groups have institutions and norms favouring either one of the two worldviews, here called narratives, they might adopt the other one due to the interaction with members from the other group. Note that we drop the word "strategy" in this work to refer to the adoption of either narrative, since individuals do not interact in strategic games nor they form expectations on their payoffs on the basis of such one-to-one interactions. Individuals are assumed to behave adaptively according to the features of the system they are embedded in and the externalities entailed in the behaviour of others. In addition, we should highlight that we do not deal with migration, i.e. shifts from one group to the other. Northerners may still decide to adopt a cognitive behaviour (e.g. Strategy 2 in [20]), but their inner nature would remain emotive. Indeed, we here disjoint the social group an individual belongs to from her narrative, so that a cognitive mind may adopt an emotive narrative without turning into an emotive mind. In the language of psychologists, each person has and ultimately maintains a certain personality, but may adopt a different narrative in the course of her life. In the longer term, a large diffusion of a cognitive narrative might even affect the personality traits of emotive minds. In the shorter term, the possibility to change narratives throughout her lifetime allows an individual to interact with different personalities, e.g. a cognitive mind interacting with an emotive one [17].

Let us think, as an example, to a meeting between a sculptor and an architect. Although the former may be an emotive mind and the second a cognitive one, the two may be able to adopt either narrative. In particular, they may attempt to converge on a common narrative when they are jointly commissioned a work from a patron. Whether it is better for their collaboration that they both maintain their narrative or coordinate on either of the two depends on the specific nature of their collaboration. It could be better for both of them to exploit their narrative divergences or, for instance, the architect might find it more fruitful to abandon his views on functionality, understanding they are constraining the aesthetic value of the work. This behavioural adaptation does not come without a cost, though. For instance, in the latter case the architect might have a harder time obtaining a commission for a bridge, which likely requires a more cognitive perspective. We do not intend this simplification of human types as a truthful description of the spectrum of features that characterises the human beings, but we rather use this example to stylise some basic narrative dynamics exploiting its convenient limitations (e.g. the presence of only two individuals makes for an easier categorisation). More in general, a cognitive mind may have appeased her emotive-behaving peers by acting emotive herself, but she may now annoy cognitive individuals she had no friction with beforehand. Indeed, individuals may not adapt their narrative depending on the people they interact with: once they adopt either narrative, they maintain it with any other agent they encounter. This derives from the fact that a narrative is neither a strategy nor an action, it is a description of the world which underlies the mind of an agent in her every action. When they abandon their narrative to adopt a new one, they will uphold it with anyone else. We characterise individuals by means of this "procedural friction" because we are interested in decisional framings and rules that have deep cultural or institutional roots in the personality of individuals, inhibiting frequent context-related adaptations.

II. THE THEORETICAL FRAMEWORK

In this work, individuals adopt the narrative which enhances their performance, given the share of members of both their social group and of the other group who adopt the same narrative. In other terms, individuals live adapting to the set of values and norms that their home narrative represents. Successful interaction with their community as a whole is also based on the extent to which they share such narrative with others. Moreover, whichever narrative they decide to adopt, they will carry with them the culture and norms it embeds also when interacting with individuals from another social group, whose population might in turn be exposed to this new set of values and practices. We can now see how every inter-group interaction is an opportunity for individuals from both groups to consider adopting a different narrative.

Analytically, we can break down what is relevant to the payoff of a narrative into three different parts: cultural and institutional environment; within-group externalities; cross-group externalities or mixed meetings. The first part refers to how advantageous it is for an individual to adopt a narrative that is embedded in the culture and in the institutions of her social group. Northerners, for instance, may have norms that arose to enhance the life of individuals with an emotive narrative, and foster the transmission of beliefs sustaining it to new individuals (for a review on social or cultural learning, see [16]). This part does not depend on how many adopt the same narrative in either group at the current time. The second part represents the influence that members of the same social group have on each other. On this regard, we keep an open mind on whether individuals may prefer that their fellow members adopt the same narrative or not. Indeed, we here investigate also scenarios in which two members of the same group, e.g. two southerners, would prefer the other to adopt different narratives. Finally, the third part concerns with the influence that members from the other social group have. For instance, a southerner may benefit from the fact that both she and the population of the North have a cognitive narrative, because she can coordinate more easily in her "mixed meetings", i.e. her exchanges. By contrast, people from the North may negatively be influenced by southerners who adopt their emotive narrative, for example because it leads to a perceived loss in *authenticity* of the narrative. Also in this case, we do not impose any structure to inter-group influence and investigate a variety of scenarios.

III. THE MODEL

As the payoffs for individuals vary depending on whether they are from the North or the South and whether they adopt the cognitive or the emotive narrative, we derive four different payoff functions corresponding to this combinations. As concerns the

North, we have:

$$E_N = a_1 + b_1 x + c_1 y \tag{1a}$$

$$C_N = a_2 + b_2 x + c_2 y \tag{1b}$$

where E_N and C_N are the payoff functions for northerners adopting an emotive or a cognitive narrative, respectively. The payoffs depend on variables x, which identifies the share of northerners adopting the emotive narrative, and y, which identifies the share of southerners adopting the cognitive narrative. To the right hand side of the equations, we may see the three parts we hinted to beforehand: the cultural environment; the within-group externalities; the cross-group externalities. In particular, a_1 and a_2 represent the fixed effect of the cultural and institutional environment of the North. Note that it affects differently individuals adopting different narratives. The intermediate terms b_1 and b_2 measure the within-group externalities: the former relates to the effect of other northerners adopting the emotive narrative and the latter refers to the cognitive narrative. Both are weighted by the share x of individuals in the North who adopt the emotive narrative, which we recall being the favoured one. The last term of the equations represents the cross-group externalities, with c_1 and c_2 measuring the effect of southerners adopting the cognitive narrative. Analogously, we derive the payoff functions for individuals from the South:

$$E_S = \alpha_1 + \beta_1 y + \gamma_1 x \tag{2a}$$

$$C_S = \alpha_2 + \beta_2 y + \gamma_2 x \tag{2b}$$

where α, β and γ are the symmetrical equivalent of a, b, and c. Note that now the share y of individuals adopting the emotive narrative in the South is used to weigh within-group externalities whereas x weighs cross-groups externalities. We now proceed to compute the difference in the narrative payoffs for the individuals in the North:

$$\Pi_N = E_N - C_N = a + bx + cy \tag{3}$$

where $a_1 - a_2 = a > 0$ represents the cultural and institutional advantage that emotive minds have in adopting an emotive narrative in the North. By contrast, $b_1 - b_2 = b$ and $c_1 - c_2 = c$ have no determined sign to allow for the greatest generality. We do the same for the payoffs of southerners:

$$\Pi_S = C_S - E_S = \alpha + \beta y + \gamma x \tag{4}$$

with $\alpha_2 - \alpha_1 = \alpha > 0$ as southerners adopting the emotive narrative are relatively at a disadvantage with respect to their peers adopting the cognitive narrative. Also in this case, $\beta_2 - \beta_1 = \beta$ and $\gamma_2 - \gamma_1 = \gamma$ have no determined sign.

A. The diffusion of narratives

The payoffs of the two narratives are relevant to determine how they will diffuse in the two social groups. Indeed, we build on the discrete choice framework described by [4], who assume that the utility of every individual is random, with a noise component that is iid and has an exponential distribution. In our work, the utility of a generic individual i, at time t, is represented by the payoff functions described in the previous paragraph plus a noise $\mu_{i,t}$.

When considering an arbitrary large (infinite) number of individuals in the group, the probability x_t for each of them adopting the emotive narrative at time t converges to the Gibbs probability of the multinomial logit model:

$$x_t = \frac{e^{\rho E_{N,t-1}}}{e^{\rho E_{N,t-1}} + e^{\rho C_{N,t-1}}} \tag{5}$$

where ρ measures the responsiveness of individuals in switching narratives. By dividing by $e^{\rho E_{N,t-1}}$, we can rewrite (5) to include the payoff difference Π_N:

$$x_t = \frac{1}{1 + e^{\rho \Pi_{N,t-1}}} \tag{6}$$

An analogous law drives the diffusion of narratives in the South:

$$y_t = \frac{1}{1 + e^{\rho \Pi_{S,t-1}}} \tag{7}$$

In order to simplify the analysis we have assumed an identical measure for the responsiveness ρ of individuals belonging to both groups. This mechanism allows individuals from both North and South to choose their narrative adaptively, i.e. according to the payoffs they yield. However, it could be noted that in this formulation swift changes in narrative adoption could occur, which would not seem realistic. In order to better describe the phenomenon, we introduce an asynchronous updating process, as in [9]. In this way, we allow for a friction (or memory) ε to make individuals less eager to sudden narrative changes:

$$T : \begin{cases} x' := \varepsilon x + \dfrac{1-\varepsilon}{1 + e^{-\rho(a+bx+cy)}} \\ y' := \varepsilon y + \dfrac{1-\varepsilon}{1 + e^{-\rho(\alpha+\beta y+\gamma x)}} \end{cases} \tag{8}$$

where the map T is defined over the domain $[0,1] \times [0,1]$ and $'$ is the unit-time advance operator.

IV. HOMOGENEOUS INTERACTIONS OR "ISOLATED GROUPS"

We now proceed to analyse the dynamics of narrative diffusion when inter-group interactions do not occur or have otherwise no effect on the payoffs. In formal terms, we set $c = \gamma = 0$, so that the term relating to mixed meetings for both southerners and northerners is null. This allows us to study the dynamics of x and y separately, focusing on the relevance of within-group externalities. On this regard, for the social group of the North we find the following result.

Proposition 1: Let $c = 0$. The difference equation that describes the dynamics of the group from the North in (8) admits a unique fixed point x^* which lies in $(0, 1)$.

- If $b > 0$, x^* is globally asymptotically stable;
- if $\varepsilon = 0$ and $b < 0$ the dynamics converges either to x^* or to a 2-period cycle around x^*;
- if $\varepsilon > 0$ and $b < 0$ chaotic dynamics may arise.

Proof: With straightforward calculation it is possible to verify that the map describing the x-dynamics has, at most, 2 stationary points and an inflection point in the interval $(0, 1)$. Considering the behaviour of the map at 0 and at 1, the result about the number of fixed points follows. Regarding the stability properties, we can observe that if $b > 0$ the x-dynamics is described by an increasing map, while if $\varepsilon = 0$ and $b < 0$ the map is decreasing.

Remark: Studying the values of ρ, it is possible to obtain further stability properties of the fixed point. However, since we focus on the relevance of parameters a and b we do not develop this point.

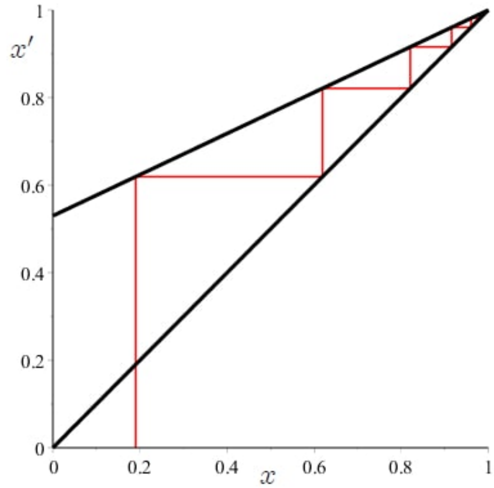

FIG. 1: Monotonic dynamics for x. All trajectories starting in the interval $[0, 1]$ converge to $x^* \in (0, 1)$. Parameter values: $a = 30$, $b = 4$, $\varepsilon = 0.47$, $\rho = 1$.

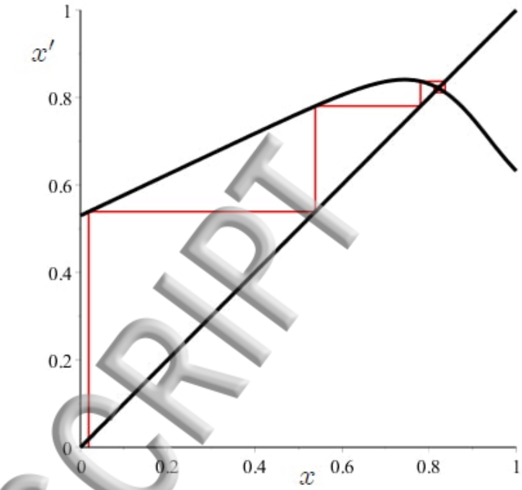

FIG. 2: Case $a > 0$, $b < 0$. Dynamics show damping oscillations around x^*. Parameter values: $a = 30$, $b = -32$, $\varepsilon = 0.47$, $\rho = 1$.

This result shows that if the northerners adopting the emotive narrative receive a positive externality from everyone else in the North who also adopts the emotive narrative, then the x-dynamics will ultimately converge to the stationary equilibrium x^*. Such equilibrium x^* will be very close to 1 if the payoffs of strategy E_N are sufficiently high with respect to those of strategy C_N. This well depicts the fact that individuals who adopt a narrative that is embedded in their culture and institutions would have no reason (save for errors) to change it if they also get positive externalities by doing so. This case, characterised by sufficiently high values of a and b, is represented in Fig. 1[2], where we can see that any initial condition would lead the population of the North to a very high x. A negative within-group externality, i.e. $b < 0$, may represent an "elitist" dynamics of narratives in the North, so that

[2] Parameters values for this and the following figures were chosen in order to illustrate salient dynamics phenomena from a mathematical or a theoretical point of view.

an increasing share of the population adopting one narrative erodes the advantages that the latter provides. In this perspective, individuals prefer to adopt either the emotive or the cognitive narrative when they are least diffused within their social group. Let us consider a couplet of narratives proposed by [7]: "Many hands make light work" vs "It's every man by himself". In a society in which the former narrative is most diffused, individuals will be highly collaborative and this will also be reflected in supportive institutions. In this context, a self-focused narrative such as "it's every man by himself" could lead an individual to relatively higher payoffs if it allows her to benefit from a supportive environment although incurring into lower personal costs for it. This is represented graphically in Fig. 2, where we can see that there is still a fixed point x^*, although it lies to the left with respect to the previous case and damping oscillations may be observed. If the negative externalities become stronger, chaotic dynamics may arise, as shown in Fig. 3. The bifurcation diagram in Fig. 4 summarises the dynamics of x for varying values of b. For values of b lower than the ones shown in the graph, we can observe alternating intervals of chaotic dynamics and low-period cycles. We underline that Proposition 1 holds symmetrically also for the South, where y and β substitute x and b, respectively.

As concerns the friction parameter ε, it is relevant to note that in this model there can also be a destabilising effect on the dynamics of the system (see [22] for a deeper analysis of the inertia). This means that if $b < 0$ and the dynamics is attracted by a 2-period cycle for $\varepsilon = 0$, an increase in ε can induce cyclical dynamics with longer period, and/or chaotic behaviour. Nonetheless, a value of ε close to 1 makes the stationary point stable. The reason is that the resistance to change, or inertia, by part of the northerners causes a continuous miscoordination within the North. In particular, if the values of a and b are not sufficiently high, the noise might induce many northerners to choose the non-favoured narrative, generating nonlinear, cyclical dynamics.

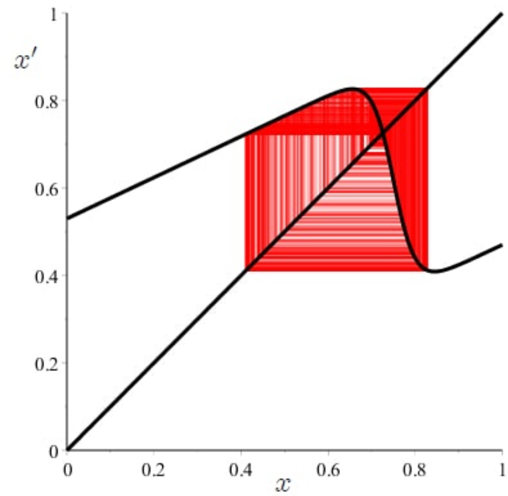

FIG. 3: Chaotic dynamics for x. In this case the narrative dynamics within groups does not reach a stationary state. Indeed, large shares of supporters for both narratives coexist, with the share adopting each narrative varying in time with an apparently random pattern. Parameter values: $a = 30$, $b = -40$, $\varepsilon = 0.47$, $\rho = 1$.

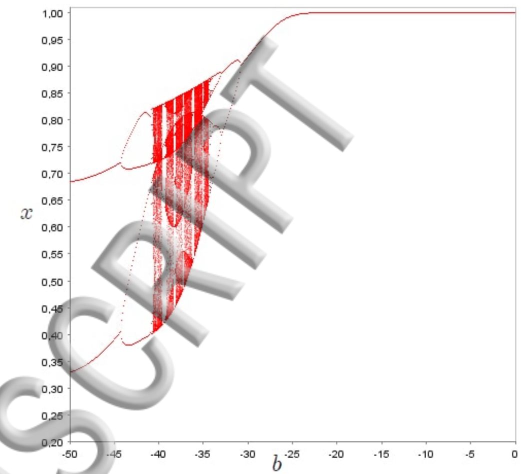

FIG. 4: Bifurcation diagram for b. We note that if the (negative) within-group externalities on payoff are sufficiently strong, chaotic dynamics may arise. Parameter values: $a = 30$, $\varepsilon = 0.47$, $\rho = 1$, b varies in the interval $(-50, 0)$.

We now analyse narrative dynamics if the two social groups are interacting and influencing each other. Formally, this translates into studying the cases in which $c \neq 0$, $\gamma \neq 0$, or both.

V. UNIVOCAL INFLUENCE BETWEEN NORTH AND SOUTH

Let us now consider the case in which only one of the social groups, say, the North, is influenced by the other group, whereas the South is indifferent to northern influences. Analytically, this is equivalent to assuming that $\gamma = 0$, so that the narrative dynamics in the South is unchanged with respect to the previous section. However, since $c \neq 0$, the dynamics of x in the North is influenced by how many of the southerners decide to adopt either narrative.

We may assume that the North and the South are isolated at first (i.e. $c = \gamma = 0$), and converging toward the fixed points x^* and y^*, respectively, presented in Proposition 1. If the North opens to influences from the South and c is positive, this phenomenon favours the process of adoption of the emotive narrative. In this case the system reaches a stationary state $(\widetilde{x}^*, y^*)$ with $\widetilde{x}^* > x^*$. Indeed, not only the emotive northerners benefit from an institutional environment designed on their narrative, but they are also advantaged by southerners adopting the cognitive narrative (the preferred one in the South). From a dynamical point of view, we note that as y converges to y^*, the asymptotic dynamics of x is described by the following map:

$$x' := \varepsilon x + \frac{1-\varepsilon}{1+e^{-\rho(a+bx+cy^*)}} \tag{9}$$

In this case, we may derive the following results on the fixed points for the North group.

Proposition 2: Let $\gamma = 0$ and y^* be the attracting fixed point for y-dynamics. For x-dynamics the following results hold:

- If $b > 0$ and either $cy^* + a > 0$ or $cy^* + a < -b$ then a unique globally asymptotically stable fixed point exists.
- If $b > 0$ and $cy^* + a \in (-b, 0)$ then one or three fixed points generally exist (that is, excluding cases in which tangent points between the graph of f and the 45-degree line exist). If three fixed points exists, the first

and the third ones are locally asymptomatically stable, the second one is unstable.

- If $b < 0$ then a unique fixed point exists which can be stable or unstable.

Proof: Notice that $f(0) = \dfrac{1-\varepsilon}{1+e^{-\rho(a+cy^*)}} \in (0,1)$ and $f(1) = \varepsilon + \dfrac{1-\varepsilon}{1+e^{-\rho(a+b+cy^*)}} \in (0,1)$. This implies that the graph of f starts above the 45-degree line and ends below it. Since f is continuous in $[0,1]$ it follows that at least one fixed point exists and generally an overall odd number of fixed points exists. In order to study their multiplicity, we consider the first and the second order derivatives of f which are $f'(x) = \varepsilon + \frac{(1-\varepsilon)\rho b e^{-\rho(a+bx+cy^*)}}{\left(1+e^{-\rho(a+bx+cy^*)}\right)^2}$ and $f''(x) = \frac{(1-\varepsilon)\rho^2 b^2 e^{-\rho(a+bx+cy^*)}\left(e^{-\rho(a+bx+cy^*)}-1\right)}{\left(1+e^{-\rho(a+bx+cy^*)}\right)^3}$, respectively. By direct calculation, we have that a) at most two local extrema exist in the interval $(0,1)$ and b) an inflection point exists in $(0,1)$ if and only if $\frac{cy^*+a}{-b} \in (0,1)$. This point, when exists, is unique. We can now distinguish two cases:

I Let $b > 0$, then f is increasing in the interval $[0,1]$. If $cy^* + a > 0$ or $cy^*+a < -b$, f'' doesn't change its sign in $(0,1)$. Therefore the uniqueness of the fixed point follows. By contrast, if $cy^* + a \in (-b, 0)$, considering the signs of the second order derivative, then one or three fixed points generally exist. The ones with an odd index are stable, the one with an even index, if it exists, is unstable.

II Suppose otherwise that $b < 0$. According to different configurations of the parameters, f may be monotone, unimodal or bimodal. If $cy^* + a < 0$ or $cy^* + a > -b$, f'' doesn't change its sign in $(0,1)$. By contrast, if $cy^* + a \in (0, -b)$ we have that $f''(0) = \frac{(1-\varepsilon)\rho^2 b^2 e^{-\rho(a+cy^*)}\left(e^{-\rho(a+cy^*)}-1\right)}{\left(1+e^{-\rho(a+cy^*)}\right)^3} < 0$ and $f''(1) = \frac{(1-\varepsilon)\rho^2 b^2 e^{-\rho(a+cy^*+b)}\left(e^{-\rho(a+cy^*+b)}-1\right)}{\left(1+e^{-\rho(a+b+cy^*)}\right)^3} > 0$. Considering the value of f at 0 and 1, in both cases a unique (stable or unstable) fixed point exists.

As outlined in Proposition 2, if the influence from the southerners adopting the cognitive narrative is comparatively disadvantageous for northerners adopting the emotive narrative, i.e. $c < 0$, more interesting dynamics may arise. Indeed, a negative c may lead to a destabilisation of the dynamics as well as to the emergence of multistability in the North. The bifurcation diagram in Fig. 5 shows the changes in the ω-limit set for the variable x as c varies. Even if in both groups the favoured narrative is widespread, i.e. x^* and y^* are stable and are very close to 1, the negative impact c from narrative divergence with the South can destabilise the North. In particular, we note that in some intervals the x-dynamics is chaotic. Moreover, for lower values of c the dynamics of x reaches lower values as well, being captured by a stable fixed point which is very close to 0. In this case, southern influence dramatically changes the dynamics in the North.

Moreover, starting from a case in which both groups are isolated and whose dynamics converge toward fixed points x^* and y^*, a negative influence from the South c may be the engine of a bistable regime. In this case, shown in Fig. 6, there exist two locally stable fixed points whose basins of attraction are separated by an unstable fixed point. Convergence towards either a lower or a higher fixed point depends on the value of x at the moment when the North starts being subject to the influence from the South. If the interaction with the South starts when the level of x is sufficiently high, then its long term dynamics will not change and it will converge to the higher fixed point. However, if this is not the case and x is sufficiently low when the interaction begins, x will converge to a stationary state with a value very close to 0. In other terms, if the share of northerners adopting their favourite, emotive narrative is high, a negative feedback from the interaction with the cognitive southerners will

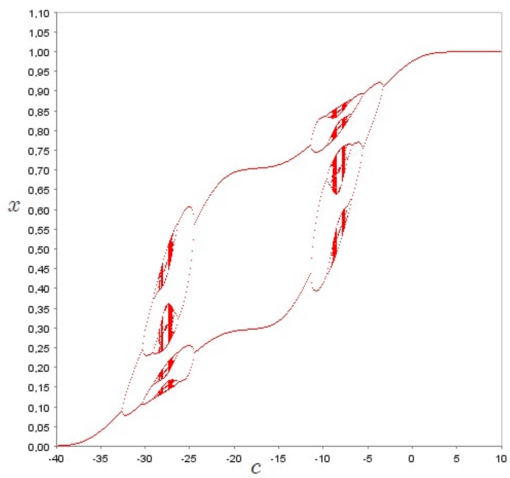

FIG. 5: Bifurcation diagram with respect to c. Parameter values: $a = 33$, $b = -30$, $\alpha = 10$, $\beta = 4$, $\gamma = 0$, $\varepsilon = 0.47$, $\rho = 1$, c varies in the interval $(-40, 10)$.

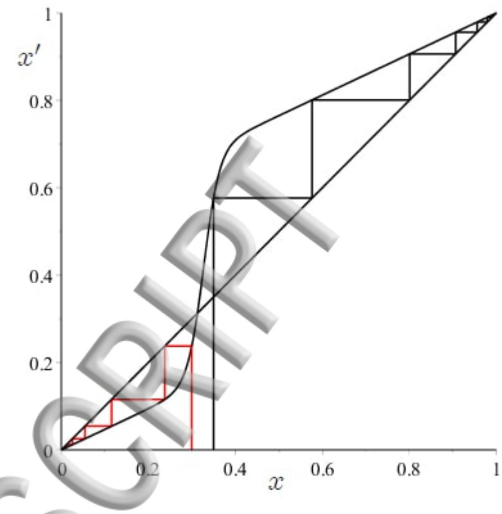

FIG. 6: Asymptotic x-dynamics (for $y = y^* \simeq 1$). The unstable equilibrium that separates the basins of attraction is $x \simeq 0.3129775350$. Parameter values: $a = 10$, $b = 55$, $c = -18$, $\alpha = 10$, $\beta = 30$, $\gamma = 0$, $\varepsilon = 0.47$, $\rho = 1$.

be absorbed. By contrast, if the population of the North is still undecided between the emotive and the cognitive narrative, convergence to the first may be reversed so that the northerners will end up adopting a cognitive narrative under the influence of the southerners.

Until now we analysed the effect of southern influence on the North when the dynamics of the two groups converges toward the fixed points x^* and y^*. In Fig. 7 we show the effect of the southern influence when the dynamics in the South is characterised by a stable 2-period cycle, whereas the dynamics in the North is attracted by a fixed point x^* close to 1. In this case, we note that for c close to, but different from 0 (negative in Fig. 7), the 2-period cycle of the y-dynamics carries over to the dynamics of x, which oscillates between two values. Moreover, since the dynamics of y converges toward a 2-period cycle, the asymptotic dynamics of x is described by the map:

$$x' = \varepsilon x + \frac{1-\varepsilon}{1+e^{-\rho(a+bx+cy_i)}} \tag{10}$$

with y_i alternating between the two values of the cycle. In Fig. 8 below we show this occurrence, which generates 2-period cycles in the x-dynamics. In particular, the figure shows in blue and in black the graphs from map (10) with y_i equal to the values of the cycle obtained in the dynamics of y : $y_1 \simeq 0.333$, $y_2 \simeq 0.668$, respectively.

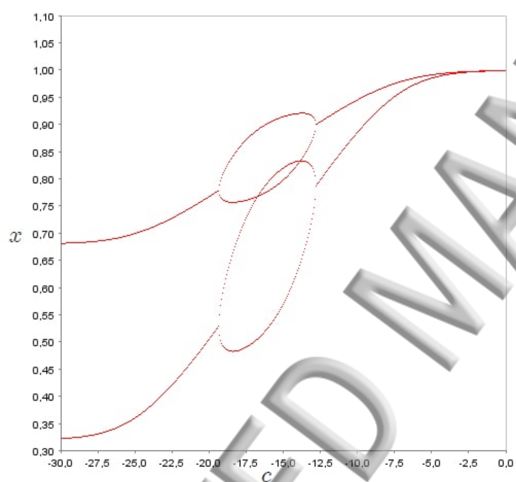

FIG. 7: Bifurcation diagram with respect to c. Parameter values: $a = 30$, $b = -23$, $\alpha = 10$, $\beta = -20$, $\gamma = 0$, $\varepsilon = 0.47$, $\rho = 1$, c varies in the interval $(-30, 10)$.

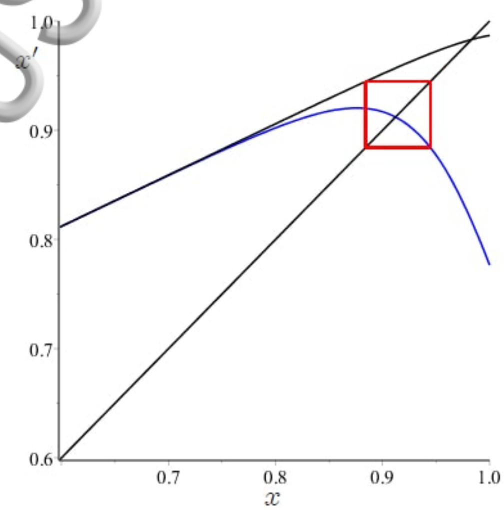

FIG. 8: Formation of a 2-period cycle in the x-dynamics. Parameter values: $a = 30$, $b = -23$, $c = -10$, $\alpha = 10$, $\beta = -20$, $\gamma = 0$, $\varepsilon = 0.47$, $\rho = 1$.

It is also relevant to note that for lower values of c, the 2-period cycle in the y-dynamics can create cyclical dynamics in x of longer period, as shown in Fig. 9.

In general, in a scenario characterised by southerners cyclically changing their narrative, inter-group externalities in the North make the payoff of northerners subject to the whims of the South. The openness of northerners towards the South drives them to inherit and possibly amplify the number of oscillations and/or their amplitude. Indeed, negative within-group externalities exacerbates the oscillations as they may push the x-dynamics towards a higher or a lower fixed point, alternately. The bifurcation diagram in Fig. 10 shows this mechanism, by which chaotic dynamics of y transfers to that of x. We note that the width of the oscillations of x may vary depending on the values of c.

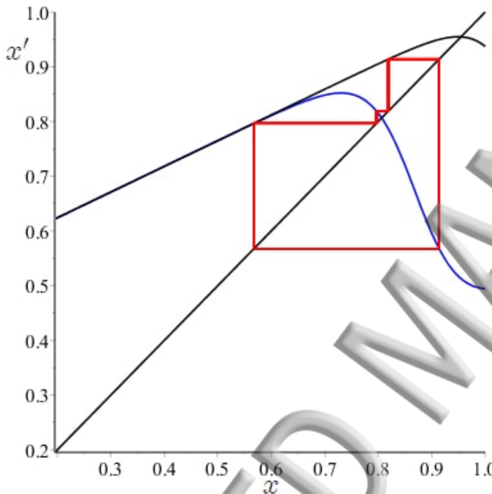

FIG. 9: Mechanism for the formation of a 4-period cycle in the x-dynamics. Parameter values: $a = 30$, $b = -23$, $c = -15$, $\alpha = 10$, $\beta = -20$, $\gamma = 0$, $\varepsilon = 0.47$, $\rho = 1$.

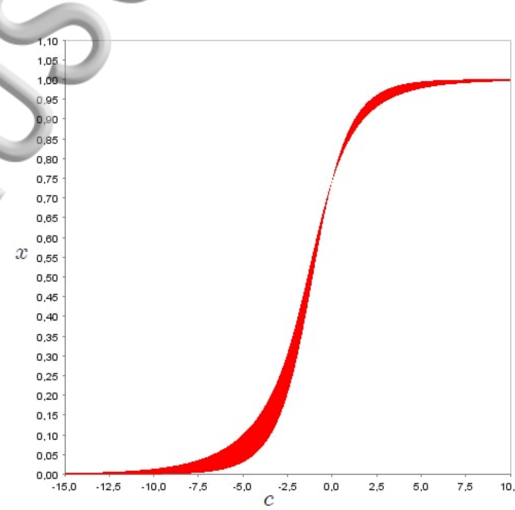

FIG. 10: Bifurcation diagram with respect to c. Parameter values: $a = 0.3$, $b = 1$, $c = -10$, $\alpha = 30$, $\beta = -40$, $\gamma = 0$, $\varepsilon = 0.47$, $\rho = 1$, c varies in the interval $(-15, 10)$.

The analysis we conducted so far in case of an univocal influence for the North holds symmetrically for the South, with γ leading to either self-enforcing or destabilising dynamics in y. As the two groups are designed symmetrically, all results holding for one also hold for the other if the parameters are opportunely inverted.

VI. BIUNIVOCAL INFLUENCE

We now investigate some crucial instances of biunivocal influence between the North and the South. Indeed, this case is much more complex than the univocal one and a complete investigation of the map is beyond the scope of this work. As we did in the previous section, we study the effect of inter-group externalities starting from a situation in which the two groups are isolated, i.e. their dynamics are decoupled. We further assume that the two groups have the same values for all parameters, so that $a = \alpha$, $b = \beta$, $c = \gamma$. This is done in order to focus on the effects of interaction rather than on the asymmetrical properties of the groups. From this assumption, it follows that the map T is symmetric, so that exchanging variables x and y leaves the system unaffected. Equivalently, $T \circ R = R \circ T$, where $R : (x, y) \to (y, x)$ is the reflection with respect to the diagonal $\Delta = \{(x, y) : x = y\}$. We note that Δ is an invariant manifold, that is $T(\Delta) \subseteq \Delta$. This implies that starting from the same initial conditions of x and y, the dynamics will lie on Δ for any iteration of the map. In this last case, the dynamics is governed by the restriction T_Δ of the map T on Δ, that is:

$$T_\Delta : x' = f(x) = \varepsilon x + (1 - \varepsilon)\frac{1}{1 + e^{-\rho[(b+c)x+a]}} \tag{11}$$

We remark that the results of section IV (concerning isolated groups) can be adapted to be valid also for the biunivocal influence case if we substitute parameter b with the parameter $b + c$. A relevant matter to investigate in bidimensional models characterised by symmetry with respect to the diagonal is whether the attractors of T_Δ are also attractors for the bidimensional map (see [2] for a detailed analysis of a symmetrical map generated by a duopoly model), that is for the trajectories in which the initial values of x and y differ. If synchronisation is reached, it means that the favoured narrative dynamics in the two groups does not show significant differences (even if this state may require a very long transient). From an analytical point of view, this information can be derived in two different ways. On the one hand, it can be derived by studying the transverse eigenvalues, in case n-period cycles exist on the diagonal. On the other hand, if a chaotic attractor lies on the diagonal, it proves particularly helpful to study the average Lyapunov transverse exponent in order to understand whether synchronisation is reached. More precisely, starting from the Jacobian matrix evaluated at a generic point on the diagonal:

$$J_{\Delta}(x,x) = \begin{pmatrix} J_1(x) & J_2(x) \\ J_2(x) & J_1(x) \end{pmatrix}, \tag{12}$$

where:

$$J_1(x) = \varepsilon + \frac{(1-\varepsilon)\rho b e^{-\rho((b+c)x+a)}}{\left[1+e^{-\rho((b+c)x+a)}\right]^2},$$

and

$$J_2(x) = \frac{(1-\varepsilon)\rho c e^{-\rho((b+c)x+a)}}{\left[1+e^{-\rho((b+c)x+a)}\right]^2},$$

we may derive its eigenvalues. By virtue of the symmetry of $J_{\Delta}(x,x)$, the eigenvalues are real and given by:

$$\lambda_{\parallel} = J_1(x) + J_2(x) = \frac{(e^{-2\rho((b+c)x+a)}+1)\varepsilon + (2\varepsilon + (1-\varepsilon)\rho(b+c))e^{-\rho((b+c)x+a)}}{\left[1+e^{-\rho((b+c)x+a)}\right]^2}$$

with eigenvector $\mathbf{r}_{\parallel} = (1, 1)$, and

$$\lambda_{\perp} = J_1(x) - J_2(x) = \frac{(e^{-2\rho((b+c)x+a)} + 1)\varepsilon + (2\varepsilon + (1-\varepsilon)\rho(b-c))e^{-\rho((b+c)x+a)}}{\left[1 + e^{-\rho((b+c)x+a)}\right]^2}$$

with eigenvector $\mathbf{r}_{\perp} = (1, -1)$. When a k-cycle $\{(x_1, x_1), (x_2, x_2), ..., (x_k, x_k)\}$ on Δ exists, the corresponding eigenvalues are:

$$\lambda_{\parallel}^{(k)} = \prod_{i=1}^{k} (J_1(x_i) + J_2(x_i)) = \prod_{i=1}^{k} \frac{(e^{-2\rho((b+c)x_i+a)} + 1)\varepsilon + (2\varepsilon + (1-\varepsilon)\rho(b+c))e^{-\rho((b+c)x_i+a)}}{\left[1 + e^{-\rho((b+c)x_i+a)}\right]^2},$$

$$\lambda_{\perp}^{(k)} = \prod_{i=1}^{k} (J_1(x_i) - J_2(x_i)) = \prod_{i=1}^{k} \frac{(e^{-2\rho((b+c)x_i+a)} + 1)\varepsilon + (2\varepsilon + (1-\varepsilon)\rho(b-c))e^{-\rho((b+c)x_i+a)}}{\left[1 + e^{-\rho((b+c)x_i+a)}\right]^2}.$$

The eigenvalue $\lambda_{\parallel}^{(k)}$ defines the stability properties of the map restricted to the diagonal Δ, whereas $\lambda_{\perp}^{(k)}$, called transverse eigenvalue, allows to verify whether such cycle attracts, at least locally, the points lying outside the diagonal. This information is obtained by studying the module of the transverse eigenvalue.

In general, an asymptotically stable attractor A for T_{Δ} is an asymptotically stable attractor also for the map T if and only if all trajectories belonging to A are transversely attracting. In order to address this issue we introduce the *transverse Lyapunov exponent* :

$$L_{\perp} = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N} \ln |\lambda_{\perp}(x_n)|,$$

where $x_0 \in A$ and $\{x_n\}$ is a trajectory generated by the map T_{Δ}. When a chaotic attractor C exists, varying initial conditions, a sequence of Lyapunov exponents is obtained:

$$L_\perp^{\min} \leq ... \leq L_\perp^{nat} \leq ... \leq L_\perp^{\max},$$

where $L_\perp^{nat}$ is associated to a typical trajectory taken in the chaotic attractor C, $L_\perp^{\min}$ and $L_\perp^{\max}$ are the extrema of the spectrum [see 5]. Different cases can be classified: 1) if $L_\perp^{\max} < 0$, then C is asymptotically stable in the Lyapunov sense; 2) if $L_\perp^{nat} < 0$ and $L_\perp^{\max} > 0$, then C is no longer stable in the Lyapunov sense, but it still attracts a large set of trajectories starting outside the diagonal. In this case, the basin of attraction has positive Lebesgue measure and C is said to be a Milnor attractor; 3) if $L_\perp^{nat} > 0$, then the set C loses its stability and becomes a chaotic saddle: according to global properties of the map, trajectories generated by initial conditions outside the diagonal and close to C can be either drawn by an attractor that envelops the saddle or captured by other attractors. As concerns the first two cases presented, we recall that an invariant set K is Lyapunov stable provided that for any neighbourhood U of K there exists another neighbourhood V of K such that any forward orbit of the map starting at V never quits U.

In case we consider two isolated groups, i.e. $c = \gamma = 0$, which show stable dynamics, and then assume the two groups start influencing each other with positive inter-group externalities for both, we observe that the result of the interaction is to push the fixed point (x^*, y^*) closer to the corner $(1, 1)$. In general, in case of two stable isolated groups, an increase in inter-groups externalities tends to promote the favoured narrative in both groups, as they are now reinforced by the presence of the favoured narrative in the other group, representing a symbiotic relationship between the two groups and the two narratives, each benefiting from a specialisation effect. In other terms, the two groups might find their favoured narrative to be complementary to the other one.

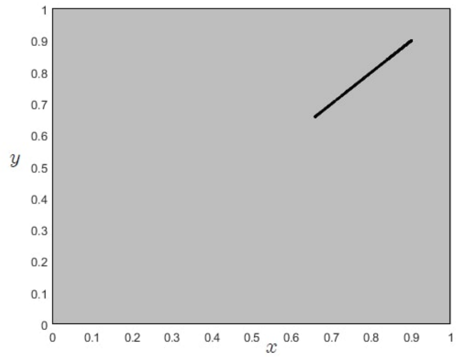

FIG. 11: A Milnor attractor exists on the diagonal. Its basin of attraction is depicted in light grey. Parameter values: $a = 34$, $b = -30.5$, $c = -7.474652$, $\varepsilon = 0.469$, $\rho = 1.2$.

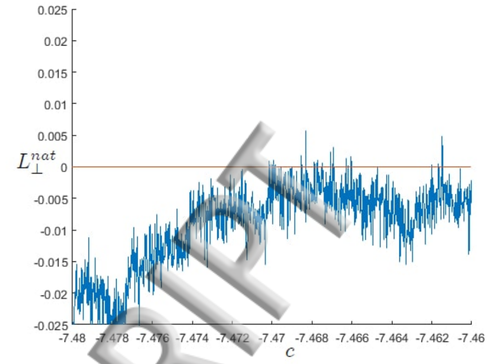

FIG. 12: Transverse Lyapunov exponent for $c \in (-7.48, -7.46)$. The other parameters are unchanged with respect to Fig. 11.

By contrast, in Fig. 11 we show how a decrease in inter-group externalities c affects the x-dynamics. After a sequence of period-doubling bifurcations, simulations suggest that a chaotic attractor exists on the diagonal for $c = -7.474652$. By further analysing this scenario, we note that the average Lyapunov transverse exponent $L_\perp^{nat}$ is very close to 0 (as illustrated in Fig. 12), which suggests that some cycles contained in the attractor are associated to eigenvalues that are greater than 1 in module. Indeed, by looking at Fig. 13, where we plot the distance between the values of x and y versus time starting from the initial condition $(0.82, 0.83)$, we observe how such value tends to 0.

However, even if this difference goes to 0, it occurs after a very long time (more than 3000 iterations) and several windows exist in which dynamics are not synchronised. This phenomenon is called on-off intermittency and here describes a circumstance in which all individuals have a comparative advantage in interacting with members from the other group adopting the same narrative whereas narratives

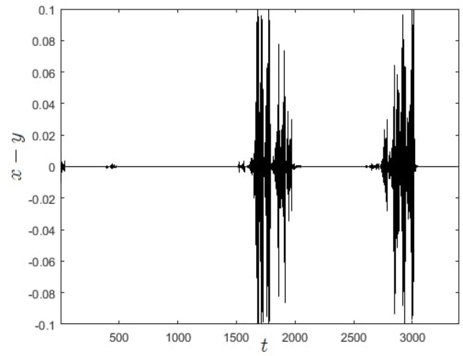

FIG. 13: On–off intermittency: synchronisation occurs after a very long transient. The values of the parameters are equal to the ones in Fig. 11.

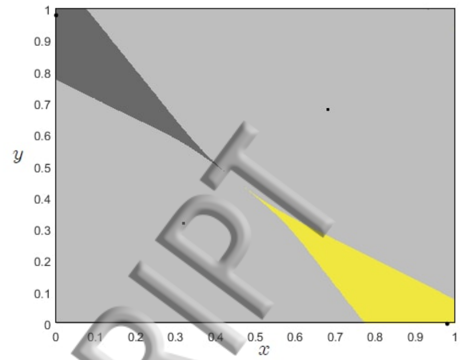

FIG. 14: Coexistence of three attractors: a two period cycle along the diagonal, and two fixed points. Different colours identify different basins of attraction. Parameter values: $a = 34$, $b = -30.5$, $c = -45$, $\varepsilon = 0.469$, $\rho = 1.2$.

show an elitist dynamics (a negative within-group externality) within the group. In other words, every individual would like to mimic what individuals from the other group are doing, but also not to do what their co-members are doing. Intuitively, these two forces drive the individuals towards opposite directions. Some individuals might be among the few to capture all the benefits of coordinating with the other group on their narrative, while also benefiting from the elitist feature of the narratives. However, as soon as the narrative they carry diffuses within their group, the corresponding payoff is undermined and some might decide to revert back to the favoured narrative. These two opposite forces make individuals particularly responsive to noise, so that it may drive the dynamics away from the attractor, although the latter is eventually reached by the system.

For values of c lower than the one we set in Fig. 13 we note that the dynamics on the diagonal tends to become regular, although multistability regimes may arise.

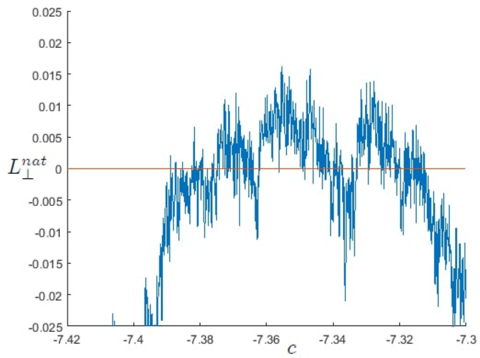

FIG. 15: Transverse Lyapunov exponent for $c \in (-7.42, -7.30)$. Other parameter values: $a = 34$, $b = -30.7$, $\varepsilon = 0.45$, $\rho = 1.2$.

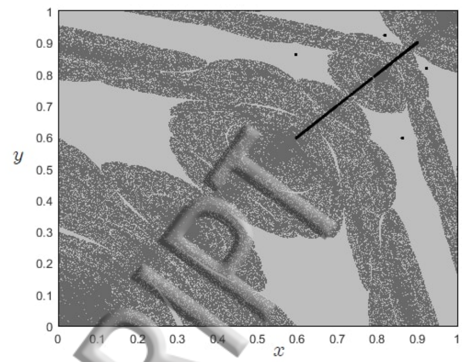

FIG. 16: Coexistence of an attractor on the diagonal and a 4-period cycle outside of it. Parameter values: $a = 34$, $b = -30.7$, $c = -7.3$, $\varepsilon = 0.45$, $\rho = 1.2$.

This is shown in Fig. 14, where two additional attractors, beyond the 2-period cycle along the diagonal, exist and are symmetrical with respect to the diagonal. Such attractors are reached by the system if the difference between x and y is sufficiently low, i.e. if the initial condition lies either in the yellow or in the dark grey zone of Fig. 14. In this case, one of the narratives is adopted by almost all individuals, whereas the other one nearly disappears. We remark that different phenomena may be observed by considering varying levels of c starting from a different parameter specification. Indeed, considering the parameter values $a = 34$, $b = -30.7$, $\varepsilon = 0.45$, $\rho = 1.2$, the study of the Lyapunov exponent presents phases in which it is positive, as shown in Fig. 15. In this case no attractor exists on the diagonal and the trajectories are attracted by a 4-period cycle outside the diagonal, illustrated in Fig. 17.

We now analyse the case in which the two groups show chaotic dynamics when isolated. This case is shown in Fig. 18, where we consider the symmetrical case in which the two groups have the same parameters. An increase in the intensity of

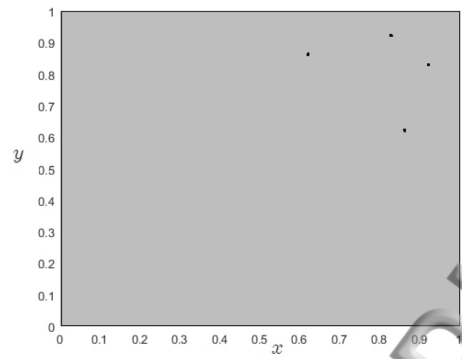

FIG. 17: A 4-period cycle outside the diagonal. Parameter values: $a = 34$, $b = -30.7$, $c = -7.36$, $\varepsilon = 0.45$, $\rho = 1.2$.

the interrelation between the two groups leads to a chaotic attractor located in the north-east region of the state plane (see Fig. 19). This is due to the self-reinforcing mechanism that may arise if the elitist dynamics described by negative values of b and β is more than offset by positive values of c and γ. In other terms, if the emotive narrative in the North yields increasing payoffs as the number of southerners adopting the cognitive narrative increases and vice versa, the shares x and y of individuals adopting the favoured narratives will be higher. As c increases further, the system may stabilise and a fixed point very close to $(1, 1)$ becomes the global attractor of the system.

In addition, we remark that by slightly changing the parameter specification, an increase in c in this stabilisation process might lead to the identification of intervals characterised by low-period cyclic attractors. For instance, the graph in Fig. 20 refers to the parameters values $a = 30$, $b = -40$, $c = 4$, $\varepsilon = 0.48$, $\rho = 1.3$, and shows a very long transient before this attractor is definitely captured by 3-period cycles, hence leading to very simple dynamics in the long term trajectory. The behaviour of the transient in this process is depicted in Fig. 21. In order to understand such

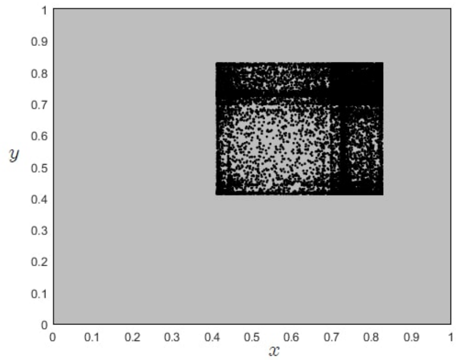

FIG. 18: Decoupled dynamics: the grey region denotes the basin of attraction of the unique (chaotic) attractor of the system. Parameter values: $a = \alpha = 30$, $b = \beta = -40$, $c = \gamma = 0$, $\varepsilon = 0.47$, $\rho = 1$.

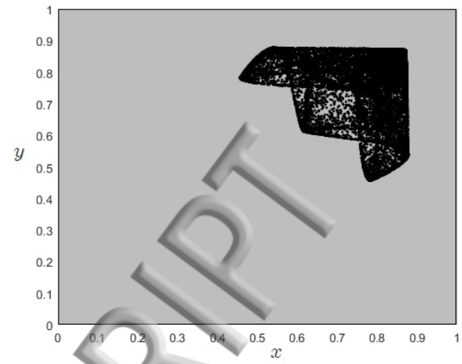

FIG. 19: Evolution of the attractor as c increases. Parameter values: $a = \alpha = 30$, $b = \beta = -40$, $c = \gamma = 5$, $\varepsilon = 0.47$, $\rho = 1$.

behaviour, we note that by virtue of the Li-Yorke theorem [14] we know that a chaotic repeller lies on the diagonal and an infinite number of cycles exists. Given the behaviour of the time series, many (potentially infinite) of these cycles are also transversally unstable.

VII. DISCUSSION AND RESEARCH DIRECTIONS

In this work, we presented how narrative dynamics in two social groups may be affected by inter-group externalities, in addition to within-group ones. Indeed, even starting from a case in which both groups have a single fixed point which is an attractor, we showed how chaotic dynamics may arise in at least one of the groups. In particular, we differentiated between the univocal case, in which only one of the groups is open to influence from the other, and the biunivocal case, in which the influence is reciprocal. On the one hand, we found that influence from another group

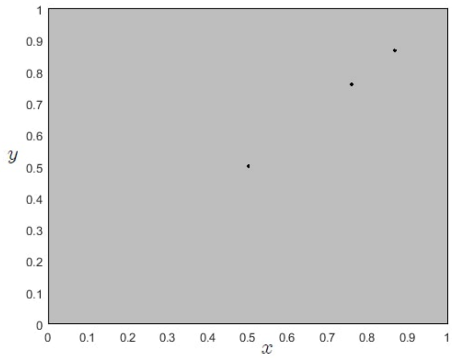

FIG. 20: A 3-period cycle along the diagonal is the unique attractor of the system. Parameter values: $a = 30$,$b = -40$, $c = 4$, $\varepsilon = 0.48$, $\rho = 1.3$.

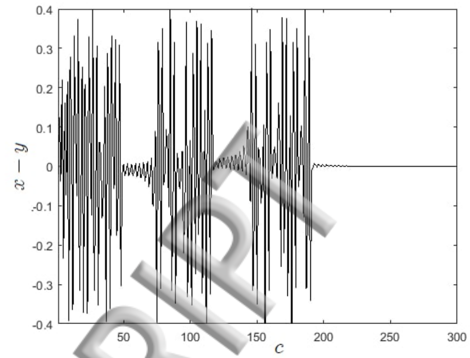

FIG. 21: The transient shows an irregular behaviour even if the trajectories are attracted by a low period cycle. This is due to the presence of a chaotic repeller.

may be strong enough to shift the dynamics in the open group away from its original attractor in which almost every individual adopts the favoured narrative toward a new fixed point in which almost nobody does so. In addition, chaotic dynamics in an isolated group may carry over to the other group if it is open to its influence. Moreover, the instability may be amplified in the open group, with larger-period dynamics. In both cases, inter-group influence is shown to be a potentially strong destabilising force. On the other hand, we analysed the case in which the influence is reciprocal and symmetrical. Firstly, we considered stable dynamics in the isolated groups and a positive reciprocal influence, so that individuals adopting the favoured narratives in both groups are advantaged from the interaction. A self-reinforcing mechanism then attracts the fixed point toward the north-east corner of the map. Secondly, although we kept considering stable isolated dynamics, we considered negative inter-group influence. In this case, the parameters allow us to identify several different scenarios, including a chaotic attractor, multistability and cycles outside the diagonals. Finally, we take into account the possibility that the isolated dynamics is chaotic in both groups and analyse the effect of a reciprocal, positive influence. We observe that also in this case the map presents a variety of scenarios, with the most relevant one being the possibility of inter-group interaction acting as a stabilisation process, leading the system from a chaotic attractor to a stable one. However, even if we analysed and presented some of the most salient results of inter-group interaction and its effects on narrative dynamics, we left some important regions of the map unexplored from which some very relevant considerations may be drawn. Further investigation of the dynamics might uncover such aspects or might focus on studying the role of numerosity and fragmentation of groups in their interaction. Indeed, although this model illustrates what are the main mechanics, it could be a scaffold toward understanding more intricate interactions between many social groups, whose internal dynamics are possibly much different from each other.

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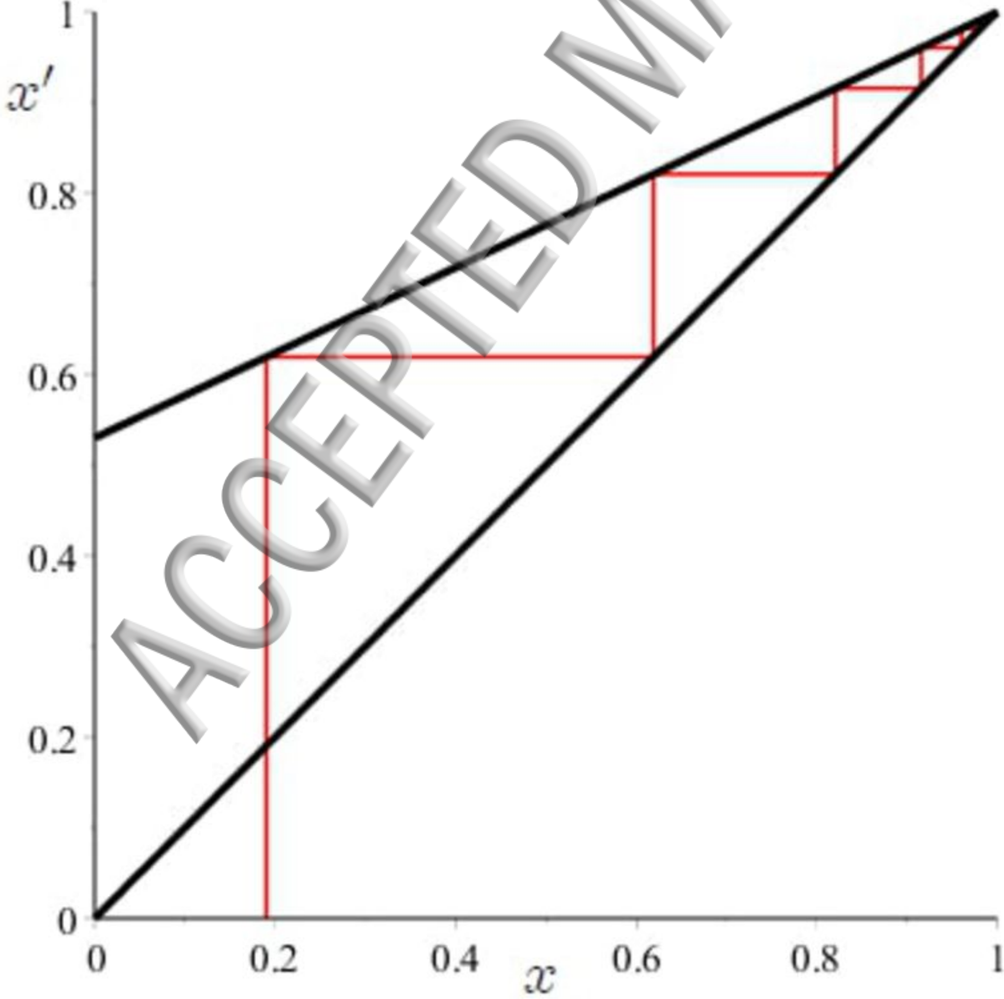

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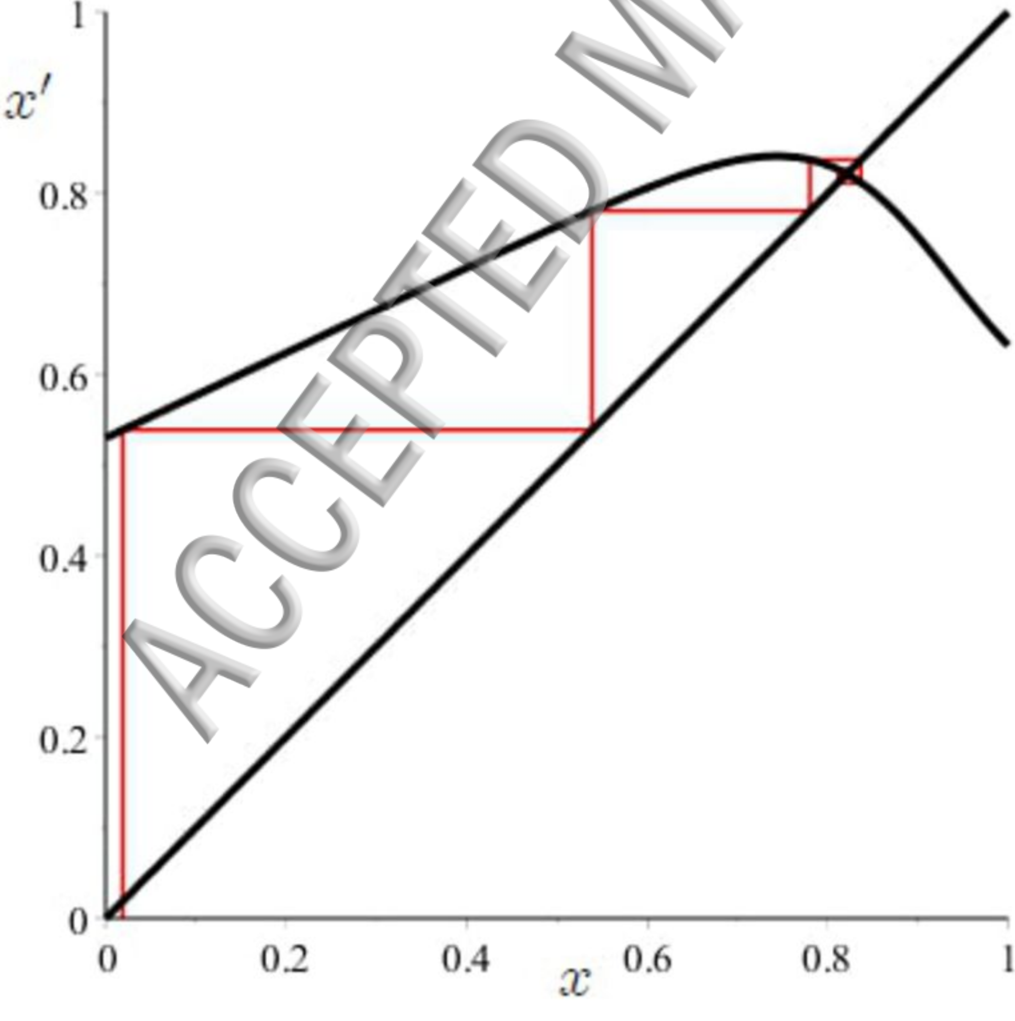
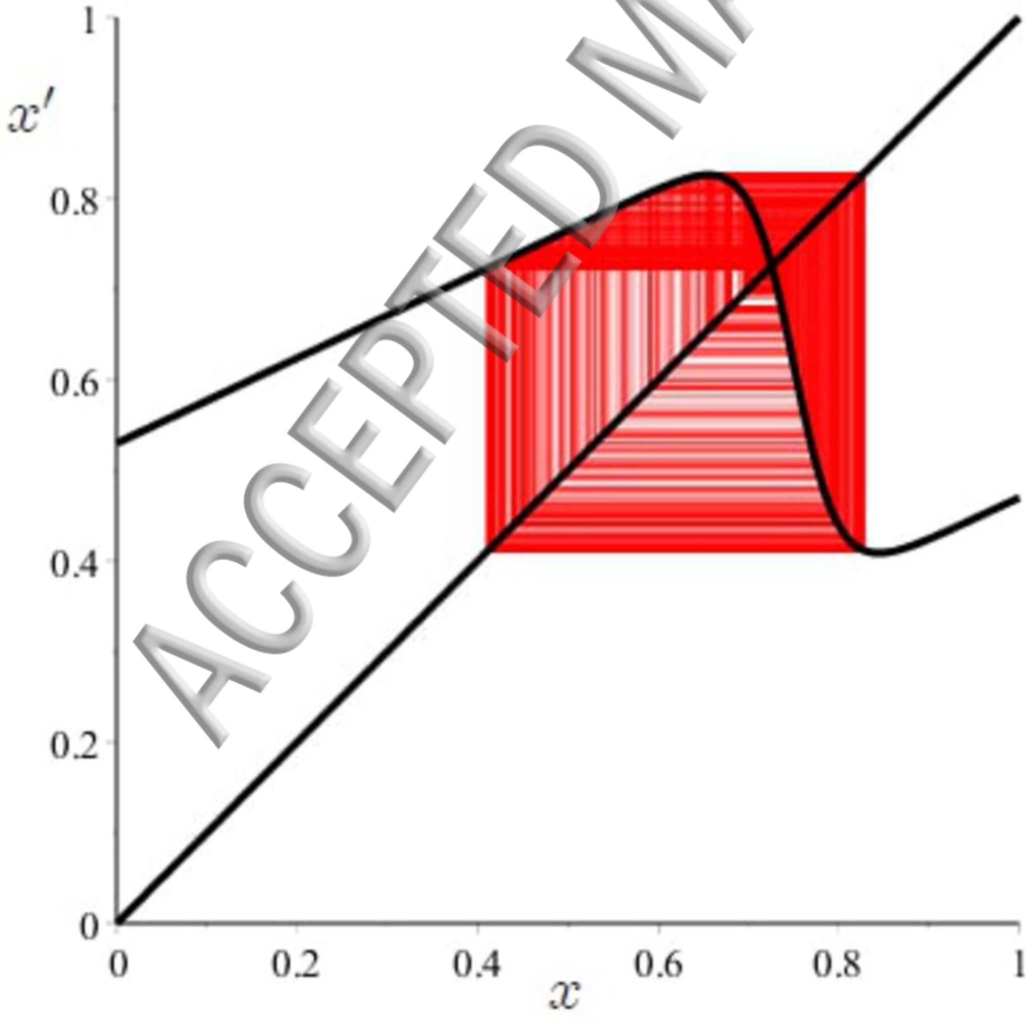
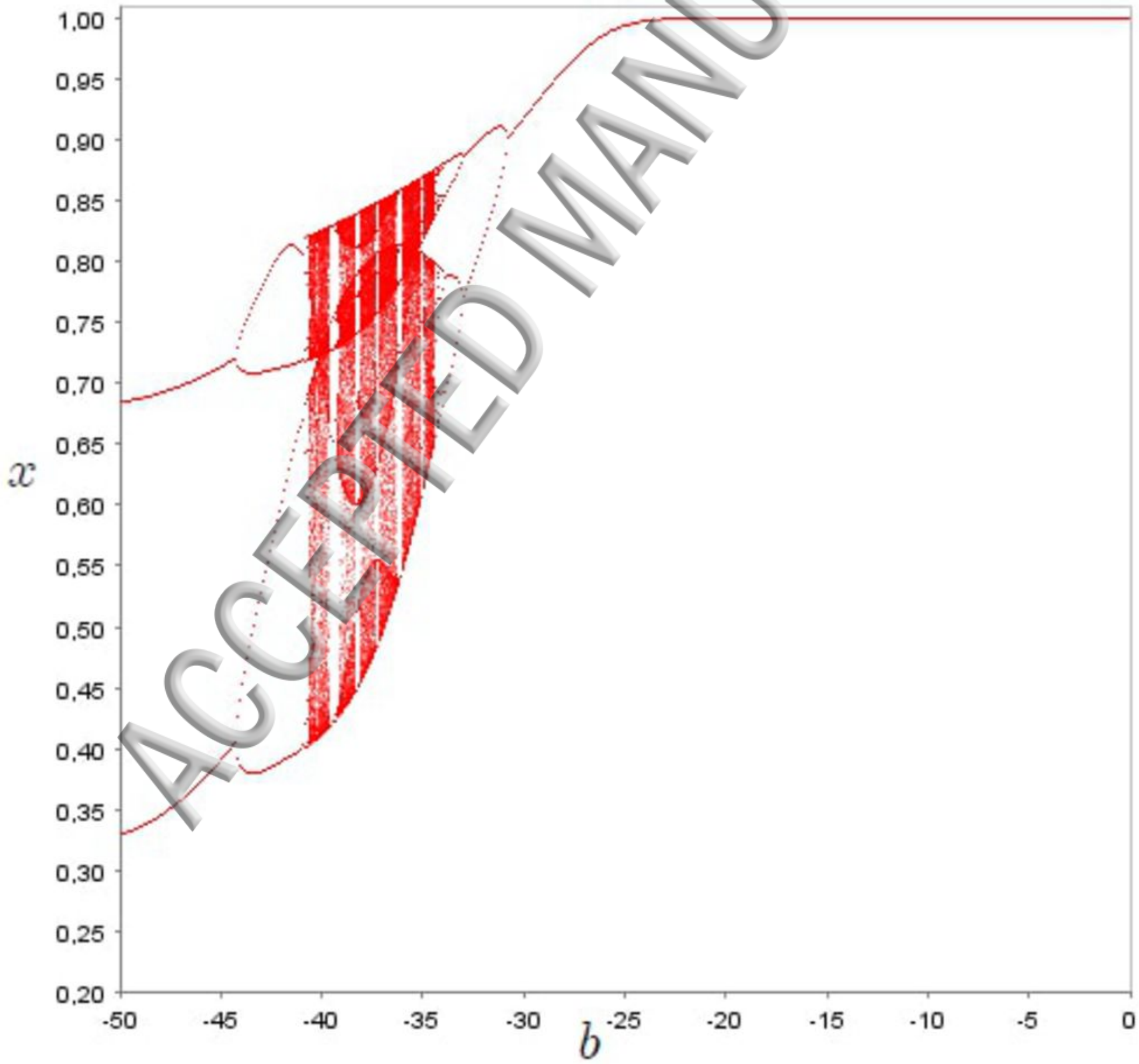
0.8
1

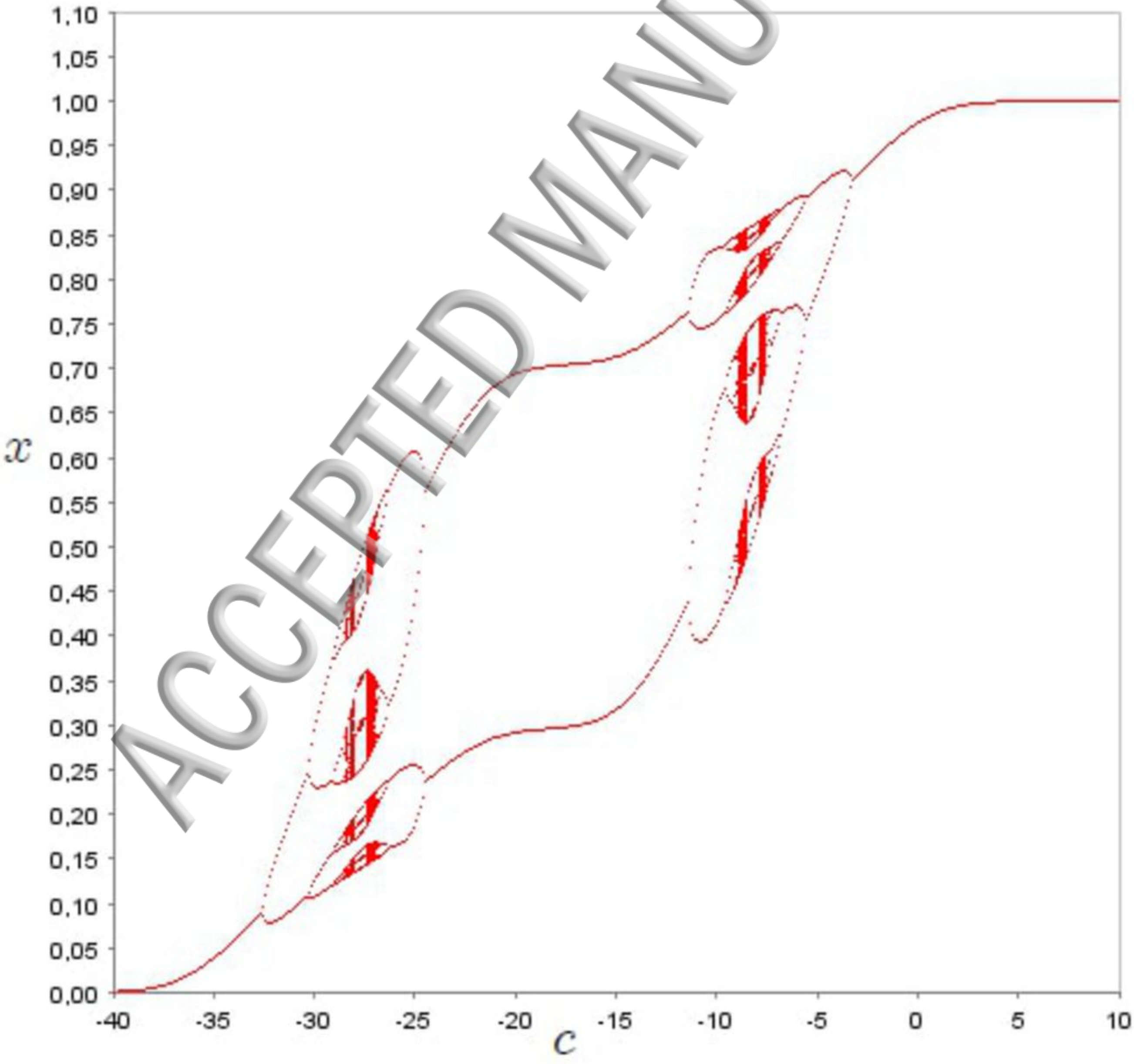


1
x'
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
x

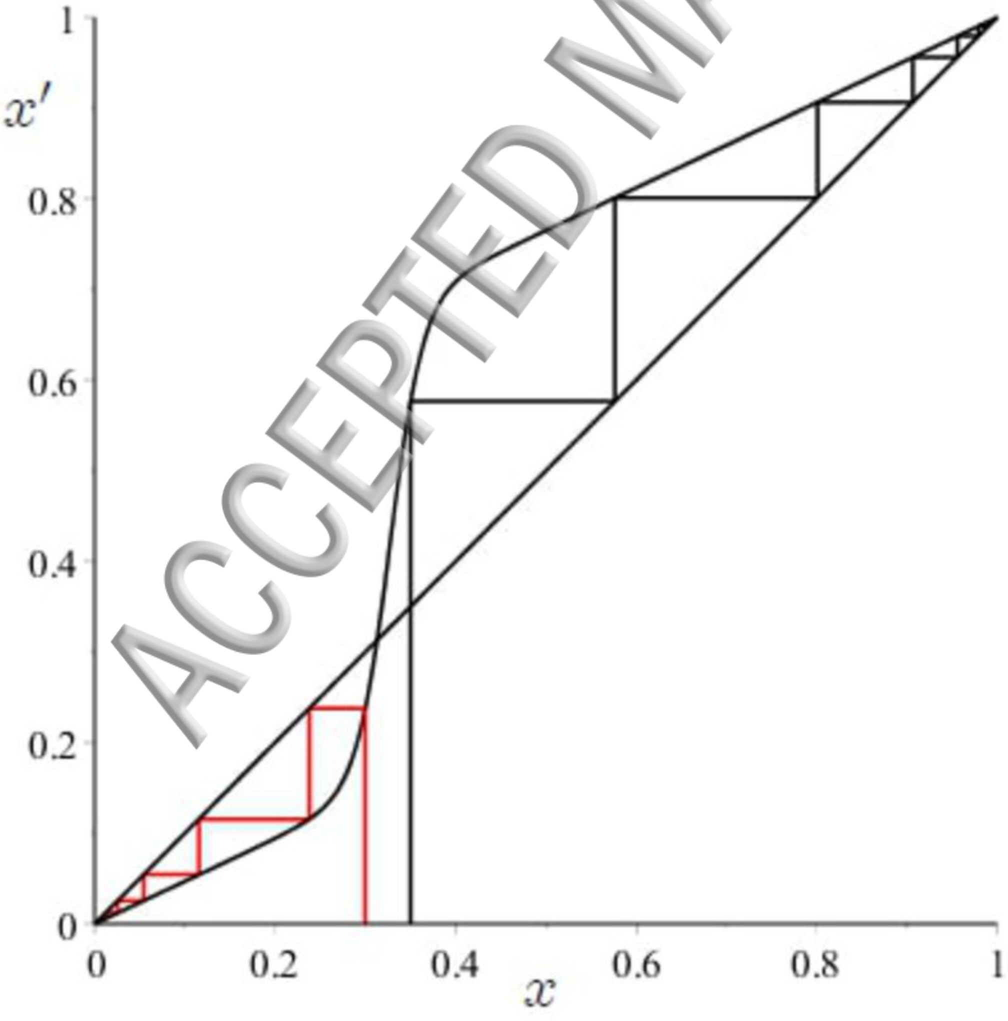


1
0.8
0.6
0.4
0.2
0
x'
0
0.2
0.4
0.6
0.8
1
x

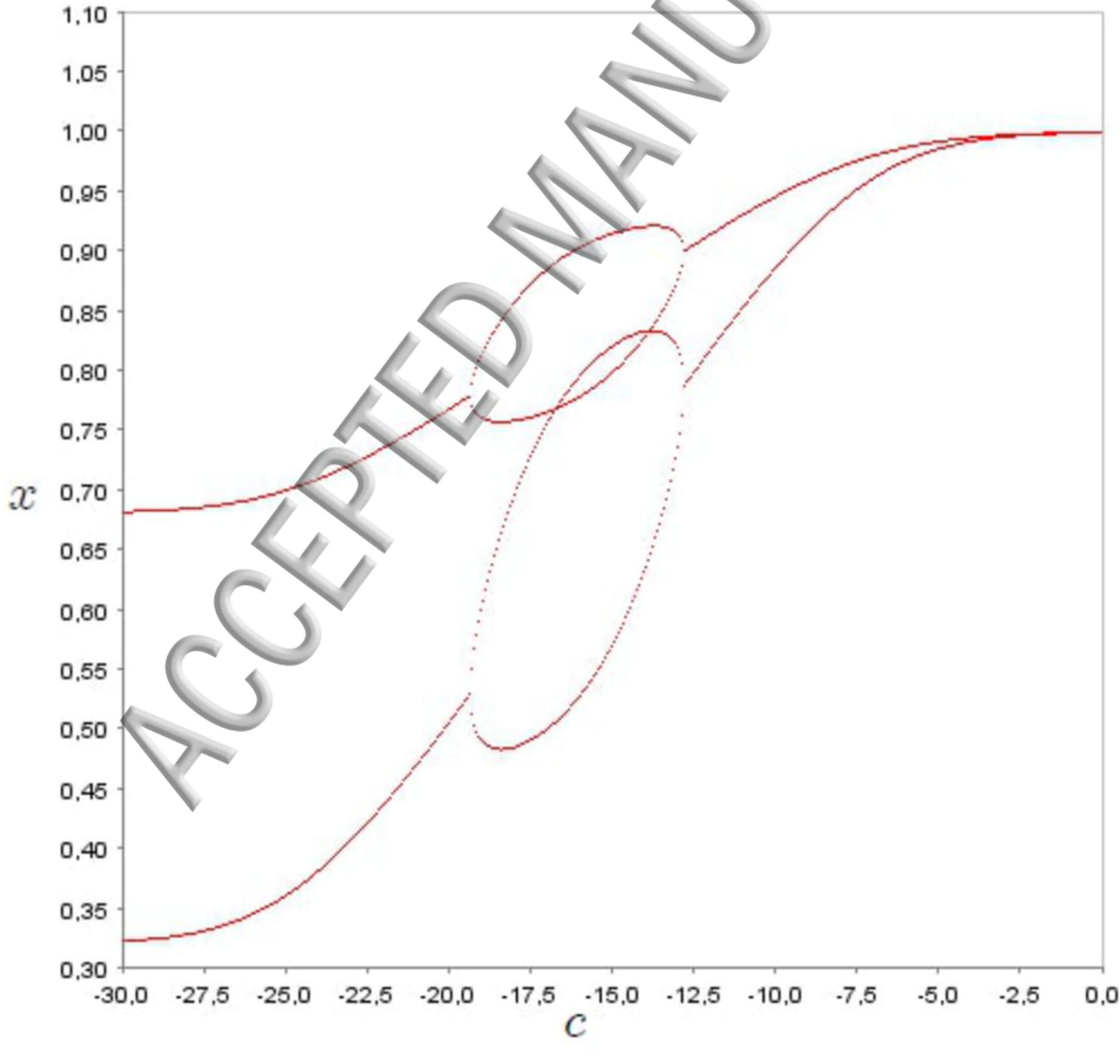

1,00
0,95
0,90
0,85
0,80
0,75
0,70
0,65
0,60
0,55
0,50
0,45
0,40
0,35
0,30
0,25
0,20
x
-50
-45
-40
-35
-30
-25
-20
-15
-10
-5
0
b

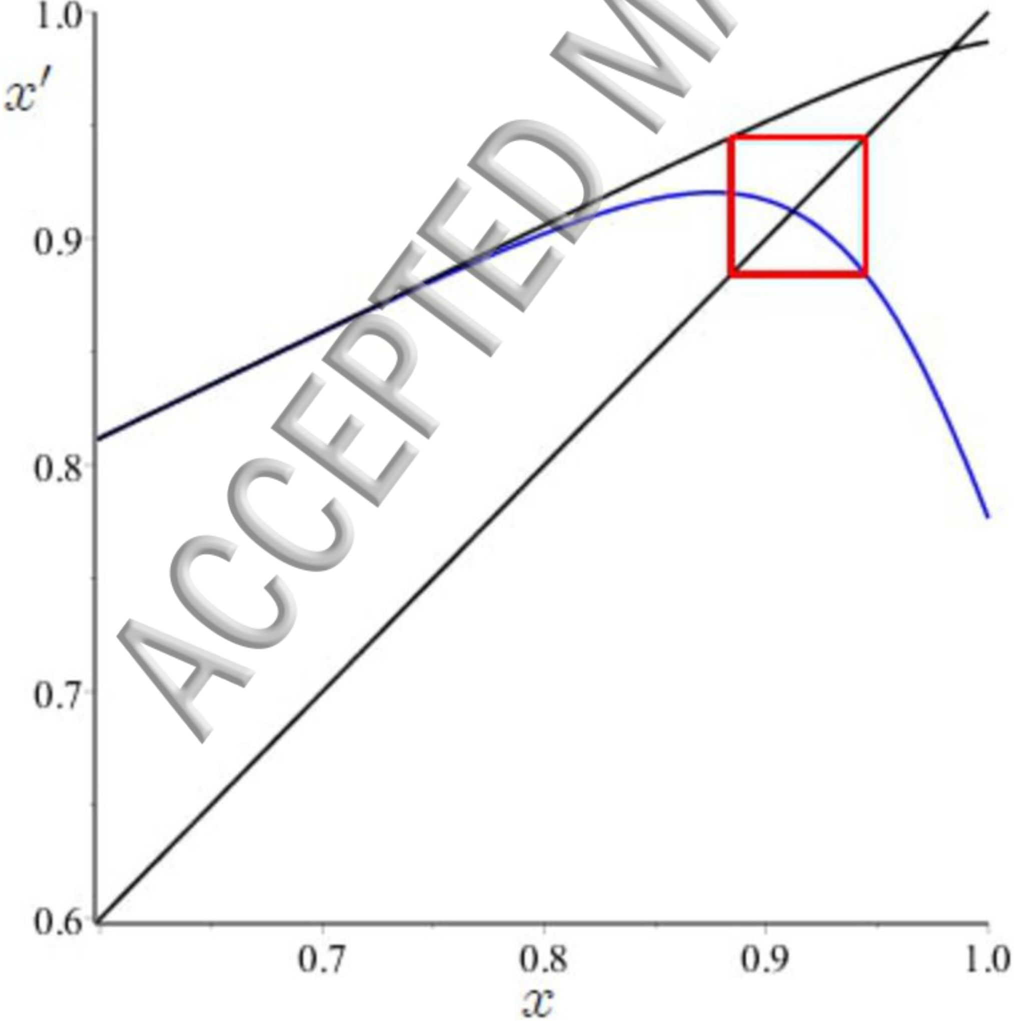


1,10
1,05
1,00
0,95
0,90
0,85
0,80
0,75
0,70
0,65
0,60
0,55
0,50
0,45
0,40
0,35
0,30
0,25
0,20
0,15
0,10
0,05
0,00
x
-40
-35
-30
-25
-20
-15
-10
-5
0
5
10
c

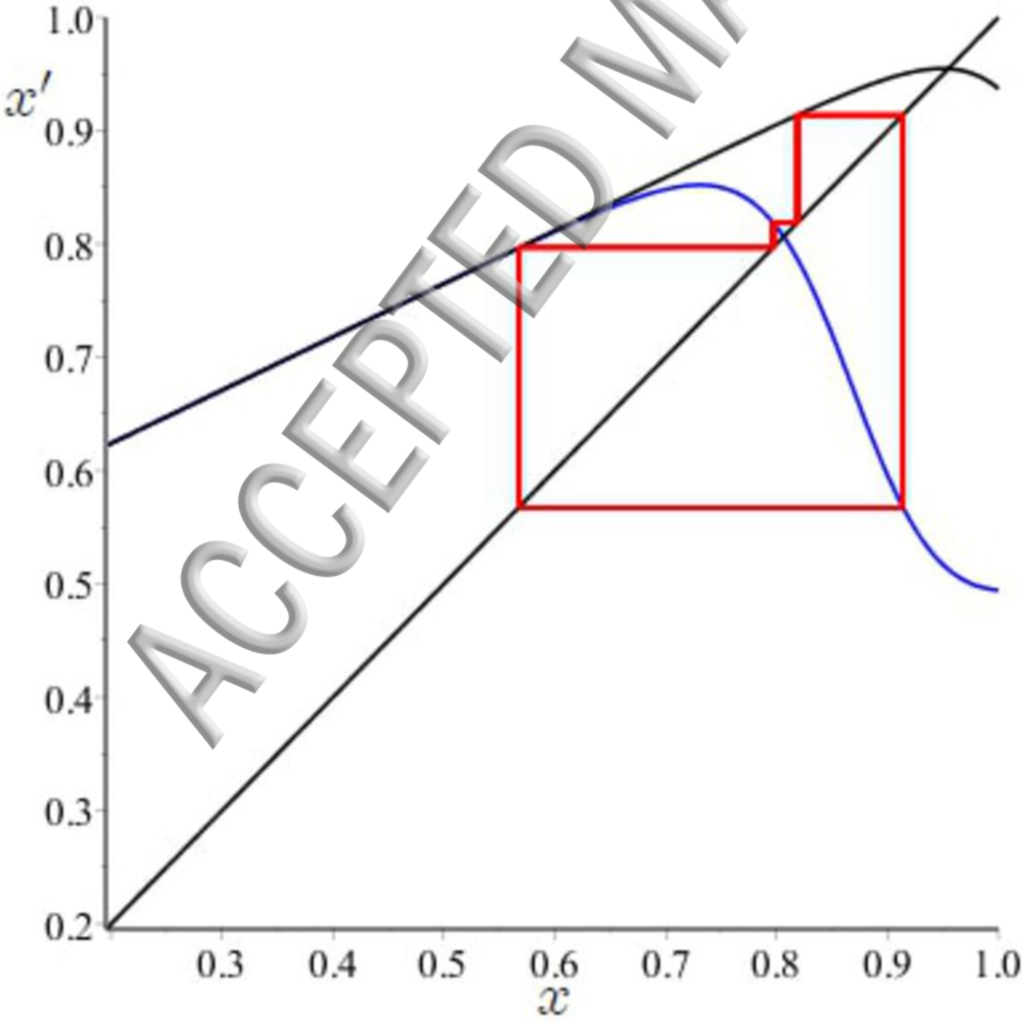


x'
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
x

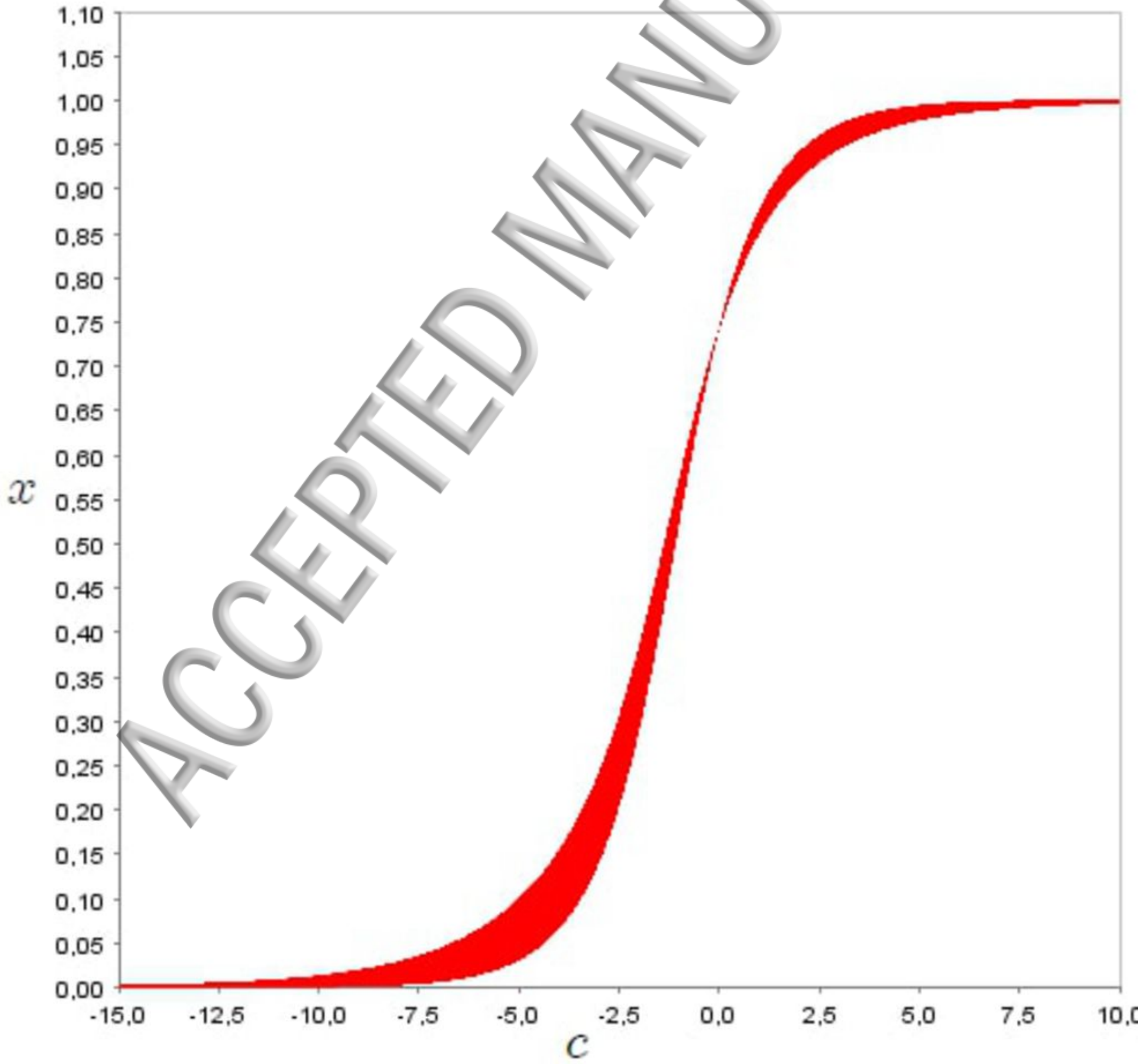

1,10
1,05
1,00
0,95
0,90
0,85
0,80
0,75
0,70
0,65
0,60
0,55
0,50
0,45
0,40
0,35
0,30
x
-30,0
-27,5
-25,0
-22,5
-20,0
-17,5
-15,0
-12,5
-10,0
-7,5
-5,0
-2,5
0,0
c

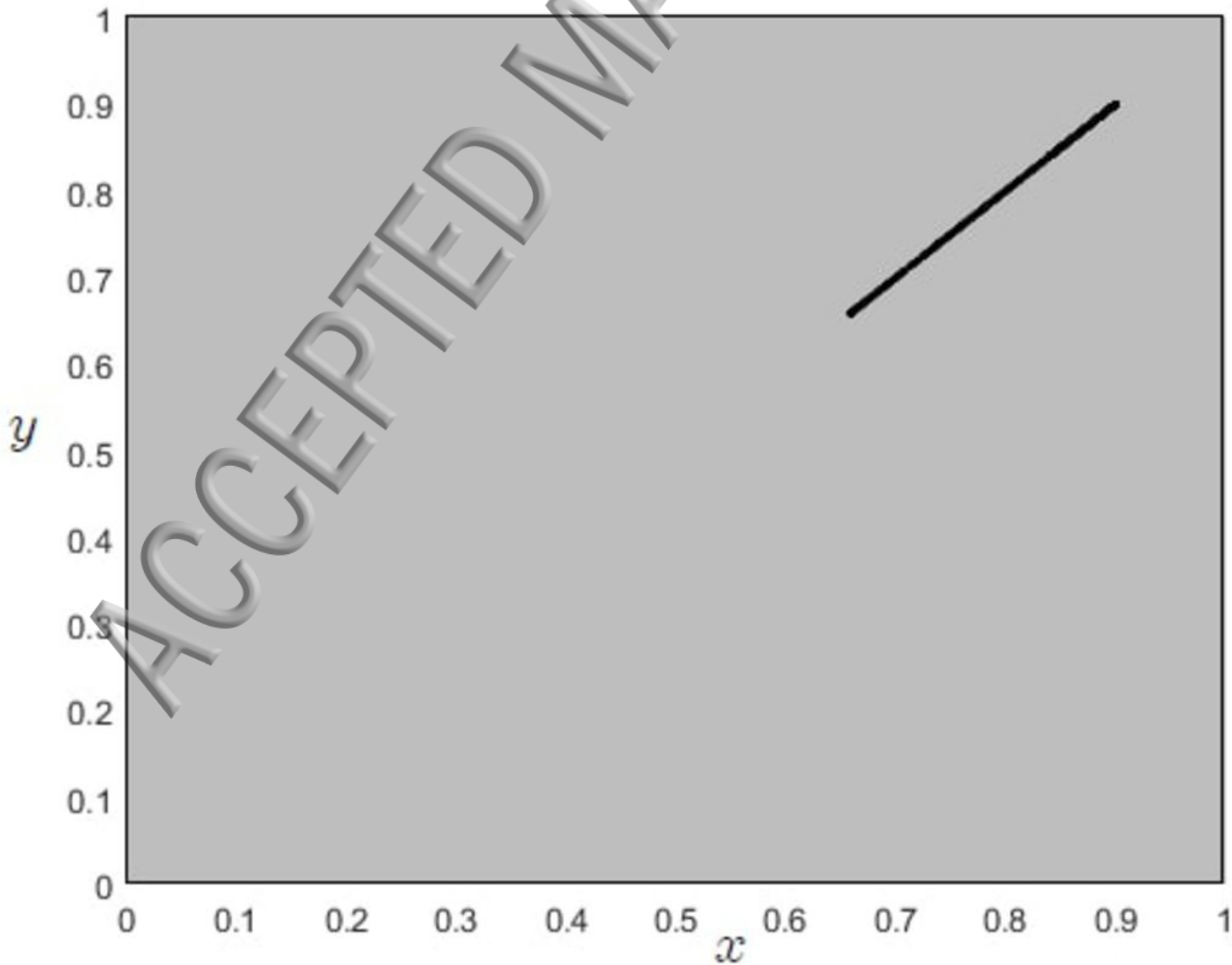


1.0
x′
0.9
0.8
0.7
0.6
0.7
0.8
0.9
1.0
x

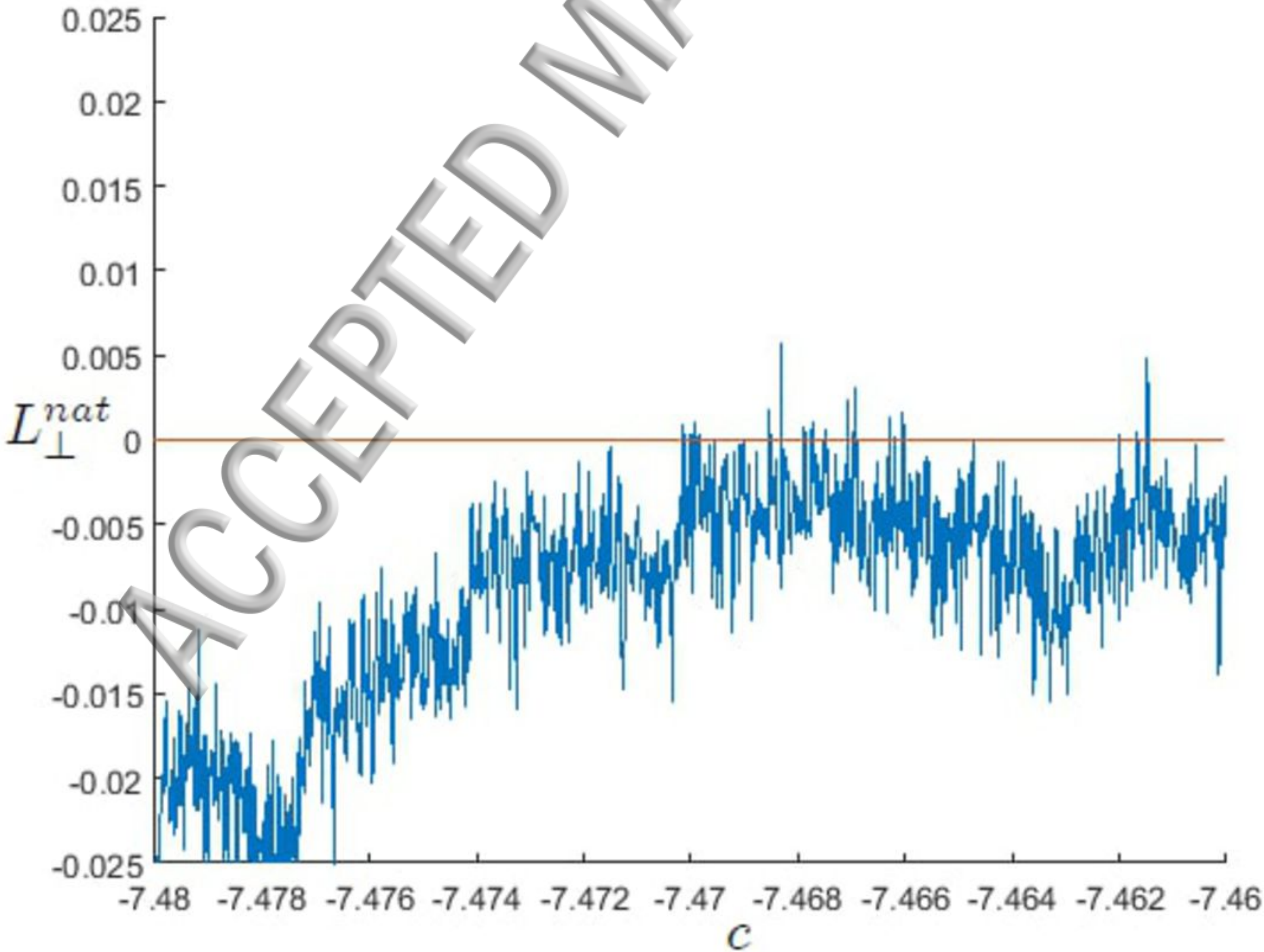


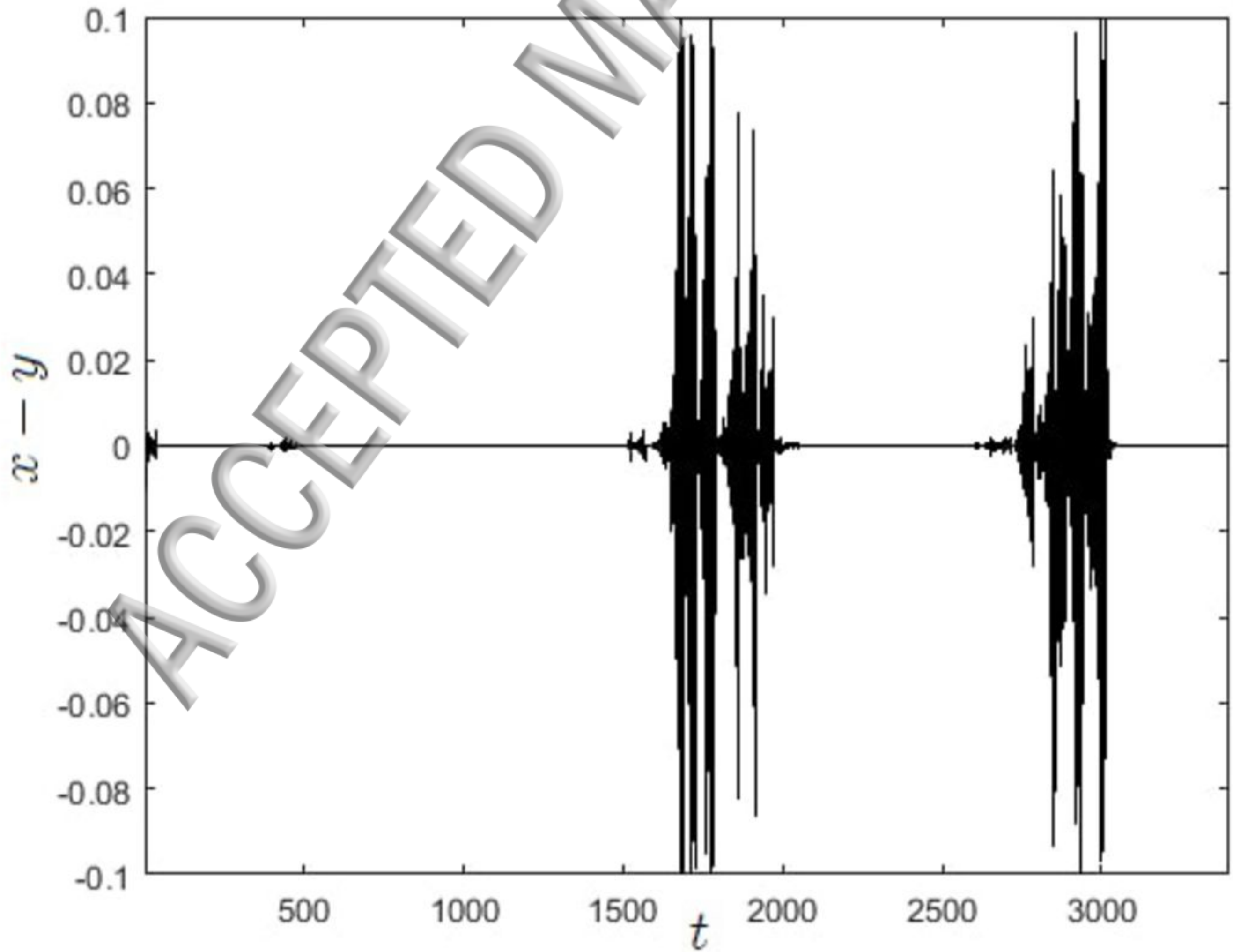
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
x'
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x

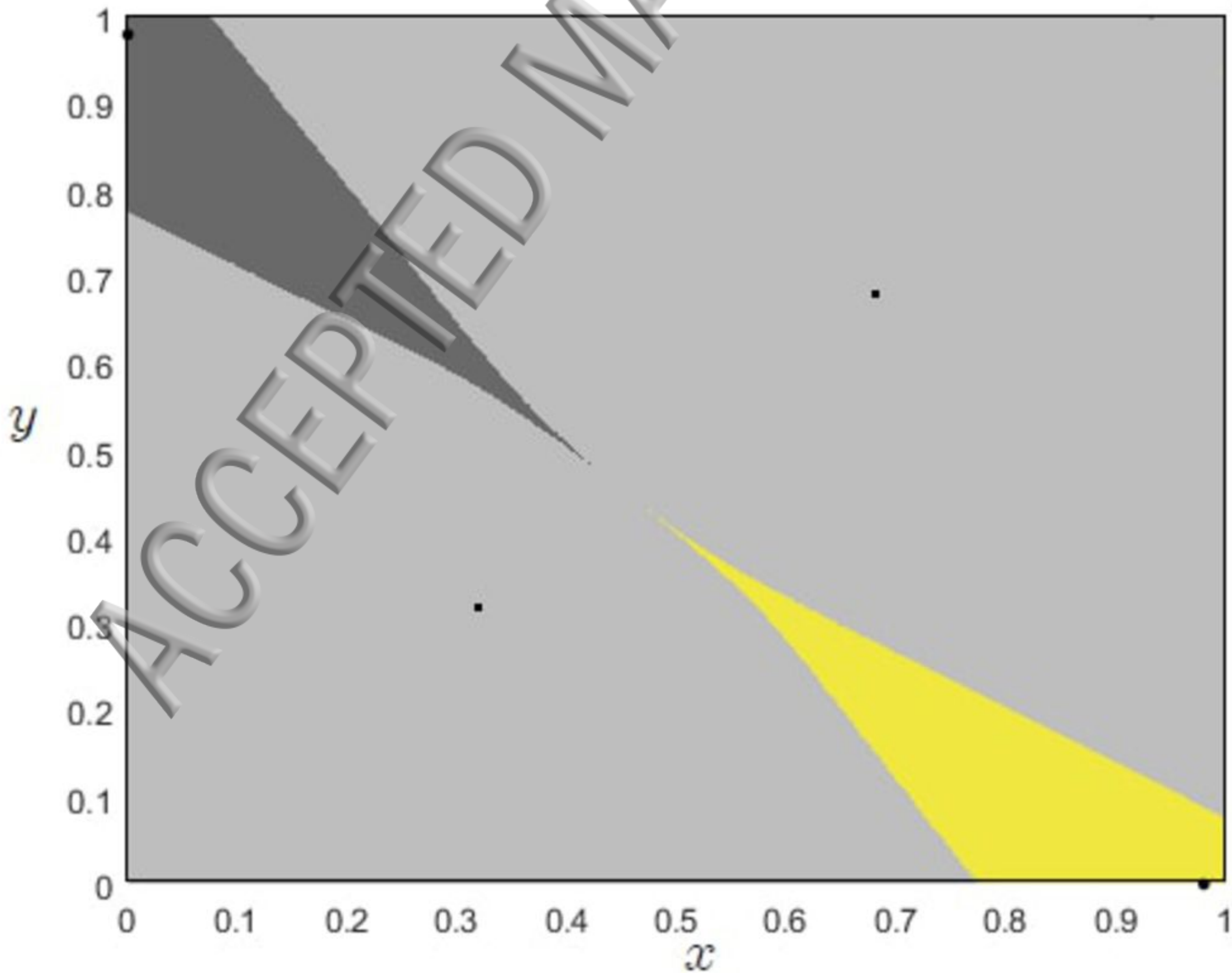

1,10
1,05
1,00
0,95
0,90
0,85
0,80
0,75
0,70
0,65
0,60
0,55
0,50
0,45
0,40
0,35
0,30
0,25
0,20
0,15
0,10
0,05
0,00
x
-15,0
-12,5
-10,0
-7,5
-5,0
-2,5
0,0
2,5
5,0
7,5
10,0
c



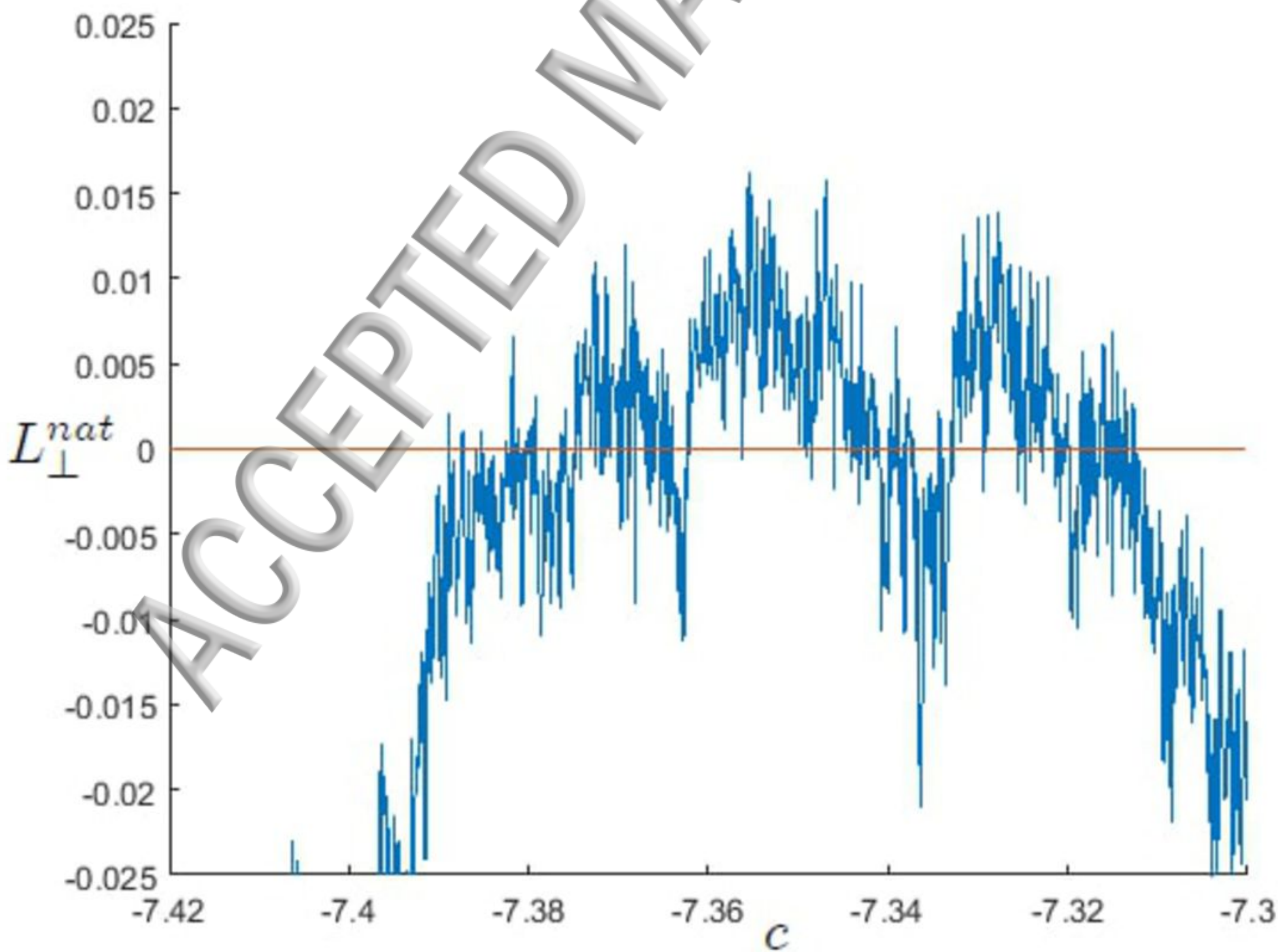
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
y
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x


0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
$L_{\perp}^{nat}$
-7.48
-7.478
-7.476
-7.474
-7.472
-7.47
-7.468
-7.466
-7.464
-7.462
-7.46
c

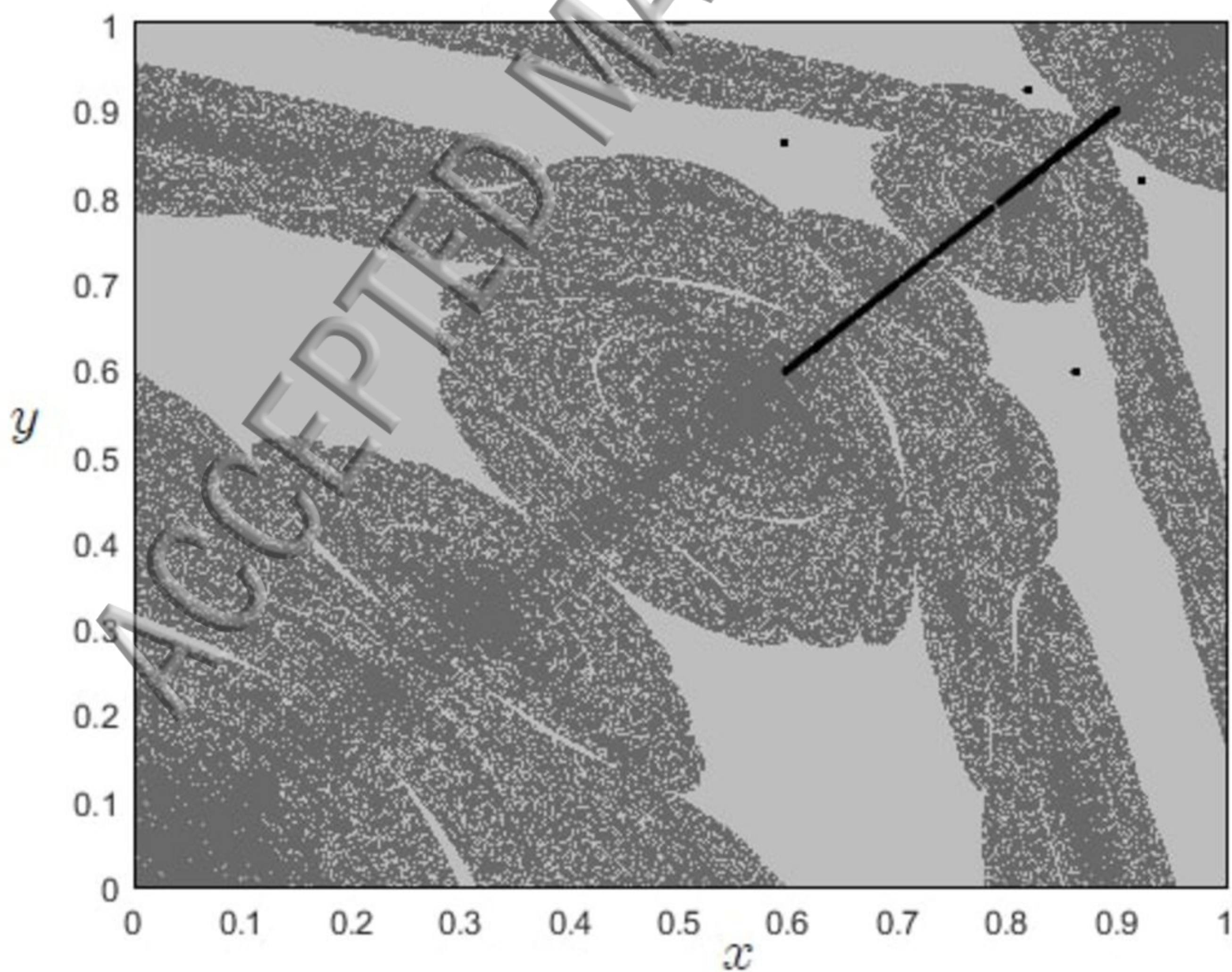

0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
$x - y$
500
1000
1500
2000
2500
3000
t



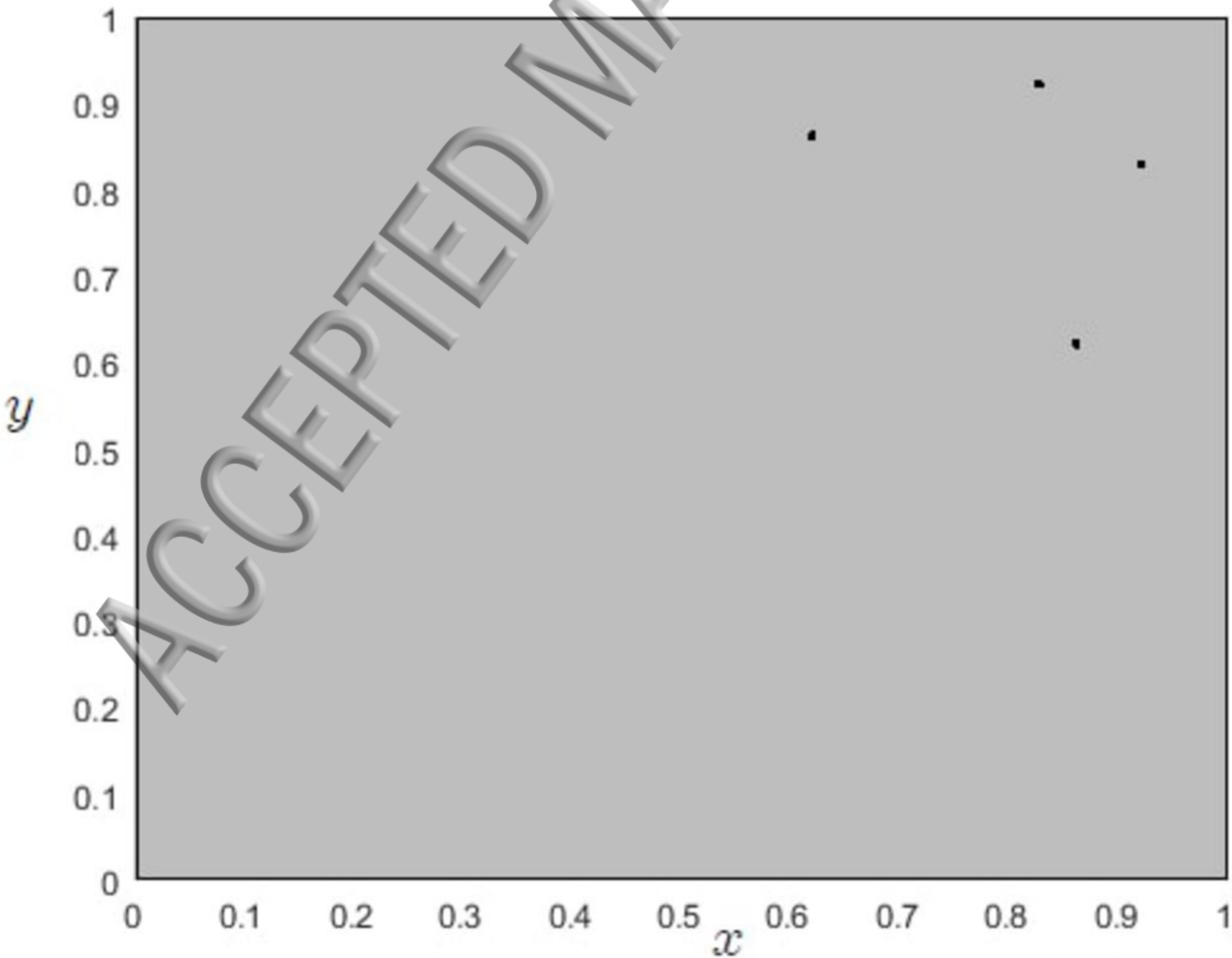
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
y
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x

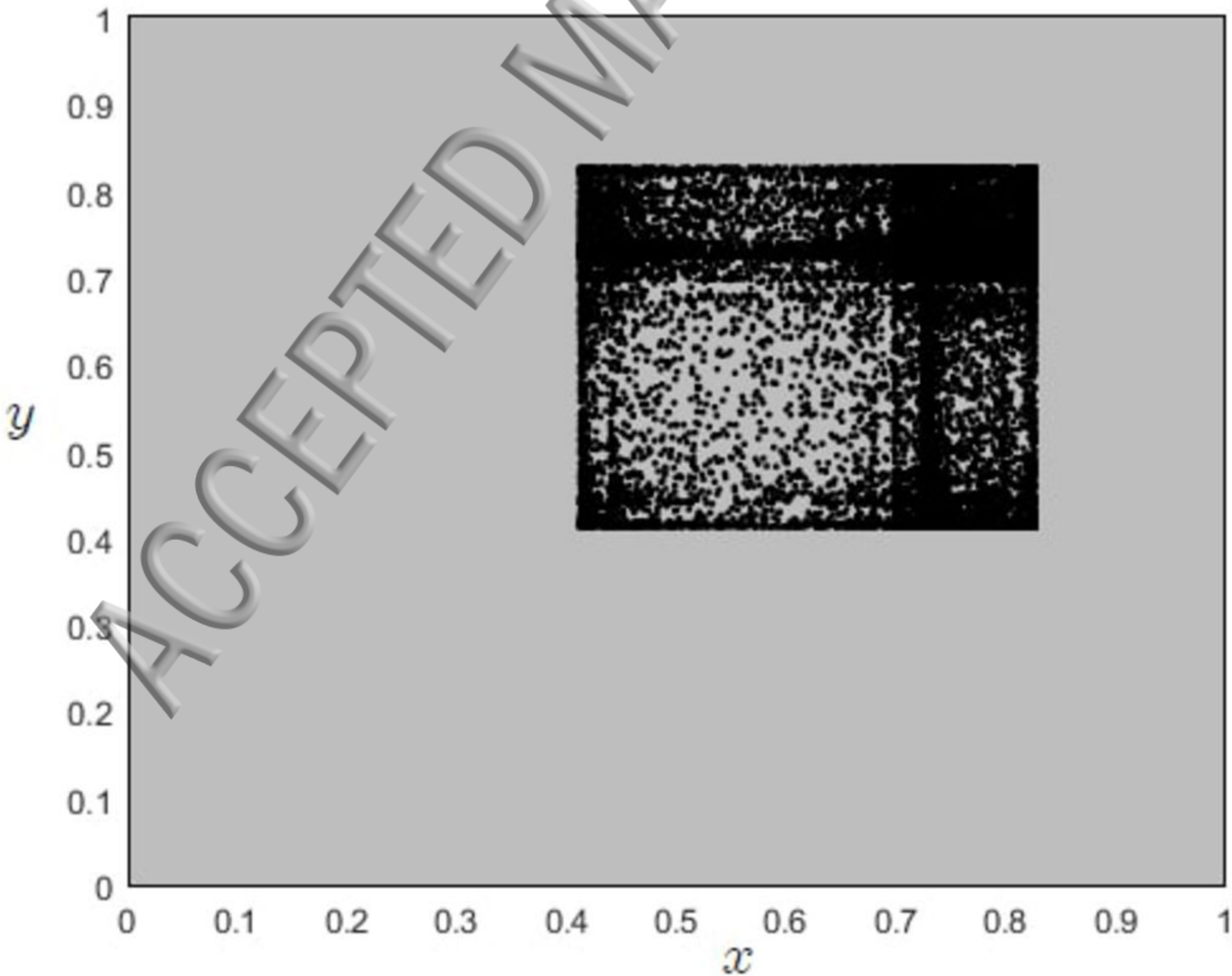


0.025
0.02
0.015
0.01
0.005
$L_{\perp}^{nat}$
0
-0.005
-0.01
-0.015
-0.02
-0.025
-7.42
-7.4
-7.38
-7.36
-7.34
-7.32
-7.3
c

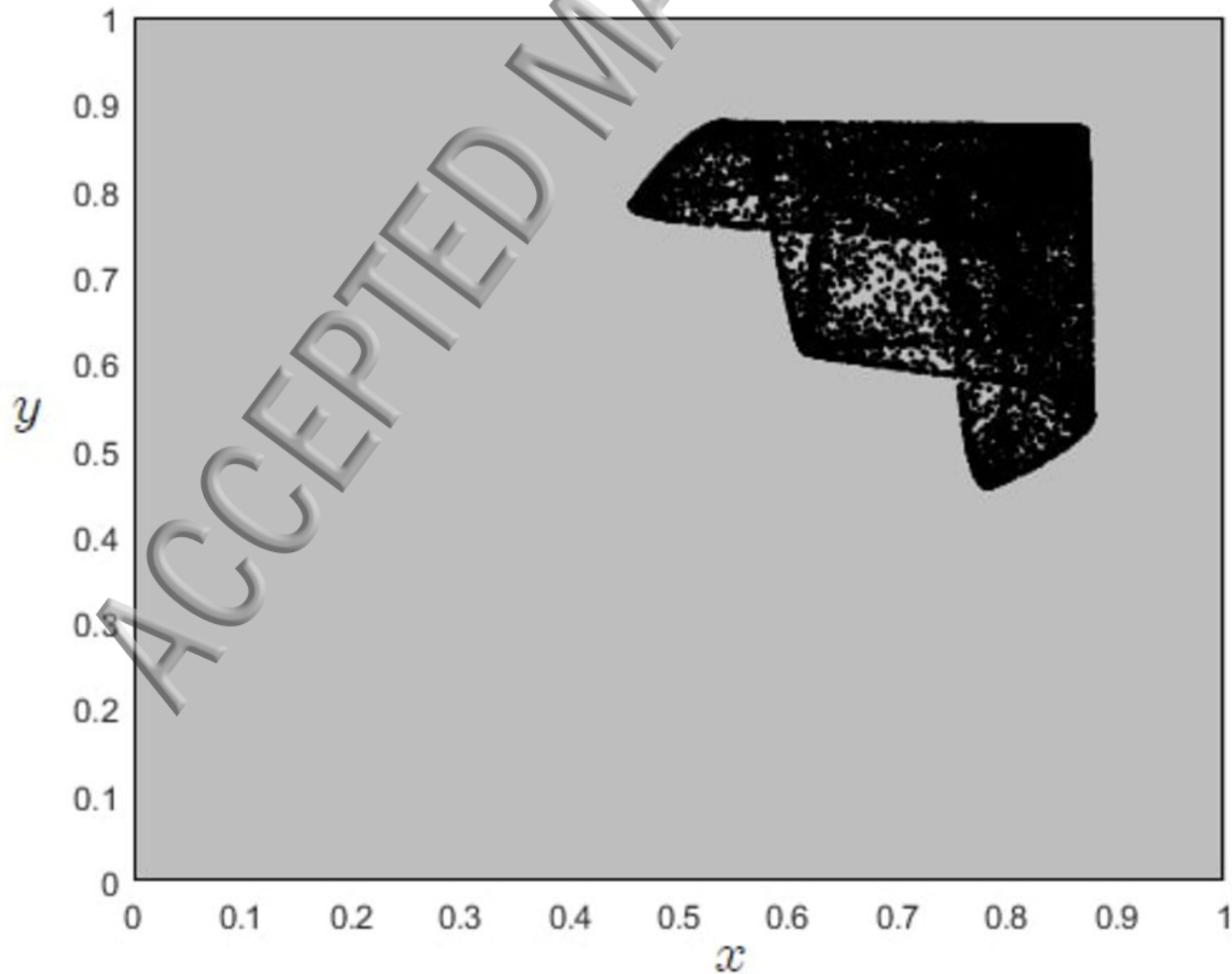


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
y
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x

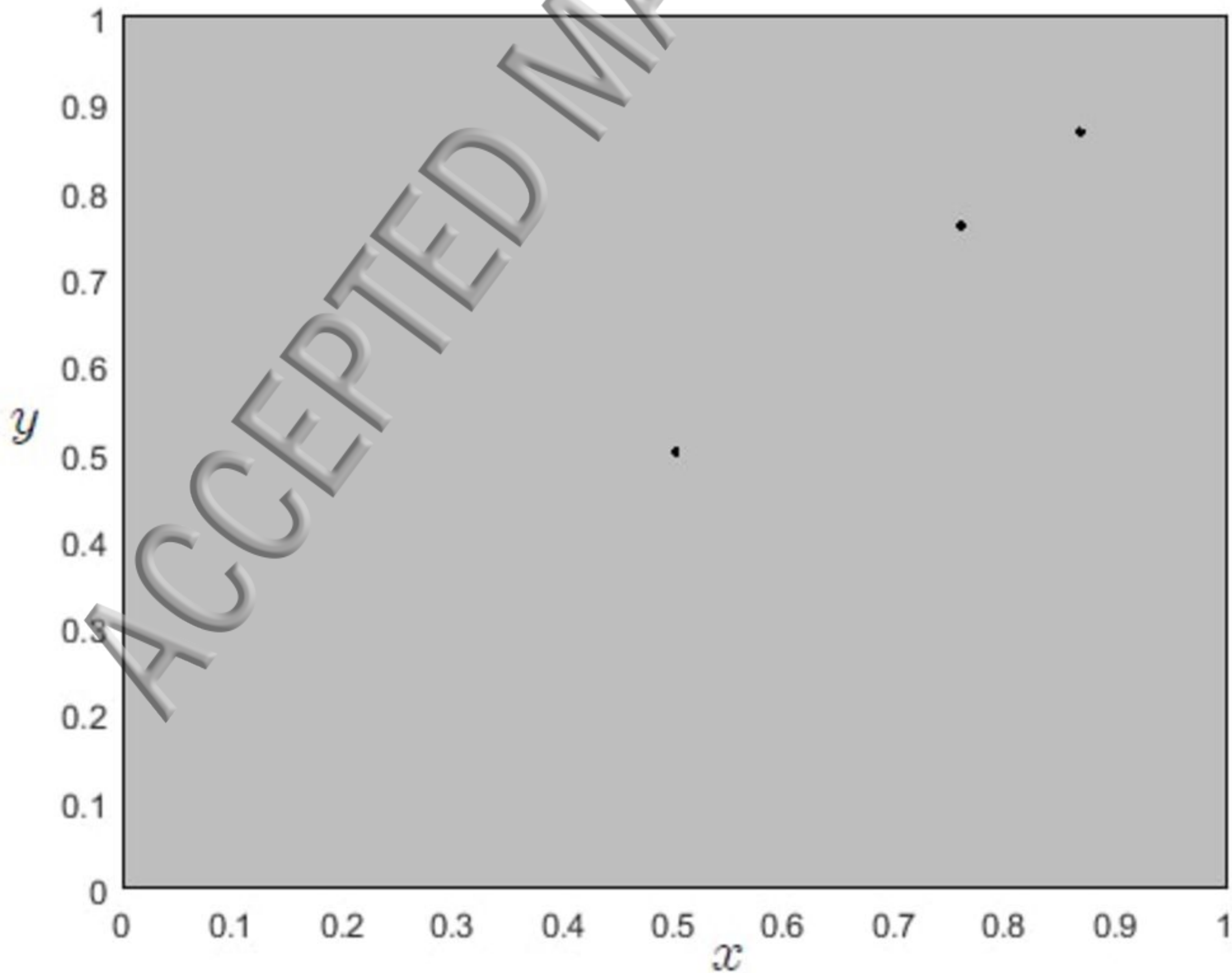

y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



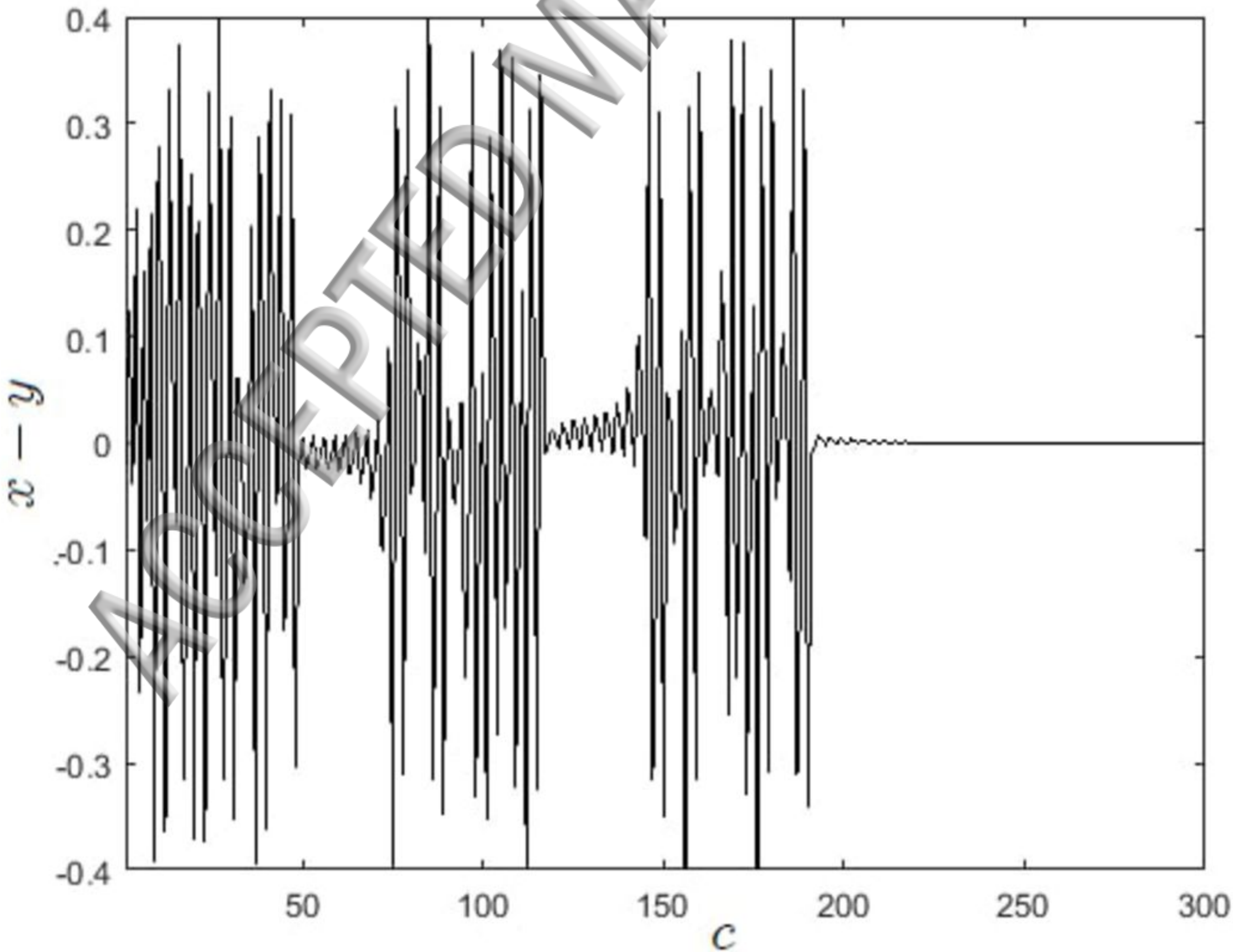
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
y
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x



0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
$x - y$
50
100
150
200
250
300
c