The Role of Relevant Knowledge and Cognitive Ability in Gambler Fallacy.

THE ROLE OF RELEVANT KNOWLEDGE AND COGNITIVE ABILITY IN GAMBLER FALLACY

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A new model by Stanovich et al. (2008) specifies the ways in which knowledge and cognitive capacity might interact in shaping reasoning performance. This model proposes that normative performance relies on knowing relevant rules and procedures (called "mindware"), detecting the need to implement them, and holding of the necessary cognitive capacity to acquire/use them appropriately. The aim of the present study was to test these assumptions investigating gambler fallacy inside probabilistic reasoning. Participants were primary (N=251) and college students (N=151). Results provide support for the claim that mindware plays an important role in probabilistic reasoning, and there is an interplay with cognitive ability. Theoretical and educational implications of results are discussed.

INTRODUCTION

According to dual-process theories, mental functioning can be characterized by two different types of process which have different strengths and weaknesses (e.g., Brainerd & Reyna, 2001; Epstein, 1994; Evans 2006; Sloman, 1996; Stanovich, 1999). Type 1 processes[1] are rapid and mandatory when the triggering stimuli are encountered, they do not require much cognitive effort, and they can operate in parallel. Type 1 processing is cognitively economical, its output is not consciously generated but seems to "pop" into consciousness (Sloman, 1996), and people "feel" intuitively that the responses are right (Epstein, 1994). Whereas Type 1 processing often leads to correct responses, in some cases they lead to systematic biases and errors. By contrast, Type 2 processes are relatively slow and computationally expensive, they are available for conscious awareness, serial, and often language based. Type 2 processes are also often associated with the use of normative rules and logical responding.

To exemplify the role of the two types of process in reasoning, imagine that in order to win a prize you have to pick a red marble from one of two urns (A and B). Urn A contains 20 red and 80 blue marbles, and Urn B contains 1 red and 9 blue marbles. When you respond to the task, you can compare the ratio of winning marbles in each urn (20% vs. 10%) which requires some time, mental effort and computations, or you can simply rely on the feeling/intuition that it is preferable to pick from the urn with

[1] Several terms have been used to refer to these two aspects of cognitive functioning (e.g. heuristic vs analytic, esperential vs rational), here we follow Evans (2006) in using the terms Type 1 and Type 2 processes.

more “favourable events”. In this example, both processes cue the normatively correct answer (that is, Urn A). On the other hand, it is possible to set up a task where Type 1 and Type 2 reasoning cue different responses. For example, if you can choose between picking a marble from an urn containing 10 red and 90 blue marbles, or from an urn containing 2 red and 8 blue marbles, the feeling/intuition that it is preferable to pick from the urn with more “favourable events” results in a normatively incorrect choice.

When Type 1 and Type 2 processes do not produce the same output, Type 1 processes usually cue responses that are theoretically incorrect and, according to dual-process theorists one of the most critical functions of Type 2 processes in these cases is to interrupt and override Type 1 processing. However, this does not always happen. In the case of a conflict between intuitions and normative rules even educated adults will predominantly produce heuristic responses.

In a recent paper Stanovich, Toplak and West (2008) outlined how people can reach a correct solution when the task besides the normative solution elicits competing response options that are intuitively compelling. They stated that people have to possess the relevant rules, procedures, and strategies, they have to recognise the need to use them, and then they have to have the necessary cognitive capacity to inhibit competing responses. In their model of reasoning, Stanovich and colleagues (2008) referred to rules, procedures, and strategies derived from past learning experiences as “mindware” (Perkins, 1995). If the relevant mindware can be retrieved and used, alternative responses became available to engage in the override of the intuitive compelling answers.

Errors can arise when we have a *mindware gap*. Indeed, when relevant knowledge, procedures, and strategies are not available, i.e. they are not learned (or poorly compiled), we can not have an override since to override the intuitive response a different response is needed as a substitute. Instead, when the relevant knowledge, procedures, and strategies can be easily retrieved, and a normative solution becomes available, errors are termed *override failures*: different alternatives are produced and there is the attempt to take the intuitive response offline, but this attempt fails since beliefs, feelings and impressions seem to be right beside rule-based considerations. So we have an override failure when people hold the rule but they do not base their answer on it.

Finally, Stanovich and colleagues (2008) addressed the role of several factors that might affect reasoning and, among them, particular attention was paid to cognitive ability. Kahneman and Frederick (2002) pointed out that “intelligent people are more likely to possess the relevant logical rules and also to recognize the applicability of these rules in particular situations […] that enable them to overcome erroneous intuitions when adequate information are available.” Thus, in both children and adults, reasoning errors are expected to be related to cognitive ability (Evans, Handley, Neilens, & Over, 2009; Kokis, MacPherson, Toplak, West & Stanovich, 2002; Morsanyi & Handley, 2008).

Starting from these premises, the aim of the present study was to test the Stanovich and colleagues' model inside probabilistic reasoning investigating gambler fallacy (Kahneman, Slovic & Tversky, 1982). Indeed, the model of Stanovich and colleagues provides a theoretical framework for integrating the educational and dual-process approaches emphasizing the role of both relevant knowledge and cognitive capacity in the development of reasoning skills. As the rules of probabilistic reasoning are very hard to derive from personal experiences (e.g., Fischbein, 1987) – that is, the actual patterns of probabilistic outcomes are "messy" or even resemble more what could be predicted based on the fallacies of probabilistic reasoning than on the relevant normative rules (see Hahn & Warren, 2009) - normative probabilistic knowledge mostly stem from what learned at school.

In details, we aimed (a) to investigate when sound probabilistic reasoning could be prevented by the lack of relevant knowledge (that we call "mindware" following Stanovich et al.'s terminology) for dealing with probability comparing different educational levels, and (b) to take into account the role of individual differences in cognitive ability and the interactions between mindware and cognitive capacity.

Committing gambler fallacy, people tend to estimate the likelihood of an event by taking into account how well it represents its parent population, i.e. a sequence of the same outcome (given two possible options) must be followed by the other outcome in order to equilibrate the proportion. In this way they do not take into account base-rates along with the independence notion. In the present study gambler fallacy was investigated in primary students since these basics of probability are taught to the fourth and fifth graders following the Italian national curricular programs[2]. Then, we compared primary school students probabilistic reasoning to college students in order to better explore the role of mindware starting from the assumption that relevant knowledge should be consolidated through education as well as the ability to recognize the need to use it in specific situations. In sum, we included three groups: students before they were taught probability issues (third graders), students who had been taught probability issues (fifth graders) and college students who had encountered issues related to probability throughout primary to high school years.

We predicted that probabilistic reasoning performance strongly relied on relevant mindware. We expected that younger primary students should perform worse due to their *mindware gap*. This difference should be observed even when individual differences in cognitive ability are partialled out. We predicted that college students would generally perform better due to their more consolidated knowledge of the

[2] Specifically, curricula include statistical surveys and their representations, some linguistic/conceptual issues related to possible, impossible, improbable events, and the development of judgment under uncertainty and estimation of odds through games of chance, inside the classical definition of probability (see http://www.rhoda.it/programm.htm). These issues were firstly included in the 1985 reform, and more recently the importance of teaching these topics was stressed in the revised government program (Moratti, 2004).

relevant rules of probability, and their higher cognitive ability. Whereas no difference should be observed among older primary students and college students once individual differences in cognitive ability are taken into account.

METHOD

Participants

The participants were 251 primary school students attending Grade 3 (*n*= 133, 68 males; mean age: 8.3 yrs) and Grade 5 (*n*= 118, 65 males; mean age: 10.5 yrs) and 151 college students (30 males; mean age: 20.3 yrs). The primary schools students were enrolled in Italian primary schools that serve families from lower middle to middle socioeconomic classes. Primary students were invited to participate. Their parents were given information about the study and their permission was requested. The college students were all students in Psychology degree program at the University of Florence (Italy). They were volunteers, and did not receive any reward for their participation in this study.

Measures and Procedure

Gambler Fallacy Task: A preliminary version of this task was used in a previous study run with children and college students (Chiesi & Primi, 2009). It consists in a marble bag game in which different base-rates in combination with two different sequences of outcomes were used. Using marbles - compared to the tossing of a regular coin traditionally employed to test gambler fallacy – the present task allows for testing this bias with both equally likely and not equally likely proportions. In detail, the task was composed of 3 different trials in which proportion of Blue (B) and Green (G) marbles varied (15B & 15G; 10B & 20G; 25B & 5G).

Before the actual task was presented, all children were shown a video in order to exemplify the concept of sampling with replacement. The bag shown in the video has a see-through part and instead of drawing a marble from the bag, the marble is pushed into the corner and then moved back inside the bag. Since the bag is closed it's a way to make clear that the number of the marbles stays always the same.

After the video, each participant received a sheet where it was written the following instruction: "*15 blue and 15 green marbles have been put into a bag such as the one shown in the video and one ball has been pushed in the see-trough part*" The first question was "*It's more likely it is...*". The following instruction explained that: "The *game was repeated with same bag and a sequence of 5 green marble was obtained*". The second question was: "*The next one is more likely it is....*". The following instruction told that: "*The game was repeated again with same bag and a sequence of 5 blue marble was obtained*". The third question was: "*The next one is more likely it is....*". After this first trial, the other two trials with different marbles proportion were presented, and for each one the three questions were asked. In sum, each participant had to answer 9 questions, three for each trial.

We formed two composite scores summing correct answers. One represents the necessary knowledge to tackle the task, i.e. how the probability of a single event changes referring to base-rates, and it was called Mindware score (range 0-3). The other, represents normative reasoning, i.e. higher the score, higher the respondent's ability to avoid gambler fallacy, and it was called Probabilistic Reasoning score (range 0-6).

After the Gambler Fallacy task, cognitive ability was measured using two short forms of the Raven's Matrices, one suitable for children, the other for adults.

Set I of the Advanced Progressive Matrices (APM–SET I): To measure children's cognitive abilities the APM–SET I (Raven, 1962) was administered as a short form of the Raven's Standard Progressive Matrices (Raven, 1941) as suggested by Nathaniel-James et al. (2004). The Set I of APM is composed by 12 matrices increasing in their difficulty level, and the items covered the range of difficulty of SPM (Raven, 1962). These items are composed of a series of perceptual analytic reasoning problems, each in the form of a matrix. The problems involve both horizontal and vertical transformation: figures may increase or decrease in size, and elements may be added or subtracted, flipped, rotated, or show other progressive changes in the pattern. In each case, the lower right corner of the matrix is missing and the participant's task is to determine which of eight possible alternatives fits into the missing space such that row and column rules are satisfied. Test adaptation to the Italian children population was done using IRT analysis procedure (Ciancaleoni, Primi & Chiesi, 2010).

*Advanced Progressive Matrices Short Form (*APM–SF*,* Arthur & Day, 1994): College students were administered the Advanced Progressive Matrices Short Form (Arthur & Day, 1994). The APM-SF is composed by 12 matrices derived from the APM. Matrices characteristic are described above. Test adaptation was done using IRT analysis procedure (Primi, Galli, Ciancaleoni, & Chiesi, 2010).

RESULTS

As expected, a differences between Grade 3 and Grade 5 was found in Mindware ($t(247)= -4.3$, $p<.001$, $d=-.55$) and Probabilistic Reasoning ($t(247)= -4.92$, $p<.001$, $d=-.62$). Older children performed better (Mindware: $M=1.90$; $SD=.86$; Probabilistic Reasoning: $M=2.83$; $SD=1.52$) than younger children (Mindware: $M=1.45$; $SD=.80$; Probabilistic Reasoning: $M=2.07$; $SD=.84$).

In order to control the effect of cognitive ability, two ANCOVAs were run in which Raven Matrices score was used as a covariate, Grade as the independent factor, and Mindware and Probabilistic Reasoning score as the dependent variables. The results showed that once the significant effect of cognitive ability was partialled out ($F(1,246)=15.16$, $p<.001$, $\eta_p^2=.06$), the main effect of educational level on Mindware was still significant ($F(1,246)=6.25$, $p<.01$, $\eta_p^2=.03$). In the same way, once the significant effect of cognitive ability was partialled out ($F(1,246)=31.24$, $p<.001$,

η_p^2=.13), the main effect of educational level on Probabilistic reasoning remained significant ($F(1,246)=6.53$, $p<.01$, η_p^2 =.03).

Starting from these results, we aimed to identify the relative weight of the two factors related to correct reasoning, that is cognitive ability and relevant knowledge. So, we conducted a hierarchical regression analysis - separately for third graders, fifth graders and college students - in which the criterion variable was the Probabilistic Reasoning, and predictors were Cognitive Ability, entered first into the analysis, and Mindware.

		Third Grade		
Step		Multiple R	*R^2 Change*	*F Change*
1	**COGNITIVE ABILITY**	0.03	/	4.64*
2	**MINDWARE**	0.00	0.00	*ns*
		Fifth Grade		
1	**COGNITIVE ABILITY**	0.19	/	26.24**
2	**MINDWARE**	0.30	0.11	18.27**
		College		
1	**COGNITIVE ABILITY**	0.05	/	4.31*
2	**MINDWARE**	0.47	0.42	68.14*

*$p < .05$, **$p < .01$

Table 1: Hierarchical regression for each student group with Cognitive Ability and Mindware entered as predictors of the Probabilistic Reasoning score.

Different patterns of results were observed for the three groups (Table 1). In third graders probabilistic reasoning was totally unrelated from mindware, and cognitive ability accounted for a little part of it (less than 5%). In fifth graders both cognitive ability and relevant mindware predicted probabilistic reasoning: cognitive ability accounted for 19% of the variance, and mindware accounted for an additional 11%. Finally, mindware explained in large part college students' reasoning accounting for the 42% of the variance.

Results show that both cognitive ability and relevant mindware would lead to increase students' reasoning performance and the relative weight of the two factors depend on educational level. Moreover, we can argue that students who correctly answered the first question not only hold the relevant mindware but the question makes them aware of the need to use it in answering the following questions. In other word, mindware was retrieved and made available to substitute the compelling intuitive response likely elicited by the sequence of five identical outcomes (i.e., all green/blue marbles).

In order to further ascertain the role of mindware on probabilistic reasoning, we compared who hold a well-learned rule and who lack or hold a poorly compiled rule. To do that, we created two groups (Hold vs Lack) using as a criterion the maximum Mindware score (that is, who always answer correctly, even when the correct answer was "blue and green are equally likely" and not a dichotomous choice between blue and green). In this way third graders were excluded from the analysis since only few students (as expected) were found to belong to Hold group. A 2X2 ANCOVA was run in which cognitive ability was used as a covariate, Educational Level (Fifth Grade vs College) and Mindware (Lack vs Hold) as independent variables, and Probabilistic reasoning as dependent variable. The results showed that once the significant effect of cognitive ability was partialled out ($F(1,199)=15.58$, $p<.001$, $\eta_p^2=.07$), the main effects of Educational Level ($F(1,199)=5.03$, $p<.05$, $\eta_p^2=.03$) and Mindware ($F(1,199)=113.07$, $p<.001$, $\eta_p^2=.36$) were significant, as well as the interaction between them ($F(1,199)=4.54$, $p<.05$, $\eta_p^2=.02$) (Figure 1). Looking at the effect sizes, we can observe that the stronger difference depends on mindware, i.e., in both groups students had a low performance when they lack, or lack to retrieve and apply, the relevant mindware.

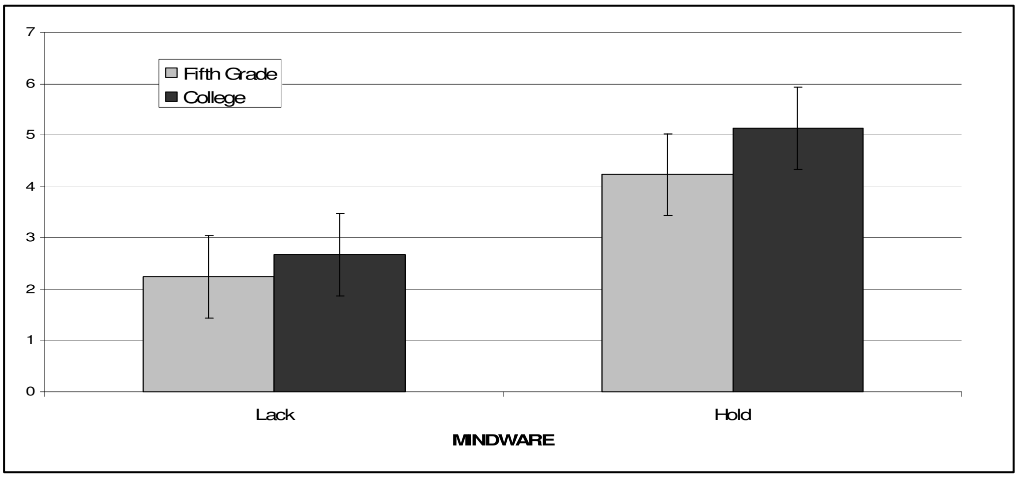

Figure 1: Means of Probabilistic Reasoning score in function of Mindware and Educational Level.

CONCLUSION

In this work we investigated the effects of relevant knowledge on primary and college students' probabilistic reasoning ability, and we examined the interactions between relevant knowledge and cognitive ability. According to Stanovich et al. (2008), the present study suggests that in solving gambler fallacy tasks the correct solution can be reached holding the relevant mindware and recognising the need to use it. Moreover, we found that individual differences in cognitive ability can be accounted for explaining sound reasoning in both primary and college students but, once the effect of cognitive ability has been taken into account, if the relevant knowledge is hold, retrieved and applied primary and college students perform equally well. In the same way, when they do not possess it or use it, their performance is equally bad.

In sum, correct probabilistic reasoning relies strongly on knowledge about rules. Since these rules are very hard to derive from personal experiences, we may conclude that normative probabilistic reasoning mostly stem from what learned at school. In this way, it becomes relevant define methods to make students aware of the need for rules even when they "feel" that these rules do not work, that is when conclusions derived from the theory are counterintuitive.

This study offers some cues to cross the bridge from a psychological approach to an educational approach. Psychological theories on reasoning assert that people are prone to rely on intuitions and that in dealing with probability intuitions seem to be right beside rule-based consideration. Nonetheless, rules are needed to reason normatively and to avoid biases. Didactical interventions have to focus on solving this puzzle.

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