The Estimation of Buried Pipe Diameters by Generalized Hough Transform of Radar Data

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Abstract

The generalized Hough transform method is applied to the measurement of the diameters of buried cylindrical pipes by Ground Penetration Radar (GPR). 600 MHz radar scans across long pipes, buried in one metre or so of soil, show complex reflection patterns consisting of a series of inverted hyperbolic arcs. The time of flight $t(y)$ as the radar probe is scanned along an axis, y, perpendicular to the pipe, shows an arc whose shape and position depends on 4 unknown variables: y_0, the position of the center of the pipe along the scan, z_0, the depth of the pipe center, R_0, its radius and V_0 the velocity in the medium. Analytic expressions for the solution of these variables have been obtained. They use sets of times t_i at corresponding positions y_i, along the arc, depending on the number of variables to be determined. In the generalized Hough method many such sets of times are chosen randomly from points on the arcs. The results are presented for example as peaks in an accumulator space for each variable. The method is demonstrated for a 0.18 m radius concrete pipe buried at a nominal 1 m depth in a road. Using data acquired at 600MHz frequency (around 0.16m wavelength in soil) the estimated radius was 0.174 ± 0.059 m.

1. Introduction

Beneath most roads are many pipes, of varying sizes, which may one day need to be dug up. The location and identification of these pipes can be very valuable. Ground Penetrating Radar is an established technique for identifying the position of these pipes [1]. A portable radar source emits a very short pulse at a typical radar frequency of 600MHz, or around 0.16m wavelength. The radar pulse penetrates the ground for a meter or so, and the reflected intensity $I(y,t)$ can be recorded as a function of the position y across the road and the round-trip time of flight $t(y)$. Figure 1 shows a typical "B-Scan" plot showing the reflected intensity gray scale as a function of the scan position y perpendicular to pipe axes, and the time of flight t. If the radar velocity V_0 is known for the soil under the road, the time of flight may be converted into range $S = {}^1/_2 V_0 t$.

The equation for the time of flight from the source to the first interface of a cylindrical pipe of radius R_0 at a position and depth (y_0, z_0) is

$$V_o t/2 = [(y - y_0)^2 + z_o^2]^{1/2} - R_o + V_o t_0/2 \tag{1}$$

where t_0 the time of flight to the surface of the road, often called the "main bang" since it gives a large easily identified radar peak. It must be measured precisely since it is seen from eq. 1 that the pipe radius adds to it directly. This equation gives rise to the characteristic overlapping inverted hyperbolic patterns that are readily identified in the figure. This equation may usefully be written in terms of two readily measurable quantities: the apex time t_a and the curvature C_a at the top of the hyperbola. These are given by [2], $t_a = 2(z_0 - R_0)/V_0$ and $C_a = 2/(z_0 V_0)$. Figure 2 shows a family of hyperbola with the same apex time and curvature, but with varying ratios of the time to cross the pipe diameter to the apex time $p = 2R_0/(V_0 t_a)$

$$t(y) = (1 + p)[t_a + C_a(y - y_0)^2/(1 + p)]^{1/2} - p t_a \tag{2}$$

It is seen that the pipes of finite diameter may be distinguished from narrow pipes provided that the hyperbola is followed for enough distance, and corresponding depth, to distinguish the asymtotic gradient defining the velocity in the soil.
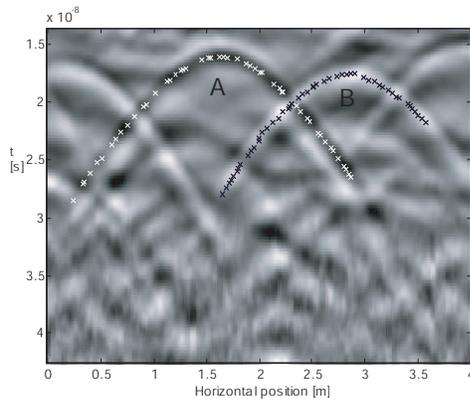
Figure 1: The time of flight t of reflections from a short radar pulse plotted as a function of the distance across the road. The pipe A on the left is a 036m diameter concrete pipe and that on right B is a 0.08m diameter electric cable.
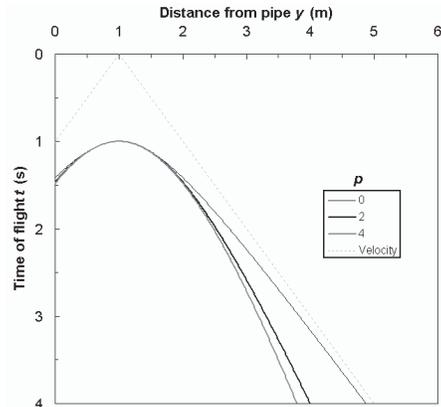

Figure 2: The calculated time of flight for pipes with the same apex position and curvature, but with differing diameters. The feint dashed line shows the asymtotic velocity, the thin line the reflection from a vanishingly small pipe.

2. The Generalized Hough Transform for Analyzing Hyperbolic Radar Reflections

Typical radar images contain overlapping hyperbolae from many buried pipes and there is no easy way to isolate the reflection from one peak from that of another.

Even if one hyperbola of the reflected pattern is isolated, as for example the peak A in figure 1, it is seen that it is overlapped by 2 or 3 major reflections. The generalized Hough transform provides a method provides a method for dealing with such cases [3,4].
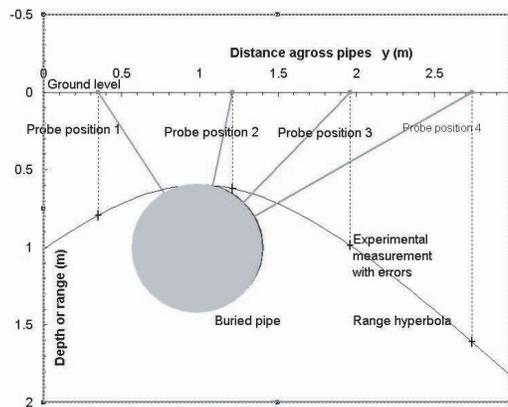

Figure 3: The generalised Hough method applied to a buried pipe of finite diameter in a medium of unknown velocity. Measurements from up to 4 probe positions are taken, taking into account the experimental errors in distance y and range s (or time t). These pairs of measurements $(y_i, s_i) i = 1, 4$ can be used to give an analytic solution of the 4 unknown parameters y_0, z_0, R_0, V_0. These parameters are used to implement an accumulator space in 1 to 4 dimensions whose peaks correspond to the most probable values of the parameters. Assumed experimental errors in the measurements Δy, Δs may be used to evaluated the probable error in the final parameters $\Delta y_0, \Delta z_0, \Delta R_0, \Delta V_0$

Figure 3 illustrates the case of a buried pipe of unknown position and diameter buried in soil of unknown velocity. From any 4 sets of probe positions y_i, $i = 1, 4$ the times of flight of the reflection from the pipe t_i may be estimated, together with the associated experimental errors in position and time Δy, Δt. These provide 4 example cases of equation 1, and these equations may be simultaneously solved to yield the four unknown parameters In general one variable at a time is eliminated from the equations. In the present case the velocity

variable V_0 was eliminated first. The equations for subsequent variables may use the solution for V_0 already found.

$$
\begin{aligned}
V_0 =& 2\{[t_1(y_4-y_3)(y_3-y_2)(y_4-y_2)+t_2(y_4-y_3)(y_1-y_4)(y_3-y_1) \\
& -t_3(y_2-y_1)(y_1-y_4)(y_4-y_2)-t_4(y_2-y_1)(y_3-y_2)(y_3-y_1)]/ \\
& [t_1t_2(t_2-t_1)(y_4-y_3)-t_3t_2(t_3-t_2)(y_1-y_4)+t_4t_3(t_4-t_3)(y_2-y_1) \\
& -t_1t_4(t_1-t_4)(y_3-y_2)-t_1t_3(t_3-t_1)(y_4-y_2)-t_2t_4(t_4-t_2)(y_3-y_1)]\}^{1/2}
\end{aligned}
\tag{3}
$$

$$
R_0 = \frac{-(1/V_0)(y_3-y_2)(y_1-y_3)(y_2-y_1)+(V_0/2)^2[t_1^2(y_3-y_2)+t_2^2(y_1-y_3)+t_3^2(y_2-y_1)]}{[t_1(y_3-y_2)+t_2(y_1-y_3)+t_3(y_2-y_1)]}
\tag{4}
$$

$$
y_0 = \frac{1}{2}\frac{(y_2^2-y_1^2)-(V_0/2)^2(t_2^2-t_1^2)-R_0V_0(t_2-t_1)}{y_2-y_1}
\tag{5}
$$

$$
z_0 = [(R_0+V_0t_1/2)^2-(y_1-y_0)^2]^{1/2}
\tag{6}
$$

These results may be used to contribute to a histogram in any desired variable whose peaks will indicate the most probable value of the variable. 2, 3 or 4 dimensional histograms may also be constructed to indicate the most probable value of all variables.
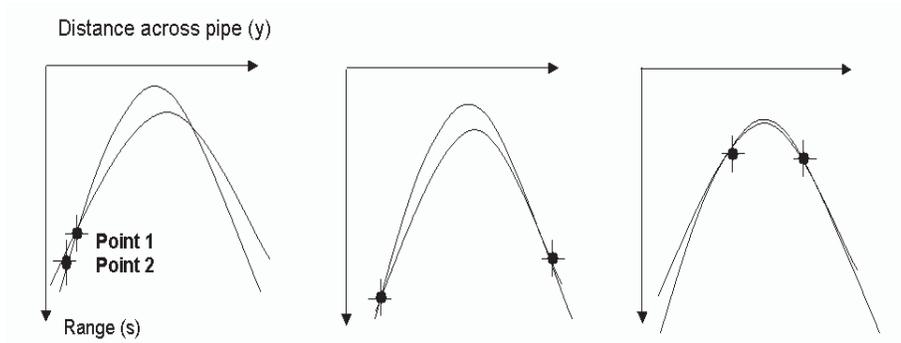

Figure 4: When experimental uncertainties are taken into account the choice of experimental points is crucial to the accuracy of the final parameters. In this two-dimensional case of determining position and depth, points (i) are "ill-conditioned", points (ii) are well conditioned for poisition, points (iii) well conditioned for depth

3. Including the Effects of Experimental Errors in the Measurement

As indicated in figure 1, errors in the position and time measurements are inevitable. Position errors occur from probe position uncertainty, probe movement irregularity and position binning. Time errors occur from experimental uncertainties, systematic errors in the analysis, and again binning errors. These will lead to errors in the solution of the unknown variables using equations 3 to 6. Much depends on the distribution of the points across the arc. Certain sets of points are "well conditioned" and will give more accurate solutions than "ill conditioned" sets. Figure 4 shows a simple example in two dimensions when two sets of position and range points y_1, s_1, and y_2, s_2 are used to determine the position and depth y_0, z_0. The equations for the position and depth of the pipe are

$$
y_0 = \frac{1}{2}\frac{(y_2^2-y_1^2)-(s_2^2-s_1^2)}{y_2-y_1}, z_0 = [s_1^2-(y_1-y_0)^2]^{1/2}
\tag{7}
$$

Differentiating with respect to the position error dy_1 gives

$$
\frac{dy_0}{dy_1} = \frac{y_0-y_1}{y_2-y_1}, \frac{dy_0}{ds_1} = \frac{s_1}{y_2-y_1},
\tag{8}
$$

As illustrated in figure 4(ii), well conditioned points for the position will have low differentials dy_0 / dy_1 and dy_0 / ds_1, which is satisfied when the separation between the points $(y_2 - y_1)$ in the denominator is large. Differentiating with respect to range error ds_1,

$$\frac{dz_0}{dy_1} = \frac{(y_1 - y_0)(y_2 - y_0)}{z_0(y_2 - y_1)}, \frac{dz_0}{ds_1} = \frac{s_{15}(y_2 - y_0)}{z_0(y_2 - y_1)}, \tag{9}$$

Summing these terms, the overall errors in y_0 and z_0 are

$$\Delta y_0^2 = \Delta s^2[(dy_0/ds_1)^2 + (dy_0/ds_2)^2] + \Delta y^2[(dy_0/dy_1)^2 + (dy_0/dy_2)^2]$$
$$\Delta z_0^2 = \Delta s^2[(dz_0/ds_1)^2 + (dz_0/ds_2)^2] + \Delta y^2[(dz_0/dy_1)^2 + (dz_0/dy_2)^2] \tag{10}$$

Experimental errors in the measurements may thus be used to give a weighted histogram when the more accurate "well conditioned" sets of points may be given a higher weight in the histogram than "ill conditioned" sets. A typical weighting is the exponential form

$$w_R = exp[-C\Delta R_0^2/ < \Delta R_0^2 >] \tag{11}$$

In practice, there will be compromise between high statistical precision at low C and low mean error at high C.
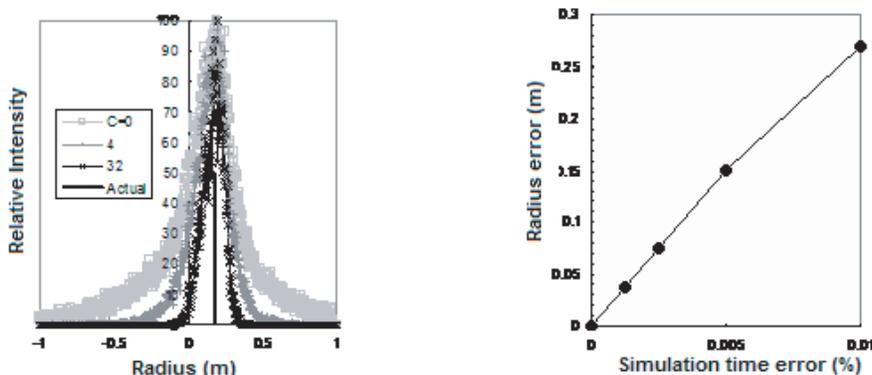

Figure 5: Left: The simulated radius distribution function for a 0.18m radius pipe at a depth of 1m. The light grey curve is the unweighted histogram, the others have weighted histotgrams according to equation 11 with values of C shown. Right: The measured width as a function of simulation error for C=4.

4. Simulated Results for a 0.18m Radius Pipe Buried at 1 m Depth

The left of figure 5 demonstrates the utility of the error-weighted Hough transform. The light gray unweighted distribution is clearly broader than the darker weighted curves with C=4 and 32. These curves have a simulated Gaussian timing error of 0.5%. In general there is a compromise to be made between good statistical accuracy in the distribution (low C) and low error in the mean radius (high C). On the right, the figure shows the increase in radius distribution peak width with increasing timing error. The correlations between radius, depth and velocity mean that the error of 0.1m in radius requires a precise 0.6% timing accuracy.

5. The Case of Two Pipes Buried Beneath a Road at About 1 m Depth

We first begin with a failure and a warning. When the full 4 unknown parameter computation was performed using the experimental data from peak A of figure 1, the radius distribution was broad with a peak at an implausible negative radius around –0.25m. Moreover the velocity distribution peaked at around 1.31 $\pm 10^8$m/s. A likely cause of the problem is the correlation between depth, radius and velocity in the 4-parameter solution. As figure 2 indicated a hyperbolic apex point and curvature is not sufficient to determine 3 parameters uniquely. It seems probable that the mean velocity in the soil well beneath the pipe, where the velocity can be most
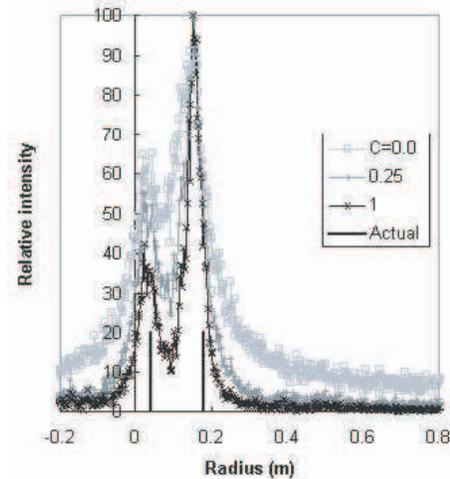
Figure 6: The radius distribution function for both pipes for a 3-parameter Hough transform in the variables y_0, z_0, R_0, Assuming a fixed value for the velocity derived from peak B. Again the light grey curve is the unweighted histogram, the others have weighted histograms with varying constants C. The vertical lines show the known actual radii of the pipes.

reliably determined, is different from that in the surface soil above the pike. Peak B is known to correspond to an electric cable with a negligible 0.04m radius. When a 3-parameter Hough analysis with parameters y_0, z_0, V_0 was performed using peak B points alone a velocity $V_0 = 1.102 \pm 0.05$ x10^8m/s was found. In figure 6 the velocity has been fixed at the value V_0, above and the radius distribution determined from the 3-parameter y_0, z_0, R_0Hough transform.

Conclusion

The generalized Hough method can be used to measure buried pipe diameters from radar measurements. Probable experimental errors can be used to weight ill-conditioned sets of points giving higher accuracy. Simultaneous position, depth, radius and velocity determination can be difficult.

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