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A practical approach based on Shape from Shading and Fast Marching for 3D geometry recovery under oblique illumination

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Abstract. Design of new industrial objects characterized by high stylistic content often starts from sketches or images of the product to be, subsequently, represented in a 3D digital form by using CAD software. To speed up this phase, a number of methods for automatic or semi-automatic translation of sketches or images into a 3D model have been devised all over the world also for reverse engineering purposes. When the image shading is a crucial information for recovering the final 3D shape, Fast Marching is recognized to be among the best method to date, especially for frontally illuminated scenes. Unfortunately, such a method cannot be directly applied when object illumination in the considered image is oblique. The present work is aimed to propose a simple, but effective, approach for recovering 3D shape of objects starting from single side illuminated scenes i.e. for solving non-eikonal SFS problems. Tested against a set of case studies, the method proved its effectiveness.

Introduction

Computer Aided Design is recognized as an essential task for all the phases characterizing the design, the development and the manufacturing of a new industrial product. However, especially for products characterized by a strong stylistic content, designers typically develop and communicate their ideas using handmade drawings, sketches, photographs or images in general. Such "hand-drawn" alternatives are, then, "translated" by CAD draftsmen into 3D models capable to provide a more realistic view of the object and to allow a deeper analysis of the stylistic design [1]. This translation process, involving a close interaction of stylistic designer and CAD operators in order to produce a CAD model carefully representing the designer's intent, is known to be considerably time consuming. This is a considerable concern to be considered when cost and time to market are crucial issues for the client and, all the more, when several alternatives have to be evaluated before proceeding with the manufacturing process.

In order to confront with these issues, in the last few years a number of Computer-based methods have been devised with the aim of speeding up the 3D reconstruction process from single images or sketches [2,3]. In case the designer produces an image, or sketch, where the three-dimensional effect of the shape of the designed object is inferable by observing the shading, the most important class of methods for performing the 3D reconstruction is the so called Shape-from-Shading (SFS). This approach is also very useful for reverse engineering purposes, when a "realistic" image of the object to be retrieved is available, but the original (actual or virtual) 3D geometry is not.

As widely known [4], SFS is a method based on the fact that, once light direction and albedo of the object are defined, it is possible to retrieve the slope of the surface with respect to the light direction in each point, by analyzing the brightness of the correspondent pixel in the image.

By assuming the image is generated by using a parallel projection (no perspective) along z direction onto xy plane, the surface of the object is completely diffusive (i.e. Lambertian), the albedo is uniform, and the light source set at infinite distance, the formulation of the problem is reduced to the well-known Partial Differential Equation (PDE), to be solved with respect to z :

$$\frac{1}{\rho} I(x,y)\sqrt{1+|\nabla z|^2} + (l_x, l_y)\nabla z - l_z, \tag{1}$$

where:

- $L = \lfloor l_x, l_y, l_z \rfloor$ is the unit-vector opposed to light direction;
- $\nabla z = \left[\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right]$ is the gradient of z;
- ρ is the albedo;
- $I(x,y)$ is the brightness of the image to be retrieved.

In case light direction corresponds to the viewing direction, the observer and the object are aligned and the surface appears frontally illuminated, the PDE (Eq.1) may be simplified into the eikonal form [5] shown in Eq.2:

$$|\nabla z| = \rho\sqrt{\frac{1}{I^2} - 1}\,. \tag{2}$$

The solution of Eq. 2 provides the z coordinates of the points belonging to the (frontally illuminated) imaged surface.

Fast Marching Algorithm

In the particular case of eikonal problem, several efficient methods have been proposed in literature [5]. Among them, the Fast Marching Method (FMM) is, probably, the fastest method to solve eikonal forms and, at the same time, the one that provides the best results in terms of accuracy.

Developed in the nineties by Sethian *et al.* [6], this method considers a special case of solution front evolution, which "actually performs a plane sweep along the lighting direction". For each time step, this plane crosses the surface of the scene $z(x,y)$ with a 2D closed curve. Seen from the image in the light source coordinates $I(x,y)$, the projection of this curve on the image plane moves as the plane sweeps over the surface.

More precisely, the FMM algorithm proceeds according to the following pseudo-code:

STEP 1 – Retrieve a "cost map" $C(x,y)$ using Eq.3. This transformation is allowed by the fact that the pixel brightness value $I(x,y)$ varies with the slope of the relative portion of the surface.

$$C(x,y) = \sin(\cos^{-1}(I(x,y))) \tag{3}$$

STEP 2 – Set a starting region corresponding, on the image, to white pixels relative to the maximum or minimum in height of the expected surface. All the points (x_0, y_0) of such a region, called "solved pixels", are crossed at time $T(x_0, y_0)$. The staring region coincide with the minima values of the cost function corresponding to an absolute maximum or minimum of the expected surface.

STEP 3 – Analyze the *k*-neighborhood of the solved pixels (i.e. the so called "front pixels") where *k*, typically, is set equal to 4 or 8. The first pixel to be crossed by the wave is the one with smaller value in terms of cost. Such a pixel, whose coordinates are generically set equal to (x_1, y_1) is crossed at time $T(x_1, y_1)$ provided by Eq.4:

$$T(x_1, y_1) = T(x_0, y_0) + dT\,, \tag{4}$$

Where $dT > 0$ is the transition time from pixel to pixel (it can be set, for instance, by the user). The pixel with coordinates (x_1, y_1) can be considered as a new solved pixel.

STEP 4 – Update the starting region with pixel with coordinates (x_1, y_1) and the new starting time is set to $T(x_0, y_0) = T(x_1, y_1) + dT$.

STEP 5 – Update the cost map by subtracting to all the remaining neighbors of the original starting region the value dT:

$$C_{new}(x, y) = C(x, y) - dT\,. \tag{5}$$

STEP 6 – Iterate steps 3-5 until all the pixels are solved. The final result of this procedure consists of a surface S resembling the expected ones.

Unfortunately, the FMM has the two main following drawbacks: 1) it is able to retrieve only monotonic surfaces and 2) it is able to correctly solve only frontally illuminated images. The first issue has been successfully overcome by segmenting the image in monotonic areas and in recomposing the obtained surfaces to obtain the entire domain [7]. This work aims to confront with the second issue, proposing a new approach to solve SFS problems with oblique lighting, i.e. non-eikonal problems. The devised method, described in the following Section, does not claim to be a rigorous mathematical procedure but, instead, to be a practical and effective method for solving the oblique-illumination problem, allowing to reliably retrieve a surface resembling the expected one. This is in fact demonstrated by the successful application of the proposed method to a number of case studies.

FMM-based approach for 3D geometry recovery under oblique illumination

Let us assume that FMM provides the exact solution i.e. the actual surface generating the image $I(x, y)$, in case of frontal illumination; in other words that it solves successfully the eikonal equation. Once we face up to a non-eikonal problem, i.e. when the illumination of the scene is oblique, the FMM is not able to solve the SFS problem directly. In fact, if this method is applied directly to the input image, pretending the illumination to be frontal, the obtained solution results into a distorted solution S'. This can be, for instance, seen in Fig. 1b, where the reconstruction of spherical segment under side illumination is provided and compared with the ground truth (Fig.1a). Moreover, the solution would not coincide with the one obtained by a rotation of S' surface to align observer with illuminant.

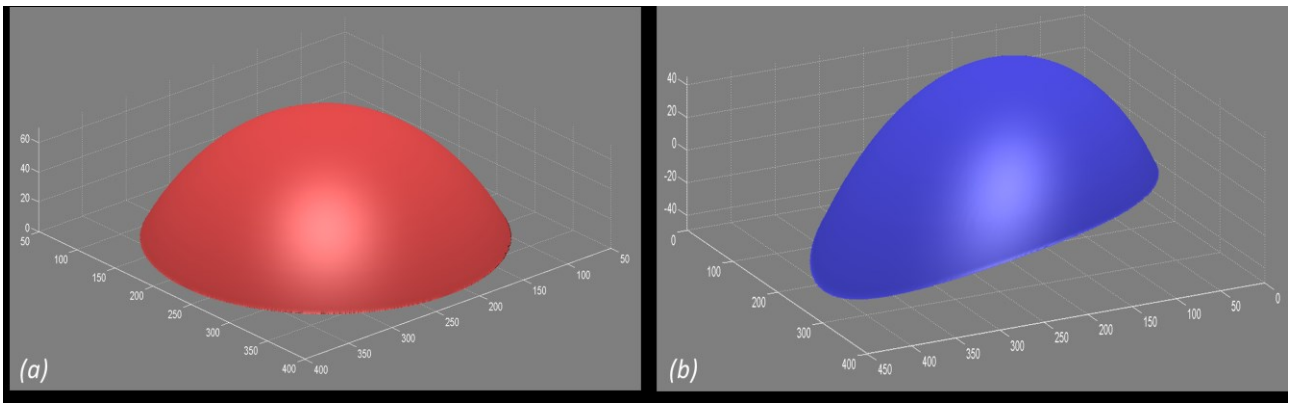

Fig. 1. (a) Spherical segment ground truth; (b) spherical segment reconstructed using FMM.

Conversely, the algorithm we propose, allows to obtain satisfactory surfaces in case of oblique lighting condition, just following a few simple steps:

STEP 1 – Solve the SFS problem applying the 6-steps algorithm described above (i.e. the FMM) to the input image $I(x,y)$ (see for instance Fig. 2a), as if the scene on the image is under frontal illumination; the result consists of a surface S' (see Fig. 2b) .

STEP 2 – Get the normal map nm' from S'.

STEP 3 – Rotate each unit vector of the normal map nm' around the axis n normal to the plane identified by L and $L_0 = [0,0,1]$ by an angle α, defined, in its turn, as the angle between L and L_0 themselves (see Fig. 2c). Such a rotation allows the definition of a new normal map $nm'' = R \cdot nm'$, where R is the rotation matrix around n.

STEP 4 – Compute the image $I''(x,y)$ as the dot product between each unit vector belonging to nm'' and L_0. The new image $I''(x,y)$ resembles the starting one (i.e. $I(x,y)$) but now, actually, it represents the scene under frontal illumination. As a result, this step allows to bring the oblique SFS problem back to a SFS frontal one.

STEP 5 – Solve the new SFS problem, by applying FMM to $I''(x,y)$. At the end of this step, the final surface S is obtained (see Fig. 2d); under the hypothesis stated at the beginning of this Section, S surface illuminated by L, exactly generates the image $I(x,y)$.

In Fig. 3 the surface of the hemisphere of Fig. 1 and the error map (projected in the *xy* plane) between S and the ground truth are shown, demonstrating the effectiveness of the reconstruction.

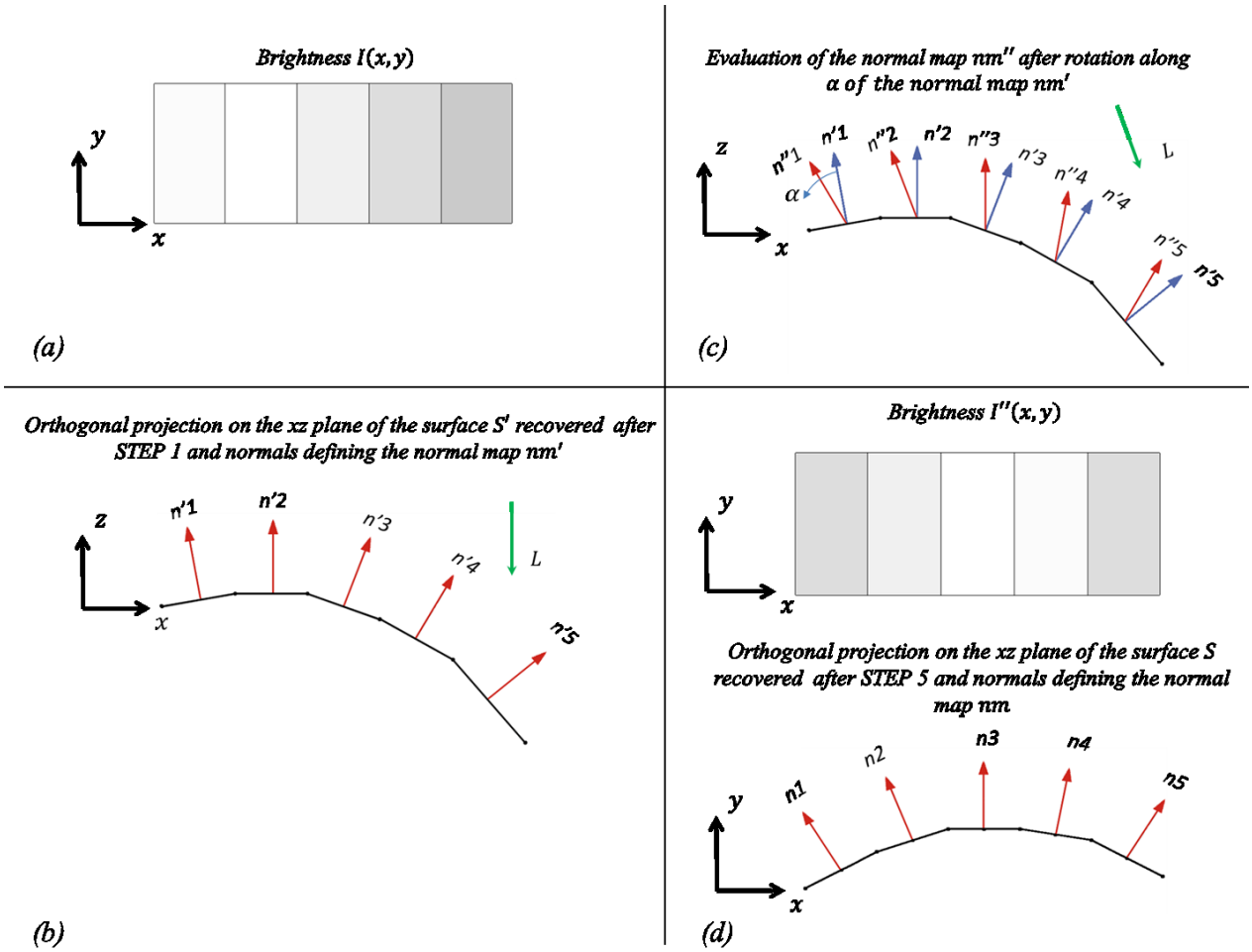

Fig. 2 – (a) Starting exemplificative image; b) surface S' recovered after STEP 1; (c) Evaluation of the normal map nm'' after STEP 3; (d) Final surface S after STEP 5.

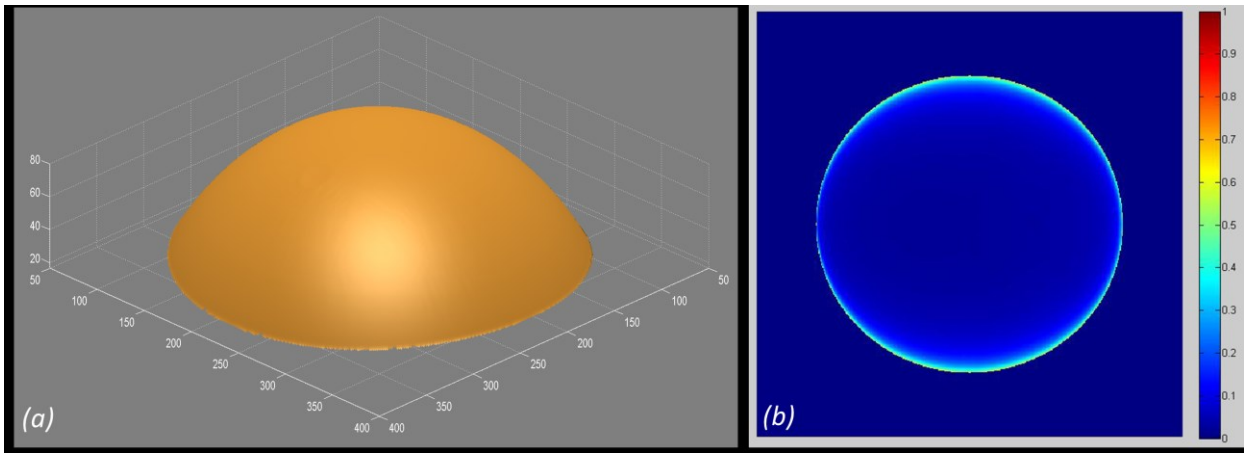

Fig. 3. (a) Spherical segment reconstructed using the proposed algorithm; (b) Error map in the plane xy.

Case Study

The procedure described above, providing a practical method for solving non-eikonal problems, has been tested on an extensive set of case studies. For this purpose, the procedure has been implemented using Matlab® programming environment. In the present section an exemplificative case study is shown. This concerns the 3D surface reconstruction of a commercial PC mouse starting from a single synthetic image obtained using oblique illumination (see Fig. 4a). In Fig. 4b the surface S' obtained after STEP 1 is shown; Fig. 4c shows the final surface S.

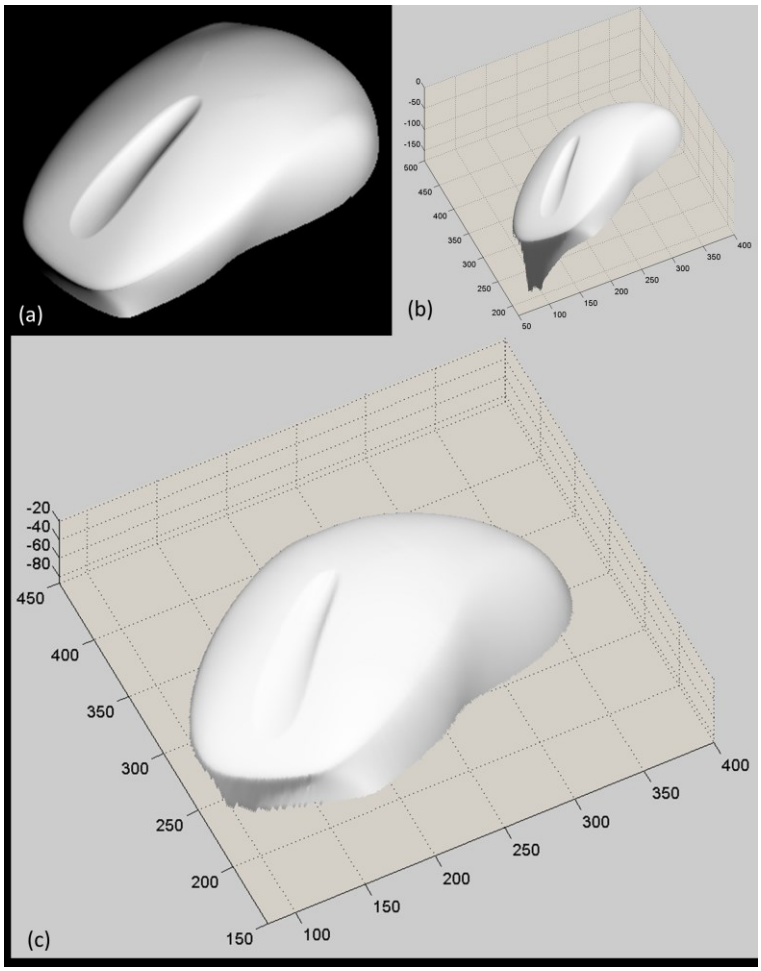

Fig.4. (a) Image of a commercial PC mouse obliquely illuminated; (b) reconstruction after STEP 1 of the proposed procedure; (c) final surface S.

Conclusions

The present paper described a simple and effective method for recovering the 3D geometry of objects starting from single images where shading is represented under oblique illumination. The method is described by a step-by-step procedure meant to simplify the whole reconstruction problem. Even if, from a strictly mathematical point of view, the proposed approach is not rigorous, it proves to be effective in performing accurate 3D reconstruction as demonstrated by a series of case studies. The transformation proposed is only valid for the pure Lambertian model and a single light source at infinity. As a consequence future works will be addressed to other illumination models. Moreover, further test will be addressed to assess the method robustness on real-life images.

References

[1] L. Governi, R. Furferi, M. Palai, and Y. Volpe: Computers in Industry (2013), in press.

[2] A. Piquer, M. Contero and F. Naya: Computers & Graphics, Vol. 29 Issue 6, pp. 892-904.

[3] M. Carfagni, R. Furferi, L. Governi, M. Palai and Y. Volpe, in: *Applications of Mathematics And Computer Engineering*, edited by Alexander Zemliak and Nikos Mastorakis, AMERICAN-MATH'11, 5th WSEAS International Conference on Computer Engineering and Applications, CEA'11 , pp. 99-104.

[4] J.D. Durou, M. Falcone and M. Sagona: Computer Vision and Image Understanding, Vol. 109 Issue 1, pp. 22-43.

[5] R. Zhang, P.S. Tsai, J.E. Cryer and M. Shah: *IEEE Transactions on Pattern Analysis and Machine Intelligence,* Vol. *21 Issue* 8, pp. 690-706.
[6] J.A. Sethian, Proc. of the National Academy of Sciences, Vol. 93, Issue 4, pp. 1591-1595.

[7] G. Zeng, Y. Matsushita, L. Quan and H.Y. Shum, in: Computer Vision and Pattern Recognition, 2005. CVPR 2005. IEEE Computer Society Conference, Vol. 1, pp. 343-350.