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# Performance prediction of radial ORC Turboexpanders

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## **Abstract**

In this paper, a zero-dimensional model for the design of radial Turbo-Expanders for ORC applications is discussed, with special reference to the estimation of losses and efficiency; a comparison between different fluids (R134a, R1234yf, R236fa, R245fa, Cyclo-Hexane, N-Pentane) is presented and discussed, referring to a typical small-size application (50 kW). In the model, different methods for the design of radial turbines are screened, with special attention to the estimation of losses, for which correlations from literature are used. Real Equations Of State (EOS) are applied to the expansion process in place of the traditionally adopted Mach relationships for ideal gas, which is a significant advancement for modeling organic fluids in ORC, often operating near to critical conditions. The results show that the total to total efficiency of the designed machines range between 0.72 and 0.80, depending on the considered fluid. Generally, higher efficiency (1.5 – 2.5 % points) can be achieved adopting backswept-bladed rotors. The most significant losses come from the rotor secondary flows, due to the high curvature of blade profiles combined to the large pressure gradient. The best performing fluids are R236fa and R245fa, followed by R134a and R1234yf.

Finally, starting from the developed design tool, an off-design analysis of turbo-expanders is presented. Once the design data are available, the characteristic curves of the expander at variable temperature, pressure and fluid mass flowrate at the expander inlet for different values of the specific speed are built. It is thus possible to evaluate the performance of the radial expanders when working far from design point. This analysis, demonstrated for R134a, shows that the total to static

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32 efficiency has a relatively modest sensitivity to the off design of the expansion ratio, especially at  
33 corrected speed below the design value.

34

## 35 **Keywords**

36 Radial Turbine Design, Expansion Efficiency Losses, Off design, Micro ORC

37

## 38 **1. Introduction**

39 Organic Rankine Power Cycles (ORC) are becoming a leading technology for energy conversion,  
40 with special reference to low size (< 100 kW) and low-temperature applications ( $T < 150^{\circ}\text{C}$ ), where  
41 the use of steam is not convenient. The thermodynamic properties of organic fluids make them very  
42 interesting for small/medium size power plants (50 to 5000 kW or more, at present); applications of  
43 ORC cycles range from heat recovery at gas turbine discharge [1, 2, 3] or internal combustion  
44 engines [4, 5], to energy conversion from biomass [6], solar [7, 8, 9], and geothermal resources [10,  
45 11, 12]. Due to the low working temperature, ORCs have typically low efficiency levels: for this  
46 reason, the accurate design of the expander is a very important issue to avoid further appreciable  
47 reduction of performance. For these applications and power range, radial (or mixed flow) turbines  
48 are usually preferred to the axial ones, because they offer several advantages: a low degree of  
49 reaction (thereby simplifying sealing), capability of dealing with large enthalpy drops with  
50 relatively low peripheral speeds, possibility of adopting a single-stage design. On the whole, this  
51 results in good performance and affordable price. The optimization of the thermodynamic cycle,  
52 with special reference to fluid selection, has been studied widely in the last years. Fluid-dynamic  
53 design of turbo-expanders can take advantage of the availability of modern CFD methods [ ...];  
54 however, there is a need for preliminary design methods, and of modeling tools capable of  
55 predicting the off-design performance (which is determined, for example, by the variation of the  
56 resource for solar-driven EGS, or by variation of ambient temperature in geothermal or heat  
57 recovery applications). In the field of low power output (i.e. up to 100 – 150 kW), radial expanders  
58 are almost the only choice, with the eventual competition of screw expanders. Generally, literature  
59 is rich of theoretical – experimental correlations for the estimation of losses in axial expanders,  
60 whereas much less data are available for radial turbines [13 – 30]. Most data available refer to ideal  
61 gas and make use of the Mach compressibility relations [13-20]. This may not be a satisfactory  
62 approximation when dealing with ORC turbines, which operate near the saturation line or close to  
63 the critical point. In the model hereafter proposed, correlations from literature are used [13-30], but  
64 real Equations Of State (EOS) are applied to the expansion process (static and total variables) in

65 place of ideal gas relations. This is an important feature, allowing to preliminary design and  
66 performance prediction of turbo-expanders that work with real substances. The correlations can be  
67 refined progressively as more data on ORC expanders become available, either from field  
68 operation, or from specific test arrangements. The thermodynamic properties of the working fluids  
69 are calculated using the libraries of the EES software, which is the programming environment  
70 adopted in this work. Making use of the design model, a sensitivity analysis investigating the effects  
71 of the different design parameters on the expander performance is presented. Finally, an off-design  
72 model has been developed and some results are discussed, in order to assess the behavior and  
73 estimate the performance of the turbo-expanders when working out of nominal conditions. This  
74 often happens when the ORC high-temperature resource is a time-dependent energy source, like  
75 solar; but also, with seasonal change of condenser conditions, on account of the heat/mass transfer  
76 performance of the cooling system (condenser/cooling tower/air cooler).

77

## 78 **2. Fundamental design concepts and parameters for Radial-Inflow Turbines (IFR)**

79 Radial-inflow turbines have been less studied than the axial ones, and have been manufactured by a  
80 limited number of companies [31, 32, 33]. The fundamental design of radial-inflow turbines is  
81 presented and discussed in books and scientific papers [13-30], most of which are based on  
82 experimental work performed at NASA between 1965 and 1975 [16, 17, 18, 20, 30]. At that time  
83 radial turbines, working with ideal gases (air, helium), were designed and tested for aerospace  
84 applications. The high values of centrifugal stresses on rotor blades and the limits on materials  
85 performance and production technology led to the design of the ideal 90° IFR turbine: that is, a  
86 rotor having radial blades at inlet. Some years later (1983) NASA researchers [18] studied and  
87 developed a radial – inflow turbine with a more performing rotor design, characterized by a blade  
88 sweep angle  $\beta_2$  at rotor inlet (IFG, figure 1). The typical velocity triangles are shown in figure 2; in  
89 both cases, the absolute velocity  $c_3$  is assumed to result axial at rotor outlet, in order to guarantee  
90 good diffuser performance (figure 2). Generally, nominal design conditions are referred to zero-  
91 incidence at rotor inlet and zero-deviation at rotor outlet; however, in radial turbines, the best  
92 efficiency values are obtained when incidence is non-zero (figure 3): this is a consequence of the  
93 rotational flow, which displaces the tangential component of the relative velocity ( $w_u$ ) in the  
94 opposite direction with respect to the peripheral rotor velocity. From the technical literature [13, 14]  
95 it appears that the best values of rotor inlet angle in the relative flow are between  $-20^\circ$  and  $-40^\circ$ ,  
96 referred to the camber line direction (the positive sign is conventionally assumed toward the  
97 direction of peripheral velocity  $u$ ).

98 In radial turbine design, some non – dimensional parameters help designers to select a geometry  
99 optimizing efficiency using a limited set of variables; the following parameters are recommended in  
100 the literature [13, 14, 17, 19, 20]:

$$101 \frac{d_{3h}}{d_{3s}} = 0.40 \quad (1)$$

$$102 \frac{d_{3s}}{d_2} = 0.70 \quad (2)$$

103

104 Another important parameter is the *isentropic velocity ratio* ( $u_2/c_s$ ), which maximizes the  
105 efficiency in the range 0.69 – 0.71 [13, 14, 17, 19, 20].  $c_s$  is the *spouting velocity*, defined as the  
106 velocity at which the kinetic energy of the flow is equal to the isentropic enthalpy drop from turbine  
107 inlet stagnation pressure  $p_{01}$  to the final exhaust pressure [14]. The definition of spouting velocity is  
108 different depending on (I) whether a diffuser is present or not downstream the turbine, see  
109 relationships (3) and (4, 5) respectively, and (II) if total (4) or static (5) conditions are considered at  
110 the turbine exit (figure 4):

$$111 c_s = \sqrt{2(h_{01} - h_{4ss})} \quad (3)$$

$$112 c_s = \sqrt{2(h_{01} - h_{03ss})} \quad (4)$$

$$113 c_s = \sqrt{2(h_{01} - h_{3ss})} \quad (5)$$

114

### 115 **3. Design Guidelines**

#### 116 **3.1 Input data and expected process output**

117 The input data consist in the expander rated power output, in the thermo-fluid dynamic variables  
118 determined by the thermodynamic cycle [36], and in a set of dimensional and non – dimensional  
119 parameters chosen by the designer (table 1)..The outputs of the calculations are the basic geometry  
120 with the related velocity triangles and the efficiency of the designed expander. The model is also  
121 able to calculate the turbine losses and their relative share in the resulting inefficiency.

122

#### 123 **3.2 Preliminary sizing**

124 The first step is the preliminary calculation of geometry, using the non – dimensional parameters  
125 listed in table 1, which determine also the isentropic nozzle and rotor enthalpy variations. The load  
126 and flow coefficients ( $\Psi = \Delta h_0/u_2^2$  and  $\Phi = c_{m2}/u_2$  respectively) are adjusted to calculate the

127 peripheral velocity ( $u_2$ ), specific speed ( $n_s$ ), speed of revolution ( $\omega$ ) and meridional component of  
 128 absolute velocity at nozzle exit/rotor inlet ( $c_{m2}$ ). These data are used for the evaluation of the mass  
 129 flow rate  $\dot{m}$ :

$$130 \quad \dot{m} = \rho_2 c_{m2} b_2 d_2 \pi (1 - BK_2) \quad (6)$$

131 The meridional component of the absolute velocity at nozzle inlet ( $c_{m1}$ ) - which is the same as  
 132 absolute velocity ( $c_1$ ) as the flow at the IGV nozzle inlet is assumed to be radial - is given by the  
 133 application of mass balance in section 1 (figure 1):

$$134 \quad c_{m1} = \frac{\dot{m}}{\rho_1 \pi b_1 d_1 (1 - BK_1)} \quad (7)$$

135 Consequently, knowing the inlet enthalpy  $h_1(p_1, T_1)$ , it is possible to calculate the total enthalpy  
 136 ( $h_{01}$ ). The thermodynamic variables at point 2 ( $p_2, T_2, h_2, s_2, \rho_2, Ma_2, Ma_{u2} = \frac{u_2}{vS_2}, Ma_{r2} = \frac{w_2}{vS_2}$ ) and  
 137 the velocity triangles at rotor inlet are determined calculating first the nozzle isentropic expansion  
 138 and then the real transformation using the nozzle loss coefficient  $\xi_N$  (figure 2):

$$139 \quad h_2 = h_{2s} + 0.5 \xi_N c_2^2 \quad (8)$$

140 Once the nozzle exit/rotor inlet conditions are known, the thermodynamic variables at point 3 (rotor  
 141 exit/diffuser inlet, figure 1) are calculated by solving at first the rotor isentropic expansion and  
 142 assuming that the difference between the absolute velocities related to isentropic and real expansion  
 143 at that point is negligible. The relative velocity at rotor output can be calculated by the conservation  
 144 of rothalpy [14], figure 2:

$$145 \quad i = h + 0.5w^2 - 0.5u^2 \quad (9)$$

146 The meridional component of the absolute velocity at point 3 is determined by the conservation of  
 147 mass (figure 2):

$$148 \quad \dot{m} = \rho_3 c_{m3} \frac{d_{3s}^2 - d_{3h}^2}{4} \pi (1 - BK_3) \quad (10)$$

149 Finally, using the rotor loss coefficient and assuming an axial discharge at rotor outlet, the static  
 150 enthalpy and velocity triangles at rotor outlet can be calculated:

$$151 \quad h_3 = h_{3s} + 0.5 \xi_R w_3^2 \quad (11)$$

152 The calculation of the real conditions at diffuser outlet is done by combining the total enthalpy  
 153 balance and the definition of diffuser loss coefficient  $\xi_D$ .

### 155 3.3 Geometry of the stator (IGV)

156 The prediction of the angle of flow leaving the bladed nozzle of a radial turbine, discussed in the  
 157 previous section, is a fundamental design topic. The next step is the calculation of the angles of  
 158 blades, which are radial at inlet for no pre swirled IGVs. At outlet, while leaving the nozzle, the  
 159 flow does not follow the vanes completely but it turns toward the meridional direction of an angle  
 160 known as deviation, due to the combined effects of boundary layer growth (limited by the  
 161 accelerating flow) and the subsequent abrupt expansion due to the trailing edge thickness. As a  
 162 simple design approach, the actual angle of blades at nozzle outlet is calculated interpolating data  
 163 from Hiatt and Johnston [13, 29], which have a rather linear behavior, approximated by the  
 164 interpolating function  $\alpha_{b2} = 0.884 \cdot \alpha_{b2} + 4.56$ .

165 Another important parameter is the number of stator blades, which directly influences the losses.  
 166 Increasing the number of blades leads to better flow guidance at the price of higher frictional losses.  
 167 A general design approach to define the stator number of blades is that it should be a prime number  
 168 compared to the rotor one. In addition to this basic criterion, the criterion of Zweifel on the optimal  
 169 ratio between the chord and the blade spacing can be adopted [14; 23]. It suggests that, in order to  
 170 minimize the losses, the ratio between the tangential load of an actual to that of an ideal blade ( $\Psi_T$ )  
 171 should be about 0.80. Knowing the absolute flow angles at nozzle inlet and outlet, and after  
 172 calculating the chord length, from the expression of blade pitch  $Z \cdot s = \pi \cdot d_2$ , the following equation  
 173 (12) gives the optimum blade spacing, from which the number of blades can be easily determined:

$$174 \quad \Psi_T = 2 \left( \frac{s}{x} \right) \cos^2 \alpha_2 (\tan \alpha_1 + \tan \alpha_2) \quad (12).$$

### 176 3.4 Optimal incidence at rotor inlet and number of rotor blades

177 Calculating the number of rotor blades is a fundamental issue in the design of radial turbines,  
 178 because it defines the basic structure of the machine and has a primary role in the estimation of  
 179 losses. There are no absolute criteria allowing an univocal evaluation of number of blades; the  
 180 design guidelines tend to avoid very low local velocities near the blade surface in the inlet region of  
 181 the rotor, with a consequent tendency to early separation [13]. On the other hand, the adoption of a  
 182 high number of blades is not so convenient, especially for small rotors: the blockage effects, the  
 183 weight and inertia of the rotor become very high. Moreover, a large number of blades is also

184 responsible for a large wetted surface area, which increases the friction losses. In the present model,  
 185 rather than using the formulation proposed by Jamieson [14, 15], which determines an exceedingly  
 186 large number of rotor blades, the formulation proposed by

187 Glassman [16] is followed:

$$188 \quad Z_R = \frac{\pi}{30} (110 - \alpha_2) \tan \alpha_2 \quad (14).$$

189 Once the number of blades is determined, the optimum rotor incidence angle can be calculated. As  
 190 previously discussed, better efficiency values are achieved when incidence is non-zero. Referring to  
 191 the camber line direction, the best values of rotor inlet angle in relative flow are between 20° and  
 192 40° counterclockwise.

193 Referring to IFR rotors (radial blades at rotor inlet), the following are the recommended  
 194 correlations:

$$195 \quad \tan(\beta_{2opt}) = \left(\frac{2}{Z_R}\right) \frac{u_2}{c_{m2}} \quad (15), [14]$$

$$196 \quad \tan(\beta_{2opt}) = 1 - \frac{0.73\pi}{Z_R} \quad (16), [13]$$

$$197 \quad \tan(\beta_{2opt}) = \frac{-1.98 \tan(\alpha_2)}{Z_R \left(1 - \frac{1.98}{Z_R}\right)} \quad (17), [13]$$

198 The above equations (15 – 17) provide similar results, even though (16) has been obtained  
 199 considering minimum Mach number conditions at the rotor inlet [13]. From these relationships, one  
 200 can notice that the optimum rotor incidence angle depends on the number of rotor blades and on the  
 201 kinematic conditions of flow.

202 In the case of the IFG design (non- radial blades at rotor inlet), the procedure suggested by Meitner  
 203 & Glassman [18] can be followed by calculating first the optimal value of the peripheral component  
 204 of the absolute velocity:

205 (18), [18].

$$206 \quad c_{u2opt} = \begin{cases} u_2 \left( \frac{[1 - \sqrt{\cos(\alpha_{b3})} / (Z_R)^{0.7}] \{1 - [(r_3/r_2 - \varepsilon_{lim}) / (1 - \varepsilon_{lim})]^3\}}{1 - \frac{tg(\alpha_{b3})}{tg(\alpha_2)}} \right) & \text{if } \frac{r_3}{r_2} > \varepsilon_{lim} \quad (18a), [18] \\ u_2 \left( \frac{[1 - \sqrt{\cos(\alpha_{b3})} / (Z_R)^{0.7}]}{1 - \frac{tg(\alpha_{b3})}{tg(\alpha_2)}} \right) & \text{if } \frac{r_3}{r_2} \leq \varepsilon_{lim} \quad (18b), [18] \end{cases}$$

207 The parameter  $\varepsilon_{lim}$  can be determined by:

$$208 \quad \varepsilon_{lim} = \frac{1}{e^{8.16 \cos \alpha_{b3} / Z_b}} \quad (19), [18]$$

209 Once  $c_{u2, opt}$  has been calculated, it is possible to calculate the relative velocity and blade angle as  
210 follows:

$$211 \quad w_{u2, opt} = c_{u2, opt} - u_2 \quad (20), [18]$$

$$212 \quad \beta_{2, opt} = \tan^{-1} \left( \frac{w_{u2, opt}}{c_{m2}} \right) \quad (21), [18].$$

213

### 214 **3.5 Expander Efficiency, Power Output, Degree of Reaction and Specific Speed**

215 The calculation of the expander efficiency, power output, design degree of reaction and specific  
216 speed are following the guidelines in [37], which are briefly recalled for completeness.

217 The efficiency can be either referred to turbine discharge or including also the diffuser; in the first  
218 case, the Total-to-Total and Total-to-Static efficiency are given by:

$$219 \quad \eta_{tt} = \frac{h_{01} - h_{03}}{h_{01} - h_{03SS}} \quad (22)$$

$$220 \quad \eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3SS}} \quad (23)$$

221 If the diffuser is included, the Total-to-Static efficiency becomes:

$$222 \quad \eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{4SS}} \quad (24)$$

223 The power output can be calculated by one of the equivalent three following equations:

$$224 \quad W = \dot{m}(h_{01} - h_{03}) \quad (25)$$

$$225 \quad W = \dot{m}(u_2 c_{u2} - u_3 c_{u3}) \quad (26)$$

226

$$227 \quad W = \frac{1}{2} \dot{m}[(u_2^2 - u_3^2) + (c_2^2 - c_3^2) - (w_2^2 - w_3^2)] \quad (27)$$

228 The degree of reaction and the specific speed are given by:

229 
$$R = \frac{h_2 - h_3}{h_1 - h_3} \quad (28)$$

230 
$$n_s = \frac{NQ_3^{1/2}}{\Delta h_{0s}^{3/4}} \quad (29) [14]$$

231 where

232 
$$\Delta h_{0s} = h_{01} - h_{03ss} \quad (30)$$

233

234 **4. Calculation of losses**

235 In order to evaluate the actual performance of a turbomachine, the contributions of different losses  
 236 must be calculated. This calculation cannot be substituted by advanced CFD methods, because it  
 237 provides vital information to the designer, about the process of loss buildup determining the final  
 238 turbomachine performance. On the other hand, advanced CFD is very useful for cross-checking the  
 239 overall results of the efficiency/loss model, and often to supplement data which would require very  
 240 detailed (often impossible) measurements.

241 Generally, the first step of the design procedure considers consequently the effects of losses through  
 242 appropriate dimensionless coefficients. The related efficiency drop  $\Delta\eta$  is then subtracted from the  
 243 isentropic value to calculate the actual efficiency:

244 
$$\eta_{act} = \eta_s - \Delta\eta \quad (31), [13]$$

245 In this model, the overall loss of the turbine is obtained by the sum of several contributions, each  
 246 one estimated through correlations which depend on kinematic and geometric parameters. The  
 247 dimensionless loss coefficients are defined in several ways, and it is important to merge them to a  
 248 common basis, in order to apply them within the same model and do some reliable comparisons  
 249 with results from literature. Referring to the  $j^{\text{th}}$  loss:

250 
$$\xi_j = \frac{h_j - h_{j,is}}{\frac{1}{2}V_j^2} \quad (32), [13]$$

251 
$$\Delta q_j = \frac{h_{0j} - h_{0j,is}}{u_j^2} \quad (33), [13]$$

252 This is the approach followed in the present model: starting from non – dimensional loss  
 253 coefficients obtained from the correlations, it calculates  $\xi$  and the related efficiency drop ( $\Delta\eta$ ), for  
 254 each kind of loss. It allows, in all kinds of expanders and operating conditions, to analyze the

255 distribution of losses and to investigate how do they affect the overall performance. The so built  
256 model provides a reliable basis to improve the design of different kinds of rotors with several  
257 possible working fluids.

258

#### 259 **4.1 Stator losses**

260 The stator losses, which are generally lower than the rotor losses, have been often evaluated with  
261 less accuracy in the literature [13, 24]. They are generally based on experimental data and make use  
262 of equations for stationary ducts. Referring to the experimental tests of Hiatt and Johnston, Benson  
263 determined the stator loss coefficients ( $\xi_N = 0.05 - 0.15$ ) [24], showing that they are very small  
264 compared to the corresponding values in the rotor. For the estimation of stator losses it is possible  
265 to apply Rodger's correlation [13]:

$$266 \quad \xi_N = \frac{0.05}{Re^{0.2}} \left[ \frac{3 \operatorname{tg} \alpha_2}{s/x} + \frac{s \cos \alpha_2}{b_2} \right] \quad (35), [13]$$

267 where:

$$268 \quad Re = \frac{c_2 b_2}{\nu_2} \quad (36)$$

269

#### 270 **4.2 Rotor losses**

271 The flow in the rotor of a radial turbine is subject to a rapid acceleration in the flow direction, and  
272 to a turn both in the meridian plane and along the camberline. These effects give rise to a complex  
273 pattern of secondary flows. The flow in the rotor of a radial turbine does not result into a high  
274 growth of the boundary layer and separation, even though, due to the three dimensional behavior, it  
275 develops a significant non uniformity of the total pressure inside the flow channel, which can lead  
276 to generation of losses. The probability of this occurrence increases when the blade loading is  
277 augmented. In the present model, the losses are divided in different contributions:

- 278 - Incidence loss;
- 279 - Skin friction loss;
- 280 - Tip clearance loss;
- 281 - Blade loading loss;
- 282 - Disk friction loss.

283

#### 284 4.2.1 Rotor Incidence loss

285 In the actual working conditions of the expander, the incidence angle of the relative flow at rotor  
286 inlet is rarely at the optimal value (equations 15 – 17). For this reason, the incidence loss appears.  
287 The recommended models for the estimation of rotor incidence loss were developed at NASA [13,  
288 14, 24]. The general approach is to assume that the kinetic energy associated with the variation of  
289 the tangential component of the relative velocity with respect to the design value, which is the result  
290 of the fluid – blade impact, is converted in internal energy of the fluid, which leads to an increase of  
291 entropy. The detailed calculation procedure is reported in [14]. The incidence losses may also be  
292 calculated using alternative approaches [13], obtained with the same conceptual assumptions and  
293 therefore formally similar:

$$294 \delta h_{0,i} = \frac{w_2^2 \sin^2(|\beta_2 - \beta_{2,opt}|)}{2} \quad (37), [13];$$

$$295 \delta h_i = \frac{(w_2 \sin \beta_2 - w_2 \sin \beta_{2,opt})^2}{2} \quad (38), [13].$$

296 The above discussed methods provide similar results, consistent with the literature. Thus, any of  
297 the two proposed correlations may be adopted leading to negligible differences.

298

#### 299 4.2.2 Friction losses

300 Friction losses can be estimated referring to a rotor-equivalent duct working on the same flow rate  
301 [26]:

$$302 \xi_{R,f} = \frac{\lambda_R L_R^*}{D_R^*} \quad (39)$$

303 Details about the calculation of the characteristic diameter and length can be found in [24] or [26].

304 As for the stator, the friction factor can be determined using Moody's diagram. The relative  
305 roughness and Reynolds number are estimated referring to the rotor-equivalent duct. Alternatively  
306 to equation (39), the frictional losses may be calculated using the following expression:

$$307 \Delta q_{R,f} = \frac{4\lambda_R [(w_2/VS_{01})^2 + (w_3/VS_{01})^2]}{4(D_R^*/L_R^*)(u_2/VS_{01})^2} \quad (40), [13].$$

308 or

309 This correlation tends, generally, to overestimate the friction losses by 70 – 80%.

310

311

### 312 **4.2.3 Tip clearance losses**

313 Tip clearance losses are due to the fluid leaking through the clearance gaps between the blade tips  
314 and the shroud. With reference to the blade geometry, in radial turbines two different types of  
315 clearances can be distinguished from the construction point of view: axial at inlet and radial at  
316 outlet [30, 38]. However, there is not a net distinction between the two kinds of clearance, but a  
317 gradual and continuous change (figure 5). Referring to studies performed at NASA [30] and more  
318 recent CFD calculations [38], it may be affirmed that the contribution of radial clearance to the  
319 overall loss is almost one order of magnitude higher than the axial one [13, 15, 30, 38].

320 Several different correlations have been proposed for tip clearance losses, some of which are  
321 specific for radial inflow turbines [...] and others are derived from centrifugal compressors [...]. A  
322 wide spread in the results can be produced using different models. Here, the model of Rodgers [13]  
323 is proposed:

$$324 \quad \Delta q_{R,cl} = 0.4 \left( \frac{\varepsilon}{b_2} \right) \left( \frac{c_{u2}}{u_{t,le}} \right)^2 \quad (43)$$

325 Equations (43) provide low values of clearance losses, which result in poor agreement with  
326 literature. Specifically, it happens when the values of axial clearance in equation are used. When the  
327 values of radial clearance are adopted, higher agreement with literature results are achieved [30] for  
328 equation (43).

329

### 330 **4.2.4 Blade loading loss (including secondary flow)**

331 Blade loading loss, including secondary flow, are caused by the high curvature of the profile and the  
332 pressure gradient in the rotor vanes. They give the largest contribution to the reduction of the  
333 expander efficiency. However, they are not extensively reported and discussed in literature, often  
334 because they are threatened in combination with other losses using experimental coefficients [18, 24,  
335 28]. The model here proposed evaluates the secondary flow losses through correlations, as functions  
336 of kinematic and geometric parameters. For the calculation of blade loading losses, the following  
337 correlation proposed by Rodgers can be used [19]:

$$338 \quad \delta q_{R,bl} = 2 \frac{\left(\frac{c_{u2}}{u_{t,le}}\right)^2}{Z_R \frac{z}{r_2}} \quad (48)$$

339 Where  $z/r_2$  is the ratio between the expander axial length and rotor inlet radius[19].

340 For the calculation of profile losses, another correlation proposed by Rodgers [40] and suitably  
341 revised by Whitfield [19] may be adopted:

$$342 \quad \delta q_{R,p} = 0.5 \left( \frac{\frac{b_2 + b_3}{r_2} \frac{b_3}{r_2}}{1 - \left(\frac{r_3}{r_2}\right)^2} \right) \left( \frac{w_2^2 + w_3^2}{2VS_{01}^2} \right) \left( \frac{VS_{01}^2}{u_2^2} \right) \quad (49).$$

343 When the results achieved from equations (48) and (49) are compared with those of literature, one  
344 must face the problem of lack of sufficient data for this kind of losses. However, comparing the  
345 values of the overall loss coefficient and efficiency with those reported in the literature, it seems  
346 that the above described correlations provide fairly reliable results.

347

#### 348 4.2.5 Disk friction losses

349 Disk friction losses are produced in the enclosure between the back disk side of the impeller and the  
350 case of the machine, where an amount of fluid can leak due to the pressure gradient and rotate  
351 around the rotor axis.. In the present model, the formulation of Whitfield [13] was adopted, which is  
352 based on the original model of Daily & Nece [...], in alternative to the model proposed by Benson  
353 which provides exceedingly large values with respect to the available test data:

$$354 \quad \Delta q_{R,df} = \frac{0.25 \bar{\rho} u_{t,le} r_2^2 K_v}{\dot{m}} \quad (50)$$

355 where:

$$356 \quad k_v = \begin{cases} \frac{\left[ 3.7 \left( \frac{\varepsilon_{ax}}{r_2} \right)^{0.1} \right]}{Re^{0.5}} & , Re < 3 \cdot 10^5 \\ \frac{\left[ 0.102 \left( \frac{\varepsilon_{ax}}{r_2} \right)^{0.1} \right]}{Re^{0.2}} & , Re > 3 \cdot 10^5 \end{cases} \quad (51)$$

$$357 \quad Re = \frac{u_2 r_2}{\nu_2} \quad (52).$$

358

359 As a possible alternative, providing similar results (in the range of 1%), the correlations proposed  
360 by NASA [14, 17, 18] can be recommended.

361

362

### 363 **4.3 Diffuser loss**

364 A diffuser is generally present in radial turboexpanders downstream of the rotor, in order to allow  
365 the partial recovery of the large kinetic energy still available through controlled diffusion of the  
366 fluid. The calculation of the diffuser loss follows the standard procedure described in [42].

367

## 368 **5. Results and parametric analysis (Design process)**

369 Making use of the above-described loss correlations, a parametric analysis has been run to assess  
370 the behavior of losses against the main design parameters and input data:

371 *a. Blade height – inlet rotordiameter ratio ( $b_2/d_2$ )*

372 *b. Flow coefficient ( $\Phi$ )*

373 *c. Load coefficient ( $\Psi$ )*

374 *d. Isentropic degree of reaction ( $R_s$ )*

375 The reference case is a 50 kW turboexpander operating with a saturated or superheated vapour ORC between  
376 the upper/lower temperature levels of 147/95 °C (referred to R134a [37]).

### 377 **5.1 Blade height – inlet rotor diameter ratio ( $b_2/d_2$ )**

378 This parameter influences the power output, mass flow rate, efficiency, blade shape and  
379 flow conditions, especially at rotor outlet (figures 6,7) for both radial (IFR) and backswept  
380 (IFG) rotor geometries. The mass flowrate (and thus the power output) increases linearly  
381 when  $b_2/d_2$  is augmented. The absolute value is strongly dependent on the considered fluid:  
382 for example, the cyclohexane flow rate is much lower than that of the other working fluids,  
383 because of the larger specific enthalpy drop. Among the fluids here considered, R1234yf  
384 shows the highest flowrate. Generally, with the reduction of density and velocity of fluids,  
385 the required blade height ratio increases at fixed expander power output. It is also important  
386 to remark that, for a fixed flowrate, expanders with backswept blades (IFG) require higher

387  $b_2/d_2$  to achieve the same flow rate as for the IFR design Generally, the rotor outlet blade  
388 height increases with increasing  $b_2/d_2$  (i.e. the ratio between hub and shroud diameters at  
389 rotor outlet,  $d_{3h}/d_{3s}$  decreases, figure 7). R134a and R1234yf show a particular trend, with  
390 remarkable differences between IFR and IFG designs (figure 7). This is due to the fact that  
391 the outlet rotor blade height is determined from the mass flowrate balance, with the  
392 constraint of axial flow. Consequently, the meridional component of the outlet rotor velocity  
393 increases with the reduction of blade height. Figure 7 reflects a widely different design  
394 geometry of expanders working with different fluids.)

395 **a. 5.2 Flow coefficient ( $\Phi$ )**

396  $\Phi$  defines the velocity triangle at impeller inlet. The flow coefficient is one of the main  
397 parameters in the design of turboexpanders, as it directly influences the mass flow rate, the  
398 performance (power output and efficiency), the geometry and the rotor number of blades. To  
399 give an idea of how  $\Phi$  influences the flow, the variation of shape of the velocity triangle at  
400 rotor inlet at two different values of  $\Phi$  is shown on figures 8 a) (radial rotor blades IFR) and  
401 8 b) (backswept rotor blades IFG). Keeping constant the other non-dimensional design  
402 parameters, the meridional component of the inlet absolute velocity ( $c_{m2}$ ) increases with  
403 increasing  $\Phi$ , whereas the peripheral velocity remains almost unchanged, as it mainly  
404 depends on the load coefficient. This results in an increase of absolute and relative velocity  
405 at rotor inlet ( $c_2$  and  $w_2$  respectively) and in a reduction of the related flow angles ( $\alpha_2$  and  
406  $\beta_2$ ). The change in  $\beta_2$  directly affects the incidence losses, whereas  $\alpha_2$  influences the number  
407 of rotor blades, according to equations (13) and (14): as  $\alpha_2$  decreases with increasing flow  
408 coefficient, the number of rotor blades is reduced, as shown on figure 9 which reports the  
409 optimized values of  $Z_R$  vs.  $\Phi$ . It must be remarked that, given the small size of the  
410 investigated expanders, it is a good practice trying to reduce large numbers of blades which  
411 results from the application of Eq. 14 (Glassman theory), in order to reduce the blockage  
412 effects. In fact, it is still possible to achieve high efficiencies also with a number of rotor  
413 blades much lower than that proposed by Eq. 14. Within the considered field of flow  
414 coefficient (typical of radial turboexpanders,  $0,08 < \Phi < 0,22$ ), no significant differences in  
415 the “optimal” number of blades was found for the different investigated fluids (R134a shows  
416 the lowest optimal number of blades, whereas CycloHexane shows the highest one, for both  
417 IFR and IFG geometries). From figure 10, it is evident that backswept bladed (IFG)  
418 expanders have a total to static efficiency ( $\eta_{ts}$ ) 1.5 – 2 points higher than radial (IFR) ones,  
419 which is in agreement with [18]. Generally,  $\eta_{ts}$  increases with  $\Phi$  (with the exception of

420 cyclohexane for IFG rotors). The highest values of  $\eta_{ts}$  are achieved by R134a and R1234yf,  
421 whereas the lowest ones are shown by R245fa and cyclohexane.

### 422 **5.3 Load coefficient ( $\Psi$ )**

423  $\Psi$  is a fundamental parameter in the design of turboexpanders, because it deeply influences  
424 their performance (rotational speed, absolute and relative flow angles at nozzle outlet/rotor  
425 inlet, overall performance) and is, with the degree of reaction, one of the main non  
426 dimensional parameters to define the different categories of rotors. Keeping constant the  
427 other non-dimensional design parameters (table 1), an increase in the load coefficient leads  
428 to a reduction of the meridional component of the absolute velocity at rotor inlet (see  
429 modification of velocity triangles in figure 11). Thus, the mass flow rate is reduced and its  
430 effect is added to the reduction of the expander total enthalpy drop, leading to an overall  
431 reduction of power output. Due to the reduction of the meridional and peripheral velocity at  
432 rotor inlet, the absolute angle  $\alpha_2$  increases, which implies a reduction of the relative velocity  
433 ( $w_2'$ ). For this reason, the load coefficient has a large influence on the incidence angle and  
434 on the associated loss. Figure 11 shows how the optimized velocity triangle for IFR tends to  
435 that of IFG with increasing  $\Psi$ . The trend of the rotational speed vs. load coefficient is  
436 shown on figure 12. It is interesting to remark the difference in rotational speed with  
437 different working fluids, which is in turn related to the total enthalpy drop and to the rotor  
438 size. Specifically, the largest rotational speeds occur for R134a and R245fa, whereas those  
439 of CycloHexane are considerably lower. Generally, a backswept (IFG) design allows a  
440 lower rotational speed. The dependence of the nozzle outlet/rotor inlet absolute velocity  
441 angle ( $\alpha_2$ ) on the Load Coefficient  $\Psi$  is shown on figure 13. These expanders are  
442 characterized by large nozzle flow angles, which become even higher in case of backswept  
443 blades. The largest values of  $\alpha_2$  for rotors with radial blades (IFR) are reached by  
444 CycloHexane and by R1234yf and R245f in the case of an IFG design. In both IFR and IFG  
445 configurations, R134a shows the lowest values of  $\alpha_2$ . The trend of the relative velocity  
446 angles at rotor inlet ( $\beta_2$ ) vs.  $\Psi$  is shown on figure 14. The highest absolute values are shown  
447 by cyclohexane and R1234yf for radial bladed rotors and R245fa and R1234yf for the  
448 backswept bladed ones. Anyway, when  $\Psi$  is within the range 1.05 – 1.15,  $\beta_2$  values are at  
449 the same levels for the different fluids in the case of backswept bladed rotors (IFG).

### 450 **5.4 Isentropic degree of reaction ( $R_s$ )**

451  $R_s$  is defined as the ratio between the static isentropic enthalpy drop through the rotor  
452 and that of the overall stage. It strongly affects the performance of the expander. When  
453 combined with the other main parameters  $\Phi$  and  $\Psi$ , it completes the definition of the

454 design geometry. When the other design parameters are fixed, with increasing  $R_s$  the  
455 isentropic stator static enthalpy drop is reduced. For this reason, the pressure at the stator  
456 outlet is higher and the related fluid density is increased. With fixed stator outlet cross  
457 sectional area, given the relatively limited change in mass flow rate, the meridional  
458 component of the absolute velocity is reduced, which offsets the increase in static  
459 pressure. Keeping constant the flow coefficient, the rotational speed is reduced. The  
460 related velocity triangle is modified as shown in figure 15. The expander power output  
461 decreases with increasing  $R_s$  of an amount variable with the different investigated fluids,  
462 as shown in figure 16. R245fa is the least sensitive to  $R_s$  because it has the lowest  
463 flowrate level and thus the lowest reduction of power output. Moreover, IFR expanders  
464 with radial blades have higher  $R_s$  than those with backswept blades. The variation of  $R_s$   
465 has strong effects on the rotor peripheral speed. Referring to figure 17, the remarkable  
466 difference in peripheral speed for the different investigated fluids and rotors can be  
467 noticed. Specifically, R245fa shows the highest values, whereas the lowest is shown by  
468 R1234yf, due to its low value of the total enthalpy drop. Finally, the backswept rotors  
469 (IFG) have a lower peripheral speed compared to the radial bladed ones. This is due to  
470 the higher load coefficient  $\Psi$  (with fixed total enthalpy drop) which characterizes the  
471 backswept geometry.

472

## 473 **6. Interpretation of results - Design process**

### 474 **6.1 IFR vs IFG design**

475 Moving from the radial (IFR) to backswept blades (IFG) configuration, the load coefficient  
476 increases. Thus, in the backswept (IFG) design the meridional component of velocity at rotor inlet is  
477 reduced to maintain the fixed flow coefficient. For this reason, in order to achieve the target 50 kW  
478 power output, the IFG design shows higher  $b_2/d_2$  ratios. An additional consequence of the higher  
479 load coefficient of backswept bladed rotors is their lower peripheral velocity (and rotational speed)  
480 for a given rotor size. Finally, backswept machines show a lower degree of reaction than the  
481 corresponding radial bladed ones. As remarked in the parametric analysis, when the overall  
482 enthalpy drop (stator + rotor) is fixed, the reduction of the degree of reaction implies a lower stator  
483 outlet backpressure and, consequently, a lower fluid density in this section. As the outlet conditions  
484 are fixed, and because the mass flowrate undergoes only limited variations, the meridional  
485 component of the absolute velocity increases to counterbalance the reduction of fluid density. Thus,

486 in order to maintain the flow coefficient unchanged, the peripheral velocity at rotor inlet increases.  
487 Finally, in order to keep the load coefficient constant, the total enthalpy drop of the expander is  
488 increased and, consequently, the related power output. In this way, by tuning  $b_2/d_2$ , the load  
489 coefficient and the reaction degree, it is possible to move across the two different configurations.  
490 Generally, backswept bladed expanders show 1.5 – 2% better efficiency levels than the  
491 corresponding radial bladed ones.

## 492 **6.2 Different working fluids**

493 A specific interpretation of the results is needed when considering the important matter of expander  
494 design with different working fluids. All those here considered are good candidates for the power  
495 cycle specifications (power output, temperature levels). On the other hand, large differences in  
496 kinematic, geometric and performance characteristics are found between the different fluids. In the  
497 following figures, the behavior of R134a, R1234yf, R245fa and cyclohexane is extensively  
498 reported. Anyhow, for sake of completeness, the analysis of different fluids has been extended to  
499 R236fa and CycloPentane. The inlet diameter of the expander is in the range 80 – 110 mm. The  
500 largest size were achieved for the hydrocarbons like cyclohexane and N-Pentane, due to their much  
501 lower density, in spite of the larger specific isentropic enthalpy drop compared to HFCs. The  
502 rotational speed is between 30000 and 50000 rpm, generally lower for backswept configurations  
503 due to the lower peripheral velocity (see the generic shape of velocity triangles in figure 15).  
504 Among the different fluids, the cyclohexane shows the lowest rotational speed, due to the much  
505 higher diameter, in spite of the high peripheral speed. The specific speed is in the 0.055 – 0.1 range.  
506 The highest value is shown by the cyclohexane because, in spite of the lowest rotational speed and  
507 the highest stage enthalpy drop, they are largely counterbalanced by the very high values of  
508 volumetric flowrate due to the lowest density. The flow at nozzle exit is generally supersonic ( $0.9 <$   
509  $M < 1.5$ ), with the exception of R1234yf due to the high values of blade height at section 2 ( $b_2$ ,  
510 table 3). The highest value of Mach is shown by CycloHexane, due to the combined effects of low  
511 density and high peripheral speed. Generally, high nozzle exit angles are found ( $77 - 83.5^\circ$ ), with  
512 larger values for backswept rotor (IFG) design. When considering the flow within the rotor, a high  
513 deflection level has to be remarked, which is in the range  $40 - 90^\circ$ , generally higher for backswept  
514 configurations. Due to the shape of velocity triangles (see velocities and angles on table 3),  
515 cyclohexane shows the highest deflection level for the IFG configurations and the lowest for the  
516 IFR ones. The hub to tip diameter ratio at rotor exit ( $d_{3h}/d_{3s}$ ) is in the 0.39 – 0.52 range and agrees  
517 with literature data [14, 17]. At rotor exit, the adoption of a diffuser for the partial recuperation of  
518 the kinetic energy may be important only for fluids like cyclohexane, which have high values of

519 absolute Mach number in this section. The total- to-total efficiency of the investigated expanders  
520 and fluids ranges between 0.72 and 0.80, generally higher for backswept configurations than for the  
521 corresponding radial ones. Generally, higher efficiencies are achieved with expanders having lower  
522 velocities and deflections. Finally, it is important to remark that, for the specific size here  
523 considered (50 kW), sub-atmospheric values of total pressure at rotor exit are not recommendable.  
524 Thus, CycloHexane and pentane are critical from this point of view, in spite of their interesting  
525 efficiency levels in ORC. On the basis of expander design and cycle performance, R245fa and  
526 R236fa represent thus the most interesting options.

### 527 **6.3 Distribution of losses and efficiency**

528 It is also important to analyze the distribution of the different losses through the expander, whose  
529 contribution to the overall reduction of efficiency ( $\Delta\eta$ ) is shown in table 3. It is practically the same  
530 for the different investigated fluids and configurations. The contribution of the *stator losses* to the  
531 overall losses ranges between 2% of R134a and 12% of cyclohexane, both referred to radial  
532 geometry. Especially in IFR configurations, the stator losses have the highest incidence for  
533 hydrocarbons (mainly cyclohexane) and R245fa compared to the other investigated fluids. It is  
534 mainly due to the high velocity in the nozzle (table 3, see also the high values of  $M_2$ ). Literature  
535 data [13] report that the *stator losses* generally range between 5 and 15% of the overall, which is in  
536 line with the results achieved with the here proposed model.

537 Under design conditions, the *incidence losses* are negligible, as the relative velocity angle at rotor  
538 inlet is the optimizing value calculated by the (15 – 17) and (21) relationships. Their contribution to  
539 the overall reduction of efficiency  $\Delta\eta_{ts,i}$  ranges between 0.03 and 0.2%.

540 The *disk friction losses* give a contribution within the 2 – 4% to the total and have a reduced relative  
541 influence on the expander efficiency ( $0.4 \% < \Delta\eta_{ts,v} < 0.8 \%$ ). The highest values are shown by the  
542 cyclohexane, mainly due to the large rotor diameter and peripheral velocity.

543 The *tip clearance losses*, here referred to an average 3% clearance fraction of blade height and to  
544 backswept configuration, give a relevant contribution on the total losses, ranging from 10% of  
545 R1234yf to 21% of CycloHexane. Generally, higher tip clearance losses are shown by fluids having  
546 higher ratio between radial clearance and inlet blade height ( $\varepsilon/b_2$ ), in agreement with eq. (43). The  
547 related overall efficiency drop ( $\Delta\eta_{ts,cl}$ ) is within the 2 - 6.5% range and agrees with literature data  
548 [30, 39].

549 The highest relative contribution to the losses (60 – 70%) is given by *secondary flows* in the rotor,  
550 which are due to the high blade curvature and pressure gradient through the blade vanes. As  
551 suggested by Rodgers [40], this contribution is shared between blade loading and profile curvature.  
552 The former represent the highest contribution to overall losses, ranging from 27% to about 50% of  
553 total. The highest values are found for R134a and 1234yf, which have the lowest number of blades  
554 and thus the highest blade loading. For this reason, cyclohexane shows the lowest relative  
555 contribution of blade loading losses, in agreement with equation (48). Their contribution to the  
556 efficiency reduction ( $\Delta\eta_{ts,cp}$ ) is variable between 8 and 13 %.

557 The contribution of profile losses to the overall turbine losses ranges from 12% of R134a to 27% of  
558 R1234yf. Their share on the overall losses is generally higher for hydrocarbons, ranging from 20 to  
559 26%. It is attributable to the combined effects of rotor geometry (i.e. higher  $d_3/d_2$ ), fluid properties  
560 (i.e. sound speed) and kinematic conditions (i.e. higher relative velocities  $w_2$  and  $w_3$ ), in agreement  
561 with eq. (49). They provide a total to static efficiency drop ( $\Delta\eta_{ts,p}$ ) variable from 3 to more than  
562 8%..

563 The *friction losses in the rotor* also represent an important contribution to the overall efficiency  
564 drop, reducing its value ( $\Delta\eta_{ts,a}$ ) from 0.9 to about 2.3 percentage points. They are generally higher  
565 in cases of turbines with larger wet surface like, for example, cyclohexane.

566 Finally, it is also important to consider the *kinetic energy loss at expander output*.. Even though a  
567 diffuser has always been considered here, this loss is representative of the difference between total  
568 to total ( $\eta_{tt}$ ) and total to static ( $\eta_{ts}$ ) efficiency of the expander. On the whole, these losses are not  
569 negligible, as they can reach up to 12% of the total. They are capable to reduce the overall  
570 efficiency ( $\Delta\eta_{ts,v}$ ) from 0.39 to 1.15 %. It is thus possible to recommend that a diffuser is always  
571 included in the design of these expanders.

572 The comparison of the results with those achieved by Rohlik [14, 17, 20] shows a substantial  
573 agreement, even though they were referred to air. Specifically, the highest contributions to the  
574 overall efficiency drop are given by blade loading and profile losses into the rotor. A similar  
575 behavior is found also for the remaining losses, even though the stator and disk friction losses  
576 calculated in this model give a generally lower contribution compared to [17]. On the contrary, the  
577 tip clearance losses are generally higher than those proposed by Rohlik [17]. Anyway, they are on  
578 line with those originally proposed by NASA [30] and successively confirmed by numerical  
579 calculations [39]. The comparison of the total to static efficiency of the here designed expanders  
580 with those achieved by Rohlik in the maximum efficiency curve [30] and with the experimental

581 results from similar machines [24], shows a good agreement regarding the design specific speed,  
582 whereas lower values of the efficiency are achieved here, especially for radial bladed expanders  
583 (figure 18). This is mainly due to the different size, geometry and working fluids considered in the  
584 present investigation, as well as to the different relationships adopted for the calculation of losses.  
585 Finally, the comparison of the results achieved in this work with those coming from experimental  
586 campaigns of Benson [24] shows a complete agreement.

587

## 588 **7. Off design performance prediction of the radial turbo-expander**

589 The above discussed design procedure can be used to build the characteristic curves of the  
590 expanders, which fundamental to predict their off- design behavior. When dealing with ORC  
591 working partially or totally with not continuously available renewables, it often happens that they  
592 work most of time under off design conditions. It is the case, for example, of solar power stations or  
593 integrated geothermal – solar binary cycles, like those proposed in [10] and [36], where the variable  
594 amount of available solar heat leads to variable massflowrate and/or thermodynamic conditions of  
595 the produced organic steam at the turbine inlet. When the characteristic curves of the designed  
596 expanders are known, they can be used to provide an estimate of their performance under variable  
597 inlet conditions (off design) and - when possible- to adjust their rotational speed in order to  
598 minimize the losses.

599 Specifically, in this chapter we analyze the buildup of characteristic curves with variable  
600 temperature, pressure and fluid mass flowrate at the turbine inlet (the latter depends on pressure  
601 drop through the expander), for different values of the corrected speed:

$$602 \quad N_c = \frac{N}{\sqrt{T_{01}}} \quad (55)$$

603 In the off design approach, the turbine geometry is specified, resulting from the design procedure  
604 described in the previous sections. Specifically, the following geometric parameters are given as  
605 inputs:

- 606 - Blade height
- 607 - Stator and rotor inlet/outlet diameters
- 608 - Passage section areas
- 609 - Number of rotor and stator blades
- 610 - Blade metal angles

611 Moreover, in the present case the rotor outlet total pressure is maintained at the design value,  
612 because it is fixed by the conditions at the condenser to satisfy the cogeneration conditions.  
613 Anyway, with the developed model it is possible to let it be variable, in order to take into account of  
614 the variable conditions at the condenser due to the timely change of the environmental conditions.

615 The loss correlations under off design conditions are the same adopted in the expander design, as  
616 well as those relating flow and metal angles. These allow the determination of the actual flow  
617 angles given the blade metal angles and the size of the expander, both coming from the design  
618 procedure.

619 In order to analyze the off-design behavior of the turbo expander, the inlet total pressure and  
620 temperature are changed. Consequently, the mass flowrate is determined from the inlet – outlet  
621 pressure drop. Another possible way of estimating the off-design behavior is to fix the expander  
622 inlet mass flowrate and total temperature and calculate the total pressure. Thus, the performance  
623 maps of the expander (i.e. characteristic curves) are achieved by building the related curves under  
624 design conditions for different values of  $N_c$ . They may be directly adopted within the  
625 thermodynamic code for the analysis of the ORC. .

## 626 **7.1 – performance behavior at off design expander pressure ratio**

627 The curves of power output, mass flowrate and efficiency vs. off design pressure ratio at different  
628 values of corrected speed  $N_c$  are shown on figures 19, 20 and 21 respectively, each one reporting  
629 the results for both radial and backswept configurations. For the sake of brevity, the off design  
630 analysis is carried out for R134a only, without losing in generality, as the behavior is qualitatively  
631 similar for all the investigated fluids. From figure 19, it is clear that the power output is reduced  
632 when the expansion ratio falls below the design value. Moreover, it is important to notice that the  
633 power output is reduced with reducing the corrected speed: this is the result of the lower enthalpy  
634 drop, due to the lower rotational speed, which is also responsible for the reduced variation of  
635 peripheral velocity (i.e. the effective component for momentum). Finally, it should be remarked that  
636 the reduction of power output with respect to the nominal value is larger than the reduction of the  
637 pressure ratio, when  $N_c$  is at the design value: for example, when the expansion ratio is at 90% of  
638 the design value, the power output is reduced to about 88% of the design value. When the  
639 expansion ratio is at 80% of design value, the power output is about 75% of the nominal. The  
640 overall behavior is found for both IFG and IFR geometries.

641

## 642 7.2 – performance behavior at off design expander corrected mass flowrate

643 The other typical turbomachinery off design parameter is the corrected mass flowrate, defined as

$$644 \quad \dot{m}_c = \frac{\dot{m} \sqrt{T_{01}}}{p_{01}} \quad (56).$$

645 Starting from the values of the total inlet pressure and temperature, from the maps of figure 20 it is  
646 possible to determine the mass flow rate with variable expansion ratio for the different values of  
647 the ratio  $N_c/(N_c)_{des}$ . It can be noticed that the radial bladed rotors (IFR) are more influenced by  
648 operation under off-design pressure ratio than the backswept ones (IFG).

649 Figure 21 shows the behavior of total to static efficiency (expressed by the off-design to design  
650 ratio) as a function of the variable (off-design) expansion ratio. It is important to remark the  
651 relatively low sensitivity to the expansion ratio for corrected speeds below the design value, due to  
652 the typical accelerating behavior of fluids into the expanders, which allows to work into a relatively  
653 wide range of incidence angles with only a limited increase of loss coefficients. Hence, the  
654 reduction of  $N_c$  below the design value could be regarded as a way to reduce the efficiency drop of  
655 the expander at reduced values of the expansion ratio. As it is seen, the investigated expanders show  
656 a relatively limited sensitivity of the efficiency to off design expansion ratio, unless it is reduced to  
657 very low values. It is interesting to notice the increasing efficiency (1.5 – 2%) at corrected speed  
658 lower than the design value. To better understand this behavior, it is important to analyze the  
659 variability of velocity triangles under the three different values of corrected speed (figure 22), Let  
660 us consider a reduction of corrected speed  $N_c$  at fixed thermodynamic conditions of the inlet fluid.  
661 As the inlet – outlet expander pressure drop is fixed, the flow rate remains practically unchanged.  
662 The lower variation of peripheral velocity leads to a reduction of stage and nozzle enthalpy drop  
663 (notice that  $R \neq 0$ ). For this reason, the pressure at nozzle exit is higher, which leads to an increase of  
664 fluid density. As the mass flow rate is constant, the meridional component of absolute velocity  
665 must be reduced, thus the related velocity triangle height decreases. When  $N_c$  is reduced, at fixed  
666 total inlet temperature ( $T_{01}$ ), the rotational speed of the expander decreases largely, and thus also  
667 the rotor peripheral velocity. The latter entails an increase of relative flow angle at rotor inlet ( $\beta_2$ ) in  
668 the same direction (see the modified red and green triangles of figure 24). Hence, the shape of the  
669 velocity triangles is modified towards the backswept configuration. This modification affects the  
670 loss coefficients of the expander. Specifically, a reduction of  $N_c$  down to 60 – 65% of the design  
671 value leads to an increase of the incidence losses and a reduction of the stator, friction and  
672 secondary flow losses, as reported on figure 25. This effect is due to the change of velocity profiles  
673 within the rotor vanes. Finally, it is interesting to analyze the behavior of the ratio of the off-design

674 to design total to static efficiency ( $\eta_{ts}/(\eta_{ts})_{des}$ ) vs the corrected speed ratio ( $N_c/(N_c)_{des}$ ), shown on  
675 figure 24 for both radial and backswept configurations, at fixed design inlet total pressure and  
676 temperature. In both cases,  $\eta_{ts}$  is maximized at values of  $N_c/(N_c)_{des}$  where the sum of losses is  
677 minimum.

678

## 679 **8. Conclusions**

680 The paper describes the features and analyzes the results of a zero dimensional model for the design  
681 of high efficiency small size ORC expanders. Two basic rotor blade geometries (radial IFR and  
682 backswept IFG) and six different possible organic working fluids (R134a, R1234yf, R236fa,  
683 R245fa, Cyclohexane, N-Pentane) have been analyzed and discussed. In all cases, the power output  
684 has been fixed at about 50 kW. The reference thermodynamic data for the specific application are  
685 taken from [36]. The relationships for the estimation of the expander losses, as well as the main  
686 design parameters, have been collected by an extensive investigation of models and experimental  
687 data available in literature for radial turbo-expanders. Generally, literature for radial expanders is  
688 much less rich than for axial turbines and, often, data and models are derived from these last and  
689 from centrifugal compressor applications, with limited adaptations. Moreover, these relationships  
690 and models are referred to ideal gases and Mach relationships, which is also the approach often  
691 applied in many CFD calculation codes. In the present work, the model applies the most recent,  
692 currently available, equations of state of the investigated real fluids expanding into the ORC  
693 turbine.

694 For the investigated fluids, the rotor diameters are in the 80 – 110 mm range and the rotational  
695 speed is variable between 30000 and 50000 rpm (specific speed is always below 0.1). The only  
696 exception is CycloHexane, which needs higher rotor diameters (190 – 200 mm) and a lower  
697 rotational speed. The designed expanders are mostly supersonic and have high values of nozzle exit  
698 angles ( $\alpha_2=77 - 83.5^\circ$ ), whereas the deflection of flow in the rotor is between 40 and 90°. The  
699 outlet rotor hub to tip diameter ratio resulting from the developed calculation code is within the 0.39  
700 – 0.52 range, which agrees with literature data. In order to partially recover the outlet kinetic energy  
701 of flow, the adoption of a diffuser at rotor outlet is proposed, which is particularly recommendable  
702 for the expansion of the cyclohexane, which has the highest value of Mach at rotor discharge.

703 The expected total-to-total efficiency of the designed units ranges between 0.72 and 0.80,  
704 depending on the considered fluid and geometry configuration. The highest contribution to the

705 expander efficiency losses is given by the secondary flows within the rotor (blade loading and  
706 profile curvature), due to the high curvature of blade profiles and the high pressure gradient.

707 The investigation, on the basis of overall thermodynamic, power plant and fluid dynamic features,  
708 shows that the most suitable fluids are R236fa and R245fa, followed by R134a and R1234yf,  
709 whereas the worst ones are CycloHexane and CycloPentane, which are further penalized by the  
710 sub atmospheric pressure at the expander output.

711 Generally, backswept bladed rotors (IFG) show 1.5 to 2.5% higher efficiencies. Moreover, they  
712 have larger values of the number of blades, load coefficient, nozzle exit angle and rotor deflection  
713 angle than the corresponding radial bladed ones. The peripheral speed and the reaction degree of the  
714 backswept configurations are instead lower. Generally, the results of the design and parametric  
715 analysis are in agreement with literature data. The proposed calculation model has been successively  
716 used to predict the off- design performance, through the construction of the expander characteristic  
717 curves (corrected mass flowrate, power output and efficiency as functions of expansion ratio and  
718 corrected speed  $N_c$ ). These last can be directly introduced into the thermodynamic ORC calculation  
719 code to evaluate the off design behavior of the expander under variable thermodynamic inlet  
720 conditions (total temperature and pressure and mass flow rate for different values of corrected  
721 speed), once the geometry of the expander is defined.

722 The calculation tool is open to improvements by the use of 2D – 3D CFD models and experimental  
723 tests on existing rotors, which potentially could allow the improvement of the less validated  
724 correlations, making it effective and reliable for the design and off-design analysis of radial turbo-  
725 expanders for small-size ORC powerplants.

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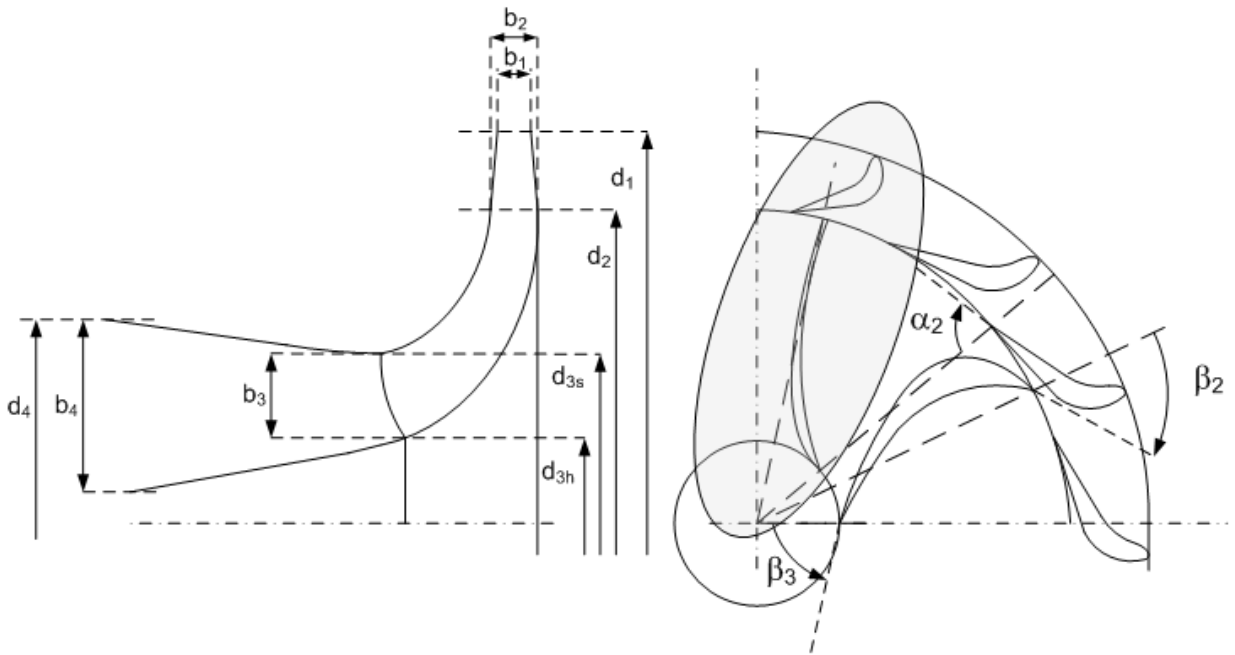
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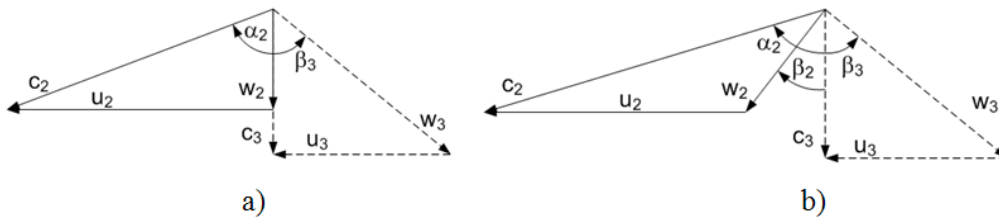


749

750 *Figure 1 – General schematic of Radial inflow turbine: 90° IFR (shaded), and General rotor shape*  
 751 *(IFG, unshaded).*

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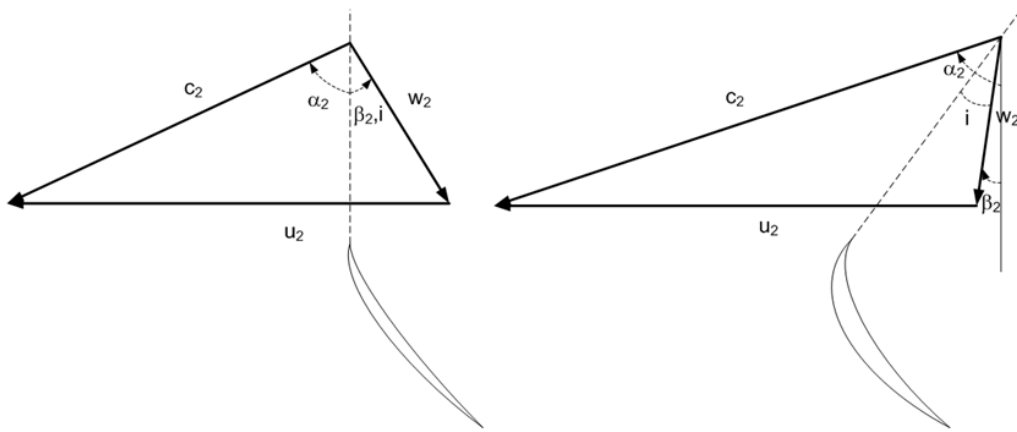


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756 *Figure 2 – Velocity triangles – nominal conditions; (a) 90° IFR (b) General (IFG)*

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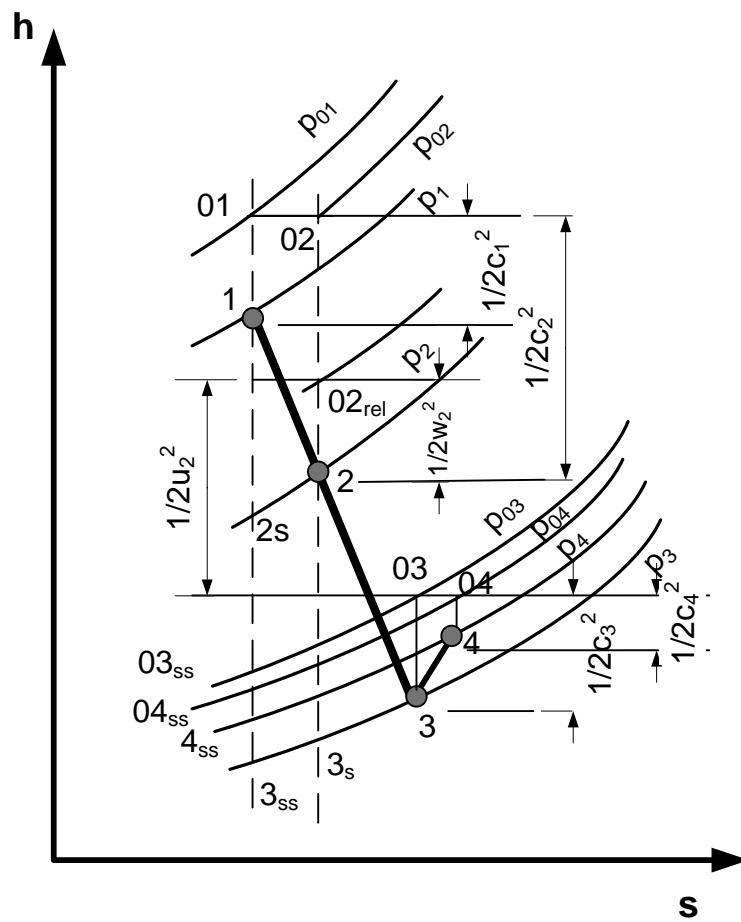
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Figure 3 – Velocity triangles – optimal incidence conditions; (a) 90° IFR (b) IFG

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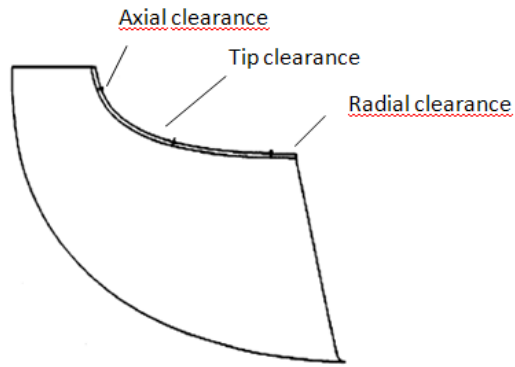


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Figure 4: Enthalpy – Entropy representation of the expansion process [14].

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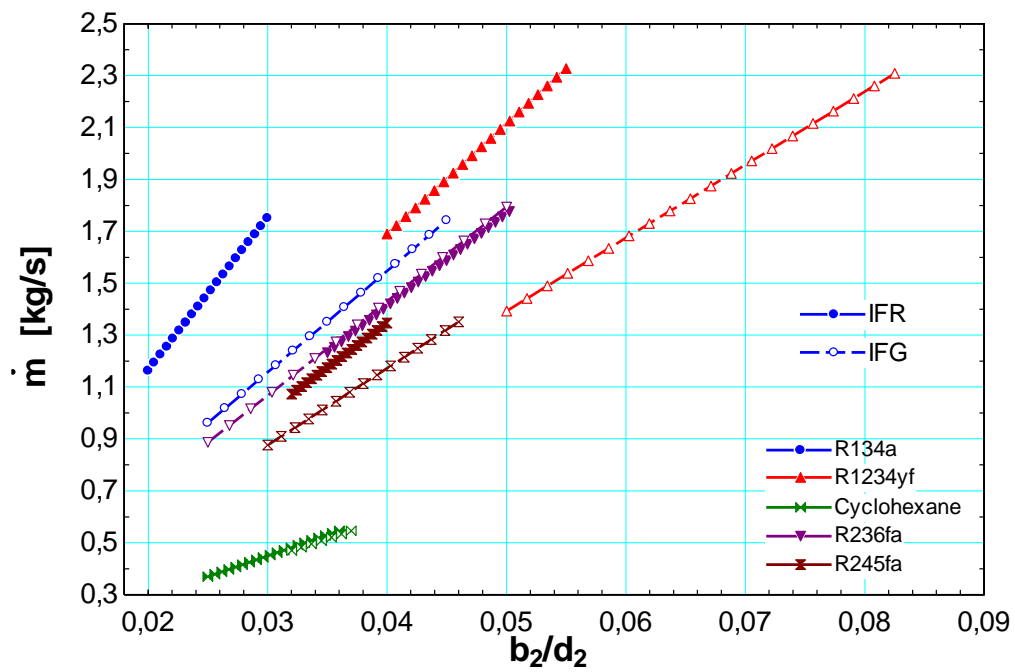
Figure 5: Tip clearance along the development of a rotor blade.

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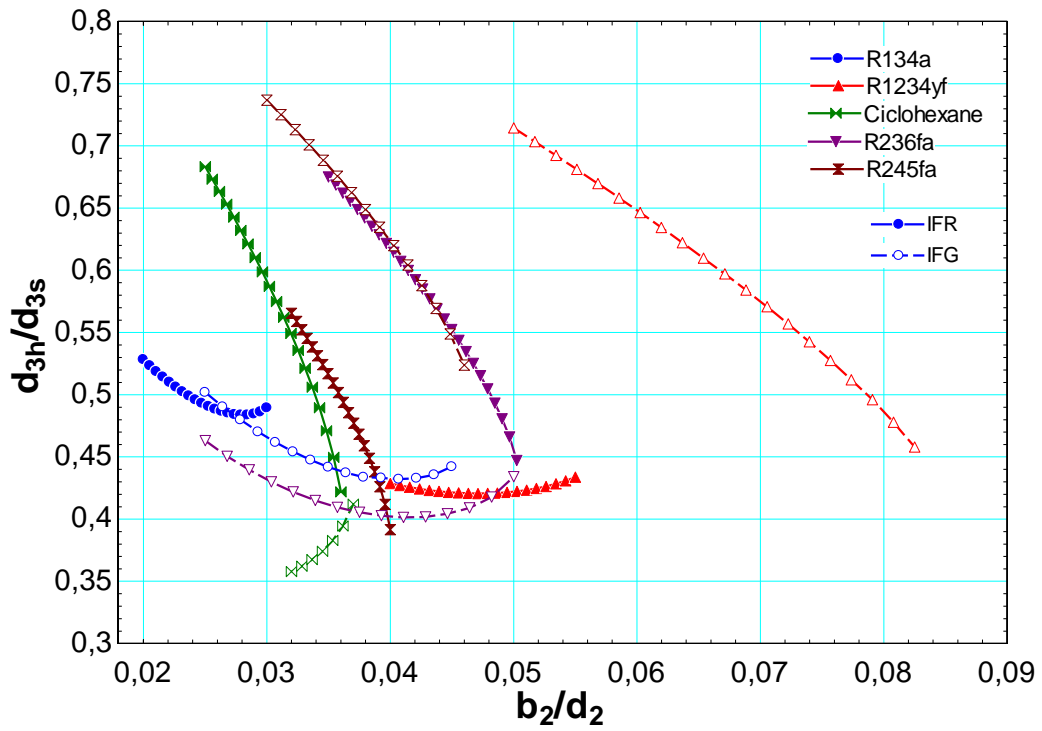
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Figure 6: mass flow rate and ratio of rotor exit diameter at hub and shroud as a function of ratio

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between blade height and diameter at rotor inlet  $b_2/d_2$  (IFR and IFG)

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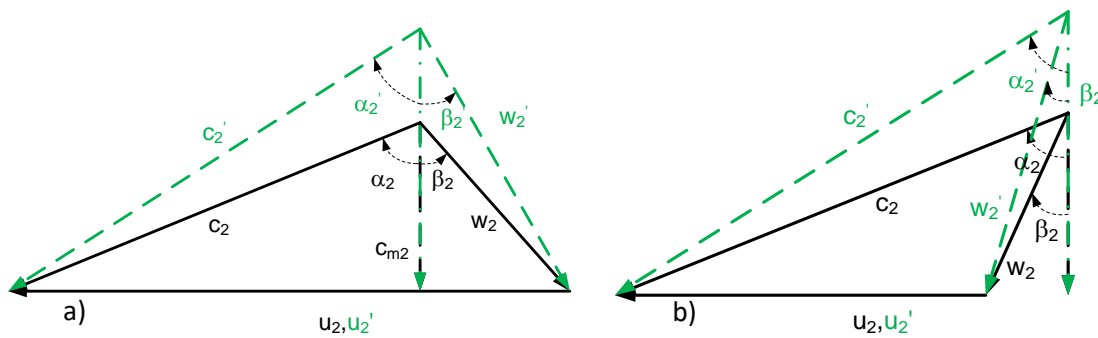
779 *Figure 7: Mass flow rate and ratio of rotor exit diameter at hub and shroud as a function of ratio*

780 *between blade height and diameter at rotor inlet  $b_2/d_2$  (IFR and IFG)*

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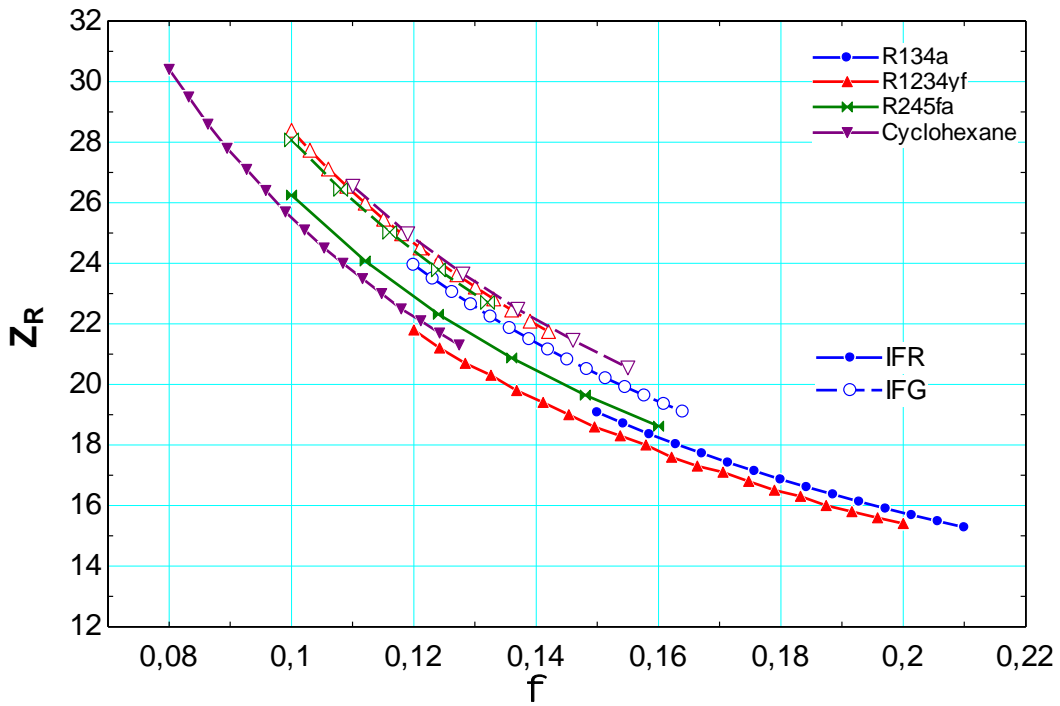
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786 *Figure 8: Variation of velocity triangles with increasing flow coefficient*

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*(from solid black to dashed green, a) IFR, b) IFG)*



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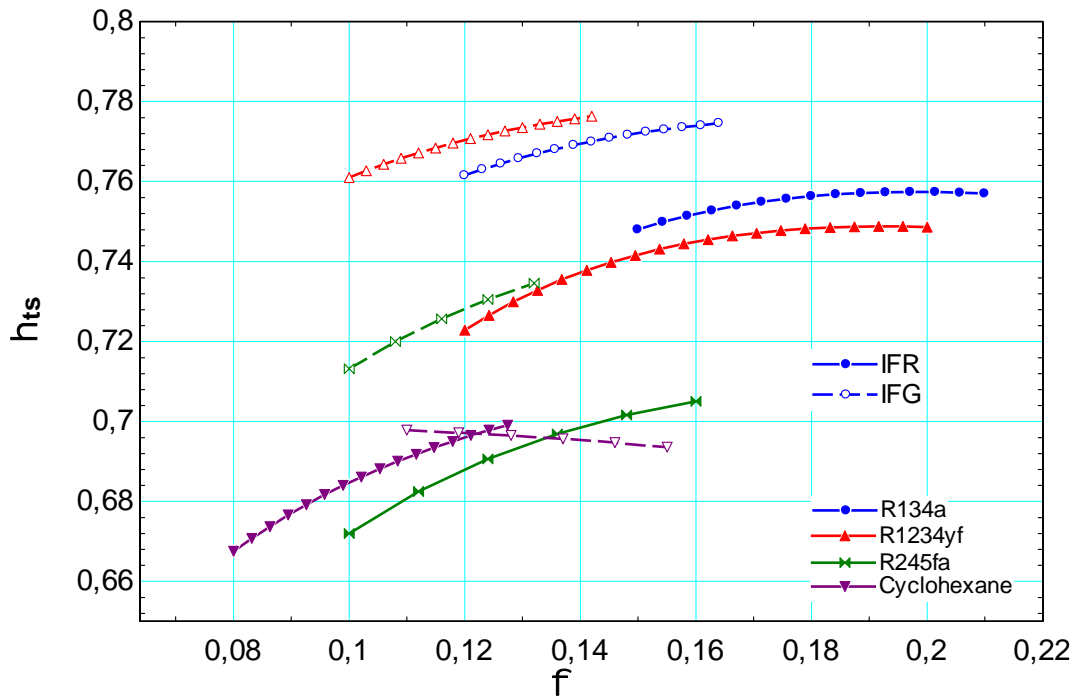
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Figure 9: Number of rotor blades as a function of flow coefficient (IFR and IFG)

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Figure 10 –Total-to-static efficiency as a function of flow coefficient (IFR and IFG)

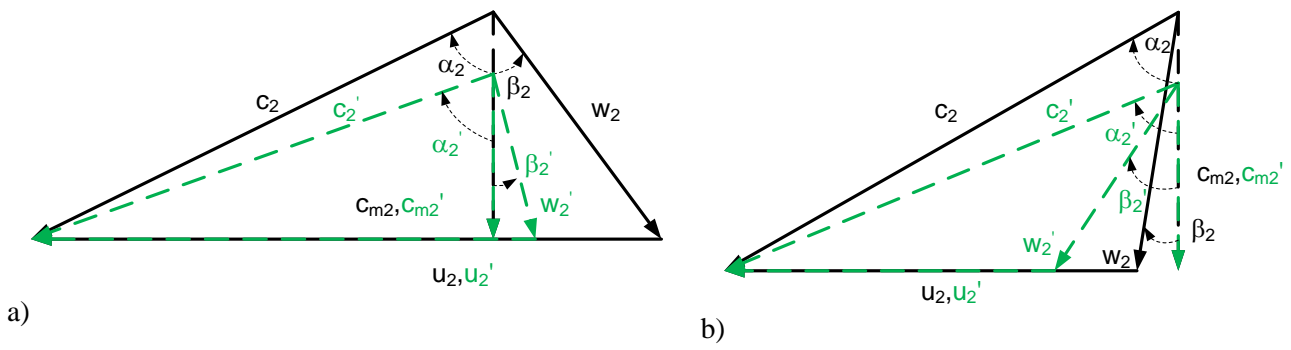


Figure 11 – Variation of velocity triangles at rotor inlet with increasing load coefficient (from black solid to green dashed) a): IFR; b) IFG

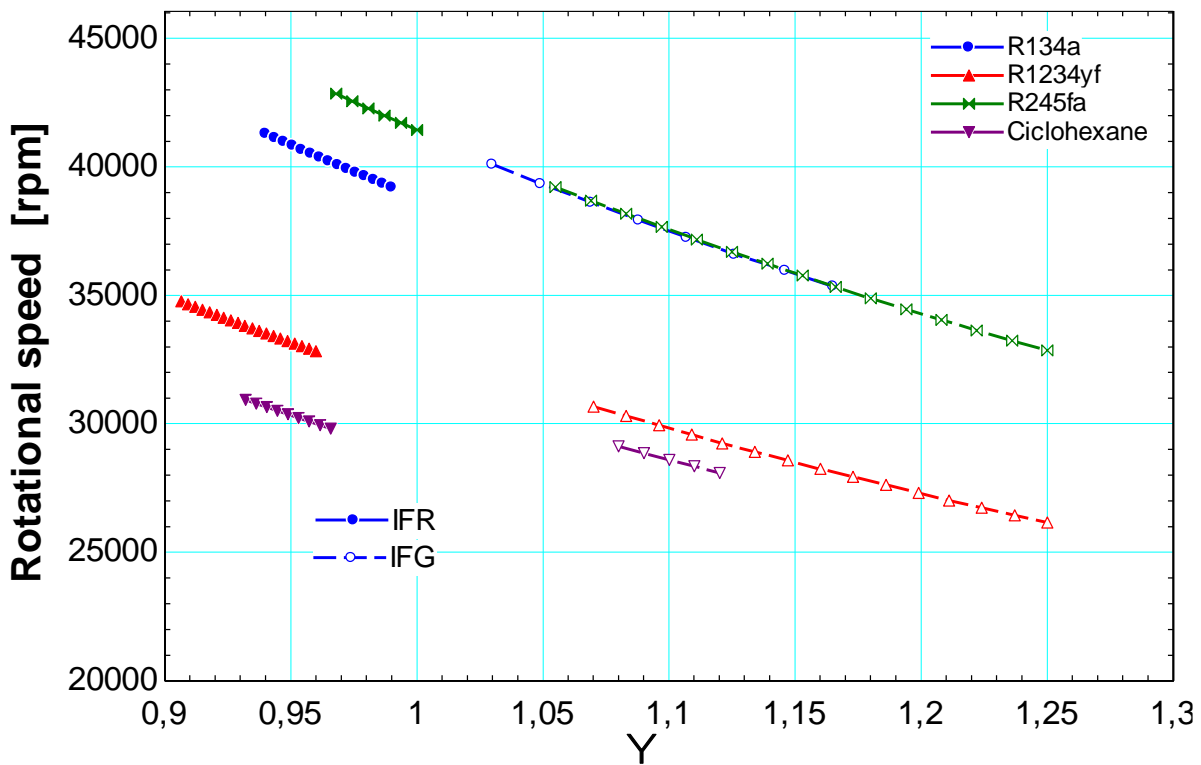
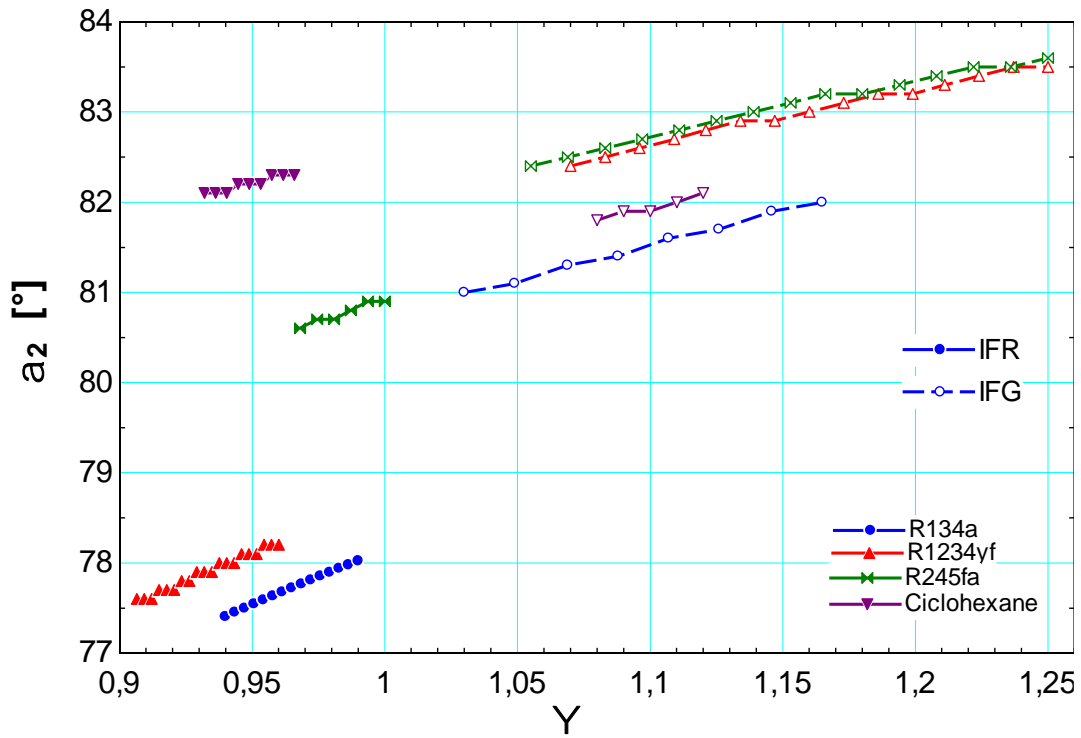
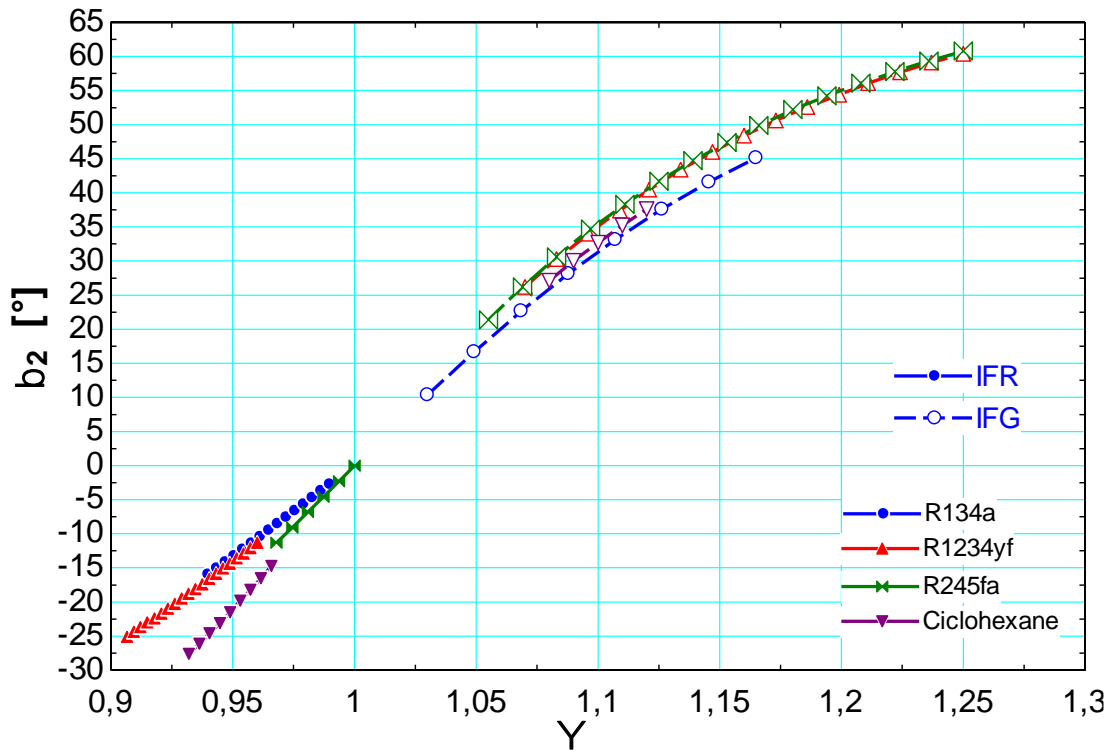


Figure 12: Speed of revolution vs. load coefficient (IFR and IFG)



803

804 *Figure 13 – Angle of absolute flow velocity ( $\alpha_2$ ) at nozzle outlet/rotor inlet vs. load coefficient (IFR*  
 805 *and IFG)*

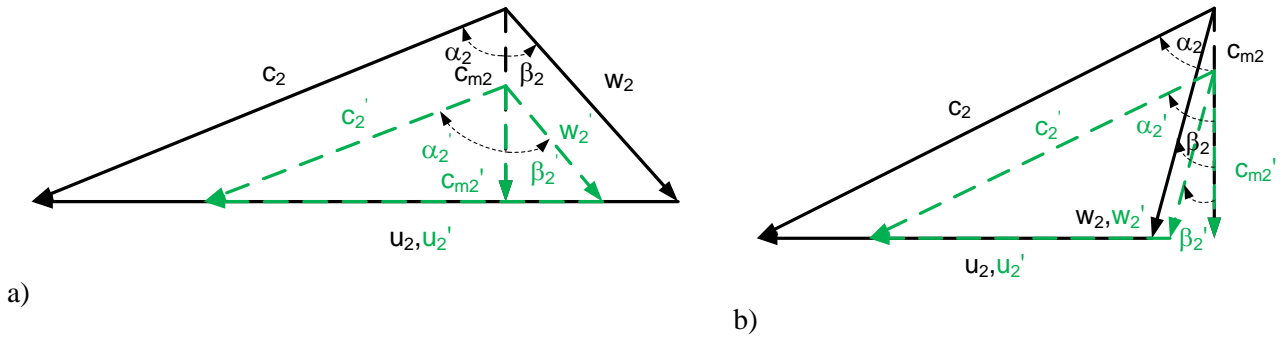


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807 *Figure 14 – Angle of relative flow velocity ( $\beta_2$ ) at nozzle outlet/rotor inlet vs. load coefficient (IFR*  
 808 *and IFG)*

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a)

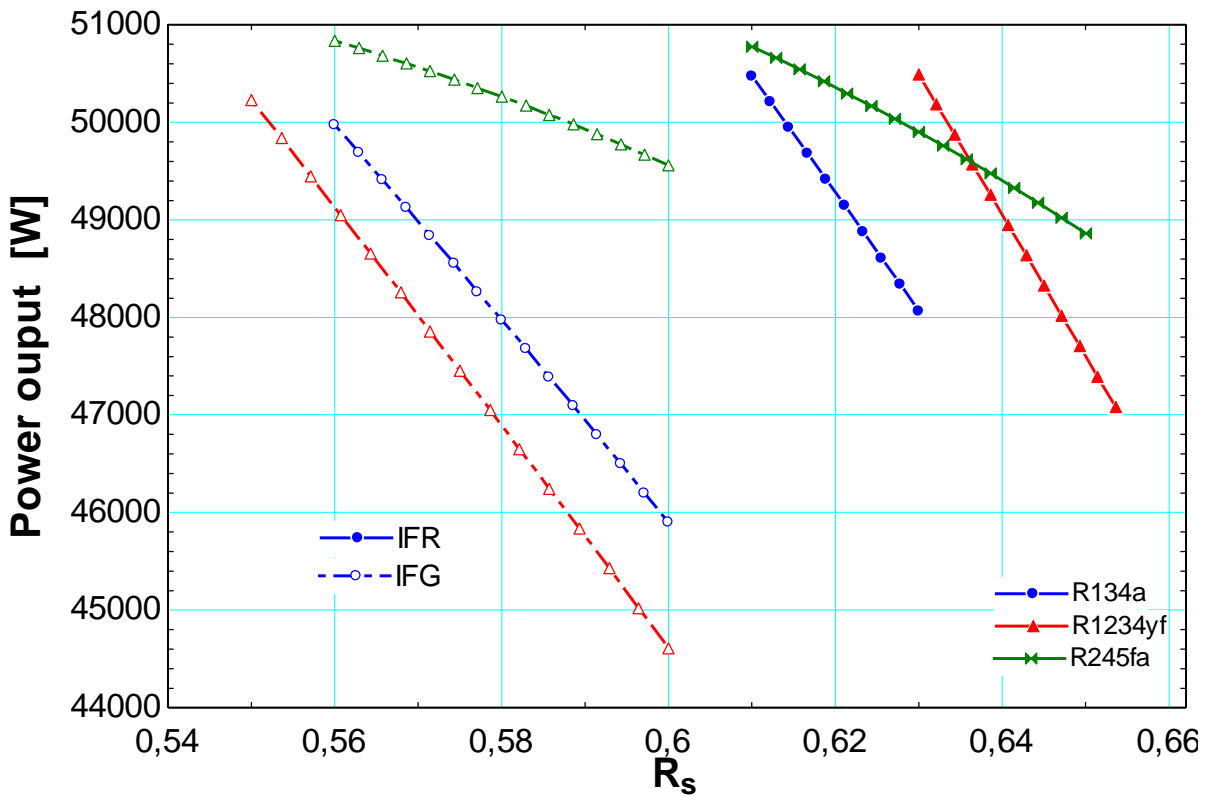
b)

Figure 15: Variation of velocity triangles with increasing in isentropic degree of reaction  $R_s$  (from black to green, a) IFR, b) IFG)

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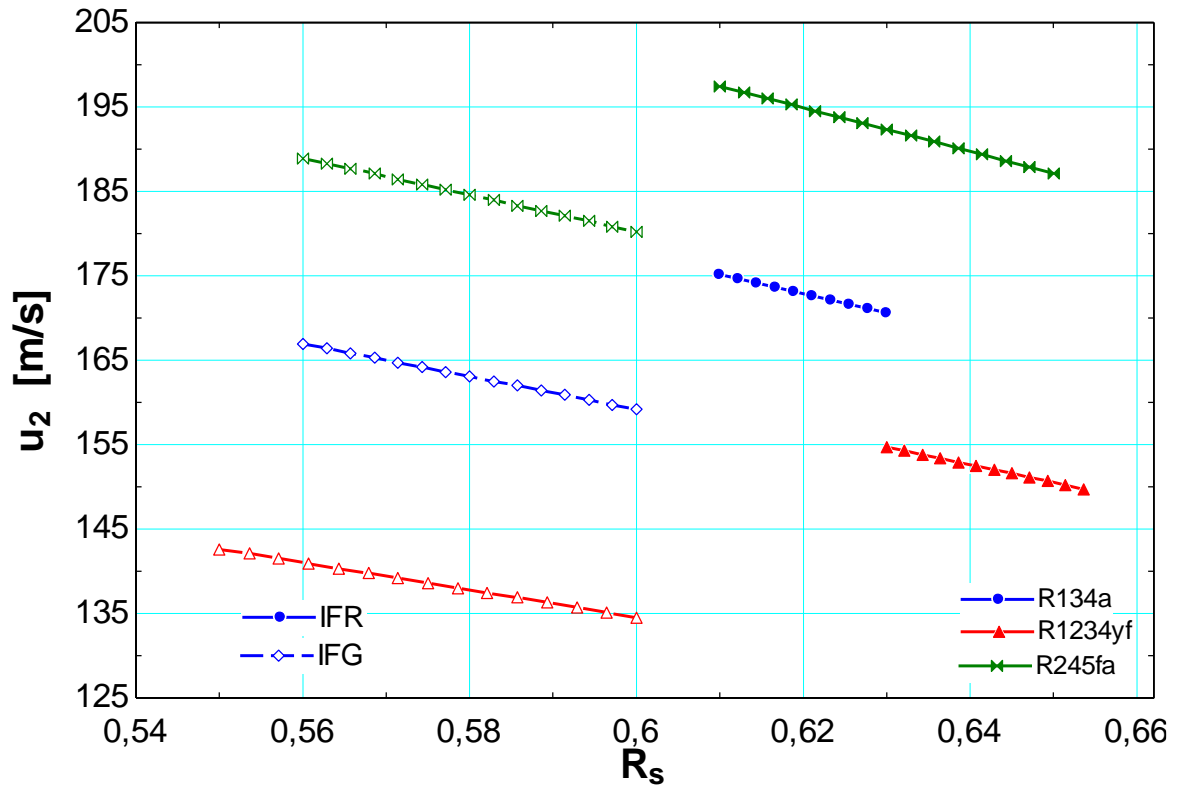
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Figure 16: Expander Power output vs. isentropic degree of reaction (IFR and IFG)

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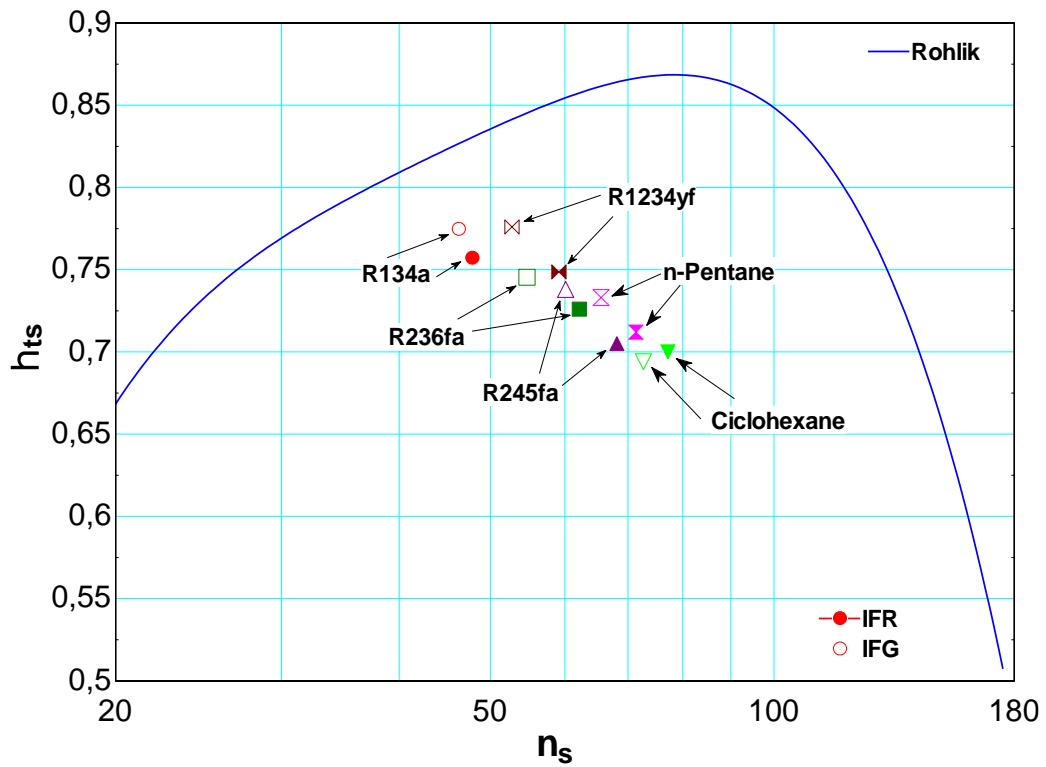
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820 *Figure 17: Expander peripheral velocity at rotor inlet vs. isentropic degree of reaction (IFR and*

821 *IFG)*



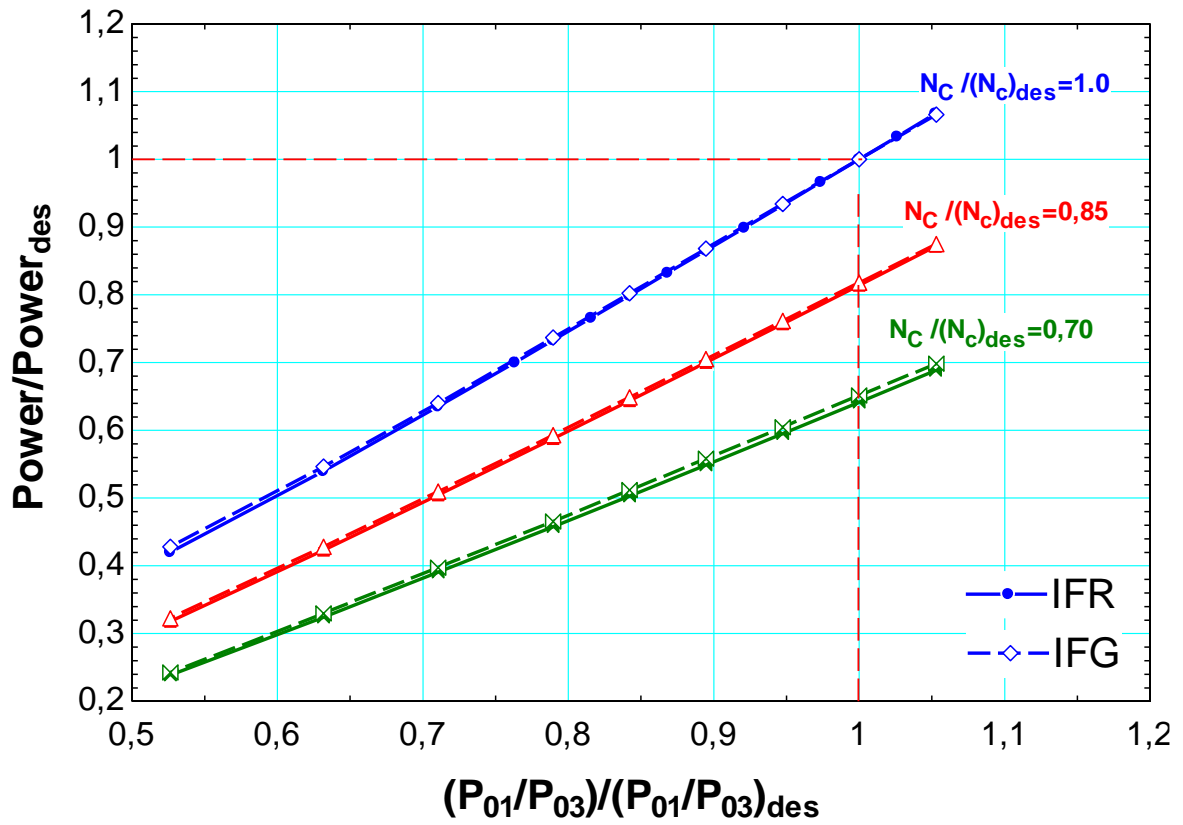
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823 Figure 18: comparison of total to static efficiency of the here designed models with the results of  
 824 Rohlik [30] (IFR and IFG)

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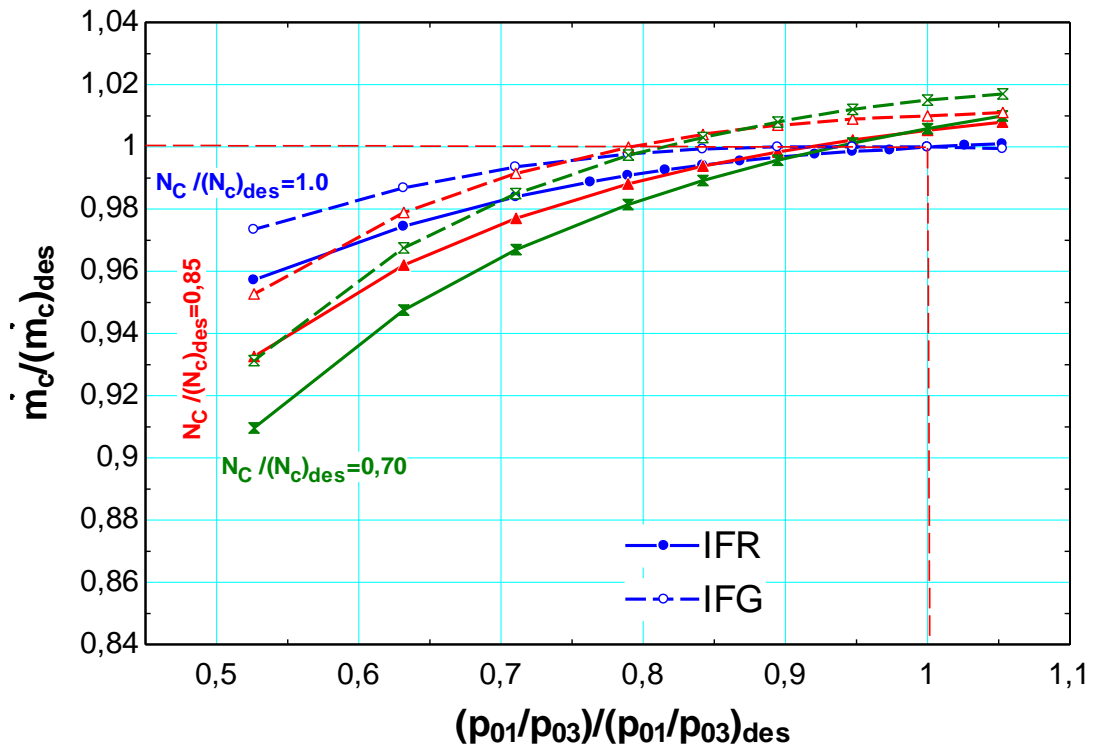


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830 Figure 19 – Power output ratio – expansion ratio characteristic curve of the expander (Referred to  
 831 R134a)

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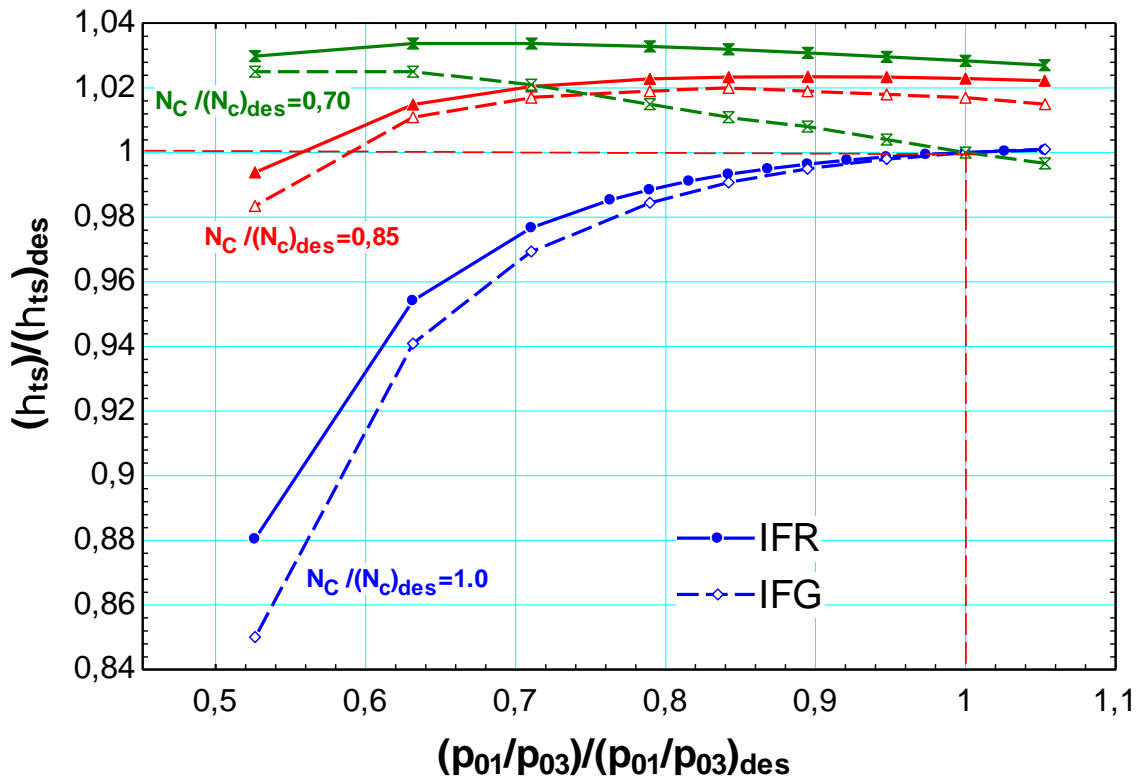


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834 *Figure 20 – Corrected flow rate – expansion ratio characteristic curves of the expander (Referred*

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*to R134a)*



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837 *Figure 21 – Total to static – expansion ratio characteristic curves of the expander (Referred to*

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*R134a)*

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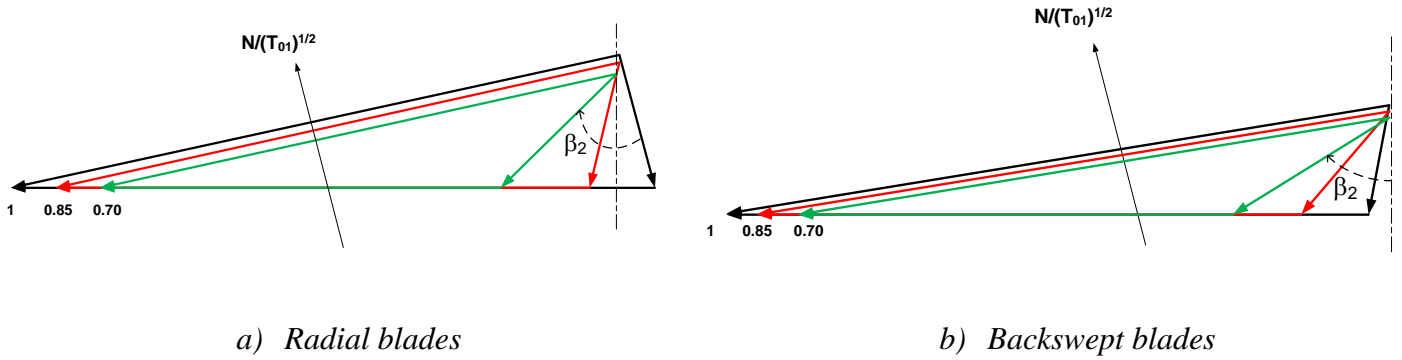


Figure 22 – Velocity triangles at rotor inlet at variable corrected speed

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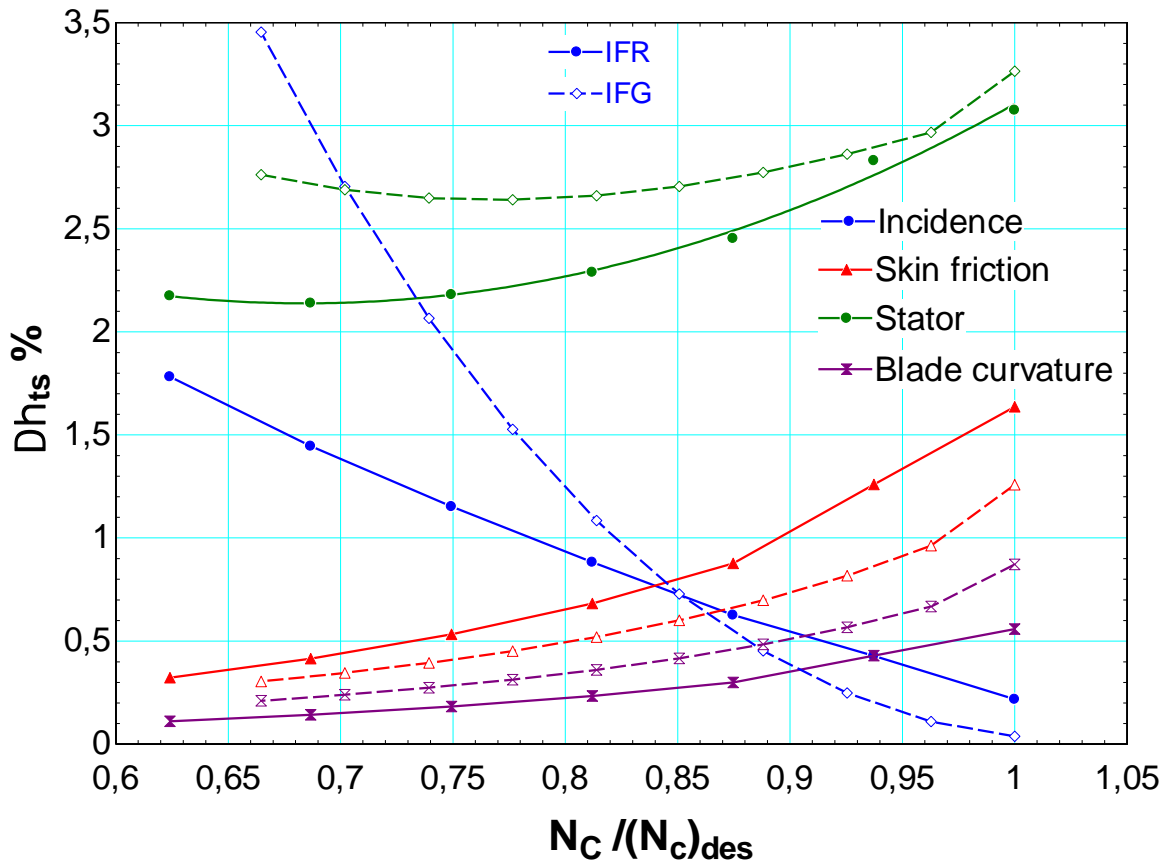


Figure 23 – behavior of total to static efficiency losses ( $\Delta\eta_{ts}$ ) with variable  $N_c$  under off design conditions

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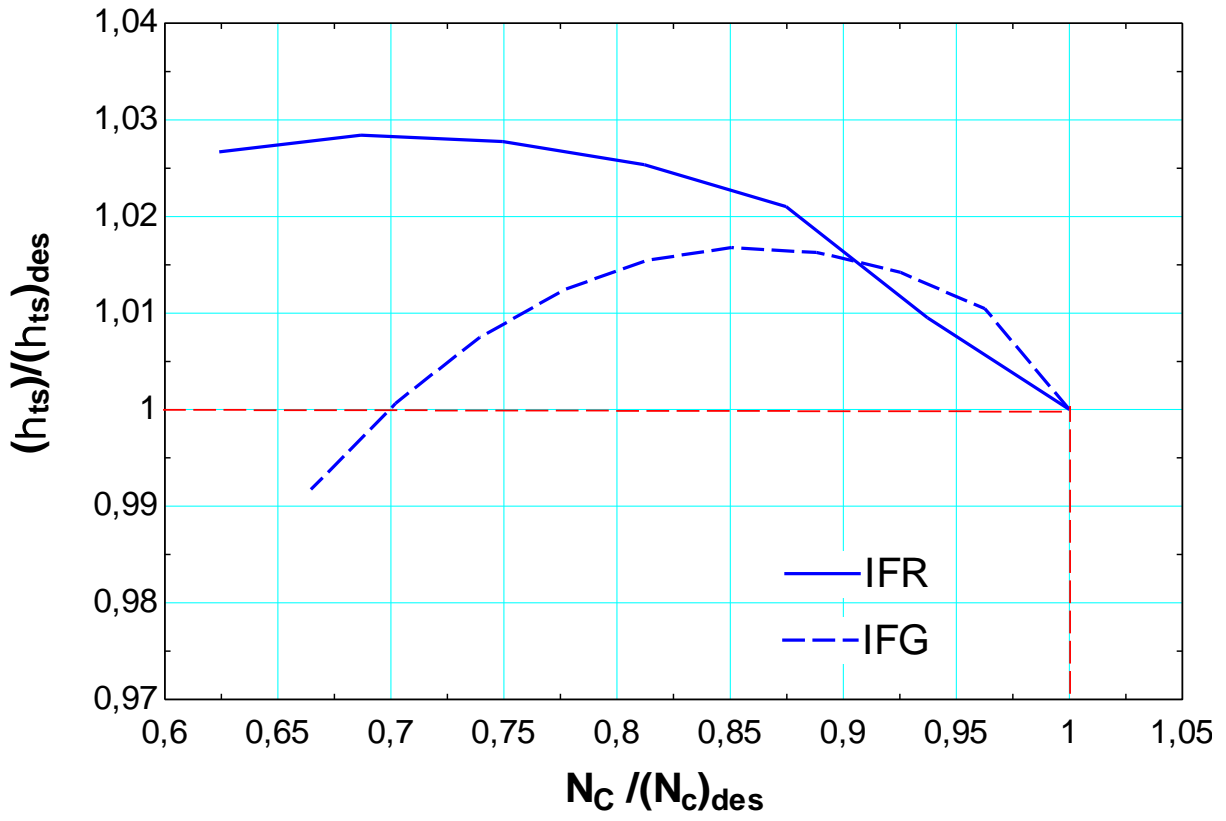


Figure 24 – Total-to-static efficiency vs  $N_c$  under off design conditions

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**List of tables**

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*Table 1: Input data and typical range of values.*

858

N.	Variable	Typical values
0	Rated Power Output, kW	Variable, typically 5 to 500 kW
1	Fluidname	R134a, R1234yf, R245fa, R236fa, Cyclo-Hexane, N-Pentane
2	Total inlet pressure $p_{01}$ [kPa]	Variable, typically 500 to 4000
3	Total inlet Temperature $T_{01}$ [°C]	Variable, typically 100 - 200
4	Isentropic enthalpy drop $\Delta h_{ss}$ [kJ/kg]	Variable, typically 28 to 130
5	Work coefficient $\Psi$	0.90-1.10
6	Flow coefficient $\Phi$	0.13-0.21
7	Isentropic degree of reaction $R_s$	0.55-0.63
8	Rotor inlet diameter $d_2$ [m]	0.08-0.195
9	Nozzle geometry ratio $d_1/d_2$	1.30-1.80
10	Rotor geometry ratio $d_3/d_2$	0.45 – 0.60
11	Diffuser geometry ratio $d_4/d_3$	1.4 – 1.6
12	Diffuser length – diameter ratio $L_d/d_3$	1.5 – 2.5
13	Rotor aspect ratio $b_2/d_2$	0.03 – 0.08
14	Nozzle height ratio $b_1/b_2$	0.8 – 1

859

Fluid Name	R134a		Cyclohexane		N-Pentane		R245fa		R1234yf		R236fa	
	IFG	IFR	IFG	IFR	IFG	IFR	IFG	IFR	IFG	IFR	IFG	IFR
Rotor geometry												
Rated power output [kW]	50	50	50	50	50	50	50	50	50	50	50	50
Total inlet pressure $p_{01}$ [bar]	38	38	5	5	10	10	31	31	31	31	30	30
Total inlet Temperature $T_{01}$ [°C]	147	147	147	147	147	147	147	147	147	147	147	147
Isentropic enthalpy drop $\Delta h_{ss}$ [kJ/kg]	38	37	131	132	105	102	54	51	30	28	38	38
Work coefficient $\Psi$	0.94	1.03	0.93	1.08	0.95	1.05	0.97	1.06	0.91	1.07	0.95	1.04
Flow coefficient $\Phi$	0.21	0.16	0.13	0.16	0.19	0.17	0.16	0.14	0.20	0.14	0.15	0.15
Isentropic degree of reaction $R_s$	0.61	0.56	0.60	0.57	0.61	0.55	0.61	0.55	0.63	0.55	0.61	0.55
Nozzle geometry ratio $d_1/d_2$	1.42	1.45	1.73	1.43	1.76	1.74	1.75	1.50	1.42	1.32	1.80	1.72
Rotor geometry ratio $d_3/d_2$	0.46	0.47	0.55	0.62	0.51	0.54	0.54	0.54	0.56	0.48	0.52	0.55
Diffuser geometry ratio $d_4/d_3$	1.40	1.40	1.60	1.60	1.40	1.40	1.50	1.50	1.60	1.60	1.50	1.50

Diffuser length – diameter ratio $L_d/d_3$	2.0	1.5	2.5	2.5	2.0	2.0	2.0	2.0	2.5	2.5	2.0	2.0
Rotor aspect ratio $b_2/d_2$	0.030	0.045	0.036	0.037	0.042	0.055	0.040	0.046	0.055	0.082	0.049	0.050
Nozzle height ratio $b_1/b_2$	1.0	1.0	0.8	0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

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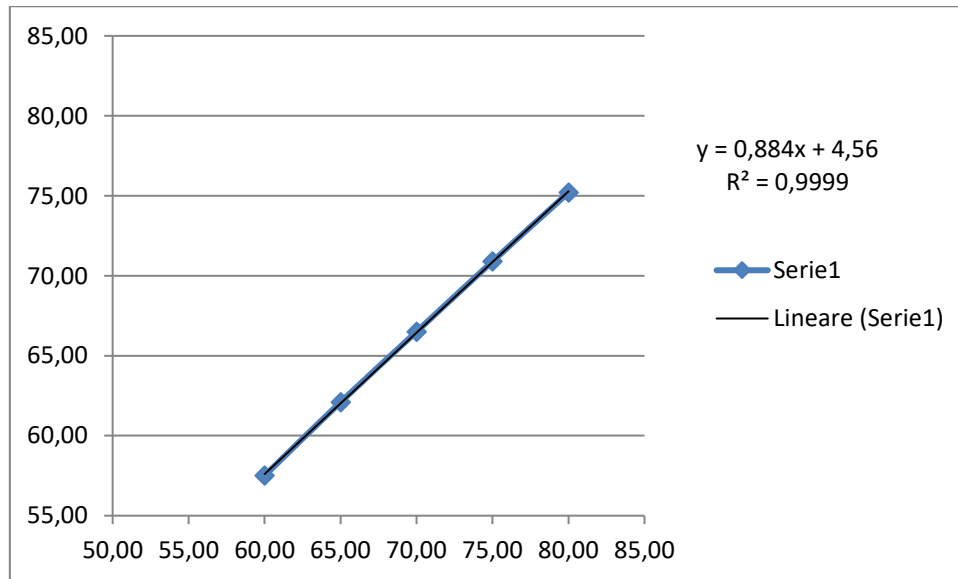
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Table 2: Nozzle setting angle as a function of absolute flow angle at nozzle outlet

862

$\alpha_2$	$\alpha_{b2}$
57.5	60
62.1	65
66.5	70
70.9	75
75.2	80

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Table 3 – design results of the ORC expander for different working fluids (IFR and IFG)

Fluid	R134a		Cyclo hexane		N-Pentane		R245fa		R1234yf		R236fa	
	IFR	IFG	IFR	IFG	IFR	IFG	IFR	IFG	IFR	IFG	IFR	IFG
$d_1$ [m]	0.115	0.115	0.335	0.273	0.188	0.189	0.154	0.138	0.121	0.117	0.157	0.160
$d_3$ [m]	0.037	0.037	0.106	0.118	0.054	0.059	0.047	0.052	0.047	0.043	0.045	0.050
$d_4$ [m]	0.052	0.052	0.170	0.189	0.076	0.082	0.071	0.077	0.076	0.068	0.068	0.075
$b_1$ [m]	0.002	0.004	0.006	0.006	0.005	0.006	0.004	0.004	0.005	0.007	0.004	0.005
$b_2$ [m]	0.002	0.004	0.007	0.007	0.005	0.006	0.004	0.004	0.005	0.007	0.004	0.005
$b_3$ [m]	0.013	0.014	0.043	0.050	0.024	0.023	0.021	0.016	0.019	0.016	0.016	0.020
$p_{04}$ [bar]	9.230	9.550	0.149	0.137	0.891	0.930	1.812	2.100	8.109	8.520	3.460	3.450
$\Delta h_{0,studio}$ [kJ/kg]	28.800	28.700	91.700	91.600	74.700	74.700	37.700	37.600	21.700	21.700	27.600	27.900
$\dot{m}$ [kg/s]	1.750	1.749	0.546	0.546	0.672	0.677	1.346	1.349	2.327	2.308	1.815	1.793
$u_2$ [m/s]	175.1	166.9	313.7	291.2	280.5	266.8	197.4	188.9	154.7	142.6	170.5	164.0
Rpm	41296	40097	30932	29115	50064	46746	42845	39211	34762	30673	37418	33672
$\alpha_2$ [°]	77.4	81.0	82.1	81.9	78.7	81.1	80.6	82.4	77.6	82.4	81.0	81.8
$\beta_2$ [°]	-15.9	10.4	-27.6	27.1	-14.7	16.9	-11.3	21.4	-25.1	26.2	-18.4	14.9
$\beta_3$ [°]	-63.9	-65.7	-57.9	-63.3	-56.0	-59.0	-60.8	-60.1	-70.4	-59.5	-59.6	-66.7
$\delta\beta_R$ [°]	48.0	76.1	30.3	90.4	41.3	75.9	49.5	81.5	45.3	85.7	41.2	81.6
$M_2$	1.047	1.077	1.500	1.626	1.321	1.374	1.441	1.493	0.921	0.982	1.276	1.325
$M_3$	0.240	0.211	0.563	0.470	0.461	0.412	0.412	0.423	0.190	0.249	0.375	0.277
$M_4$	0.120	0.106	0.191	0.166	0.216	0.197	0.171	0.174	0.073	0.095	0.157	0.119
$M_{r2}$	0.237	0.172	0.235	0.261	0.268	0.223	0.240	0.211	0.219	0.144	0.210	0.196
$M_{r3}$	0.537	0.514	1.059	1.048	0.823	0.802	0.843	0.848	0.566	0.490	0.741	0.696
$M_{u2}$	1.087	1.032	1.602	1.490	1.363	1.292	1.469	1.403	0.992	0.910	1.327	1.261
$M_{u3}$	0.480	0.468	0.897	0.936	0.682	0.688	0.736	0.735	0.533	0.422	0.639	0.639
$N_s$	0.059	0.057	0.095	0.09	0.089	0.080	0.084	0.074	0.073	0.065	0.077	0.067
R	0.52	0.48	0.55	0.47	0.53	0.49	0.52	0.49	0.54	0.47	0.54	0.48
$d_{3h}/d_{3s}$	0.490	0.443	0.422	0.412	0.393	0.490	0.391	0.520	0.434	0.457	0.479	0.434
$Z_B$	15	18	21	21	16	18	16	19	15	19	14	18
$\Delta\eta_{ts,N}$ (%)	0.55	0.87	3.75	2.17	1.785	2.37	1.753	1.87	0.692	0.85	1.625	2.02
$\Delta\eta_{ts,i}$ (%)	0.22	0.03	0.03	0.19	0.21	0.08	0.27	0.11	0.05	0.20	0.26	0.06
$\Delta\eta_{ts,cl}$ (%)	4.59	3.90	5.00	6.48	4.48	2.80	5.00	3.70	3.33	2.22	3.20	4.00
$\Delta\eta_{ts,f}$ (%)	1.62	1.25	2.05	1.43	1.80	1.28	2.32	1.30	1.36	0.89	1.32	1.25
$\Delta\eta_{ts,ke}$ (%)	1.65	1.33	3.29	2.22	3.31	2.78	2.42	2.78	1.25	2.31	2.68	1.50
$\Delta\eta_{ts,bl}$ (%)	11.9	11.27	8.1	9.12	10.8	11.09	11.0	10.63	11.2	11.20	12.77	10.90
$\Delta\eta_{ts,p}$ (%)	3.06	3.26	6.49	8.14	5.83	5.80	6.02	5.16	6.79	4.25	4.90	5.03
$\Delta\eta_{ts,df}$ (%)	0.72	0.59	1.15	0.81	0.65	0.53	0.67	0.64	0.46	0.39	0.61	0.57
$\eta_{tt}$	0.773	0.788	0.733	0.716	0.744	0.760	0.730	0.766	0.761	0.800	0.753	0.76
$\eta_{ts}$	0.757	0.775	0.700	0.693	0.711	0.732	0.705	0.738	0.748	0.780	0.726	0.745
$\eta_{ts,eff}$	0.7689	0.784	0.727	0.711	0.736	0.752	0.723	0.760	0.758	0.795	0.747	0.757

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872

873 **Nomenclature**874  $b$  blade height [m]875  $BK$  blockage factor876  $C$  absolute velocity [m/s]877  $c_s$  spouting velocity [m/s]

878	$d$	diameter [m]
879	$D^*$	characteristic diameter [m]
880	$h$	enthalpy [J/kg]
881	$I$	rothalpy [J/kg]
882	$L^*$	characteristic length [m]
883	$M$	Mach number
884	$\dot{m}$	mass flow rate [kg/s]
885	$N$	rotational speed [rpm]
886	$N_c$	corrected rotational speed [rpm K <sup>1/2</sup> ]
887	$N_s = \frac{N/60 \cdot Q_3^{1/2}}{\Delta h_{0s}^{3/4}}$	specific speed
888	$p$	pressure [Pa]
889	$q$	loss coefficient
890	$Q$	volumetric flow rate [m <sup>3</sup> /s]
891	$r$	radius [m]
892	$Re$	Reynolds number
893	$R$	Degree of reaction
894	$s$	blade spacing [m]
895	$T$	temperature [K]
896	$u$	peripheral velocity [m/s]
897	$w$	relative velocity [m/s]
898	$\bar{w}$	average relative velocity [m/s]
899	$VS$	sound speed [m/s]
900	$V$	Velocity (general) [m/s]

901	$x$	chord [m]
902	$z$	axial length of rotor [m]
903	$Z$	number of blades
904		
905	<u>Greeks</u>	
906	$\alpha$	absolute angle (from radial direction, positive with u) [°]
907	$\alpha_b$	actual angle of blades at nozzle outlet
908	$\beta$	relative angle (from radial direction, positive with u) [°]
909	$\Delta, \delta$	variation
910	$\varepsilon$	clearance[percentage of the blade height]
911	$\varepsilon_{ax}$	axial disk clearance [m]
912	$\xi$	loss coefficient
913	$\eta$	efficiency
914	$\lambda$	frictional factor
915	$\mu$	dynamic viscosity [Kg/s-m]
916	$\nu$	kinematic viscosity [m <sup>2</sup> /s]
917	$\rho$	density [kg/m <sup>3</sup> ]
918	$\bar{\rho}$	average density [kg/m <sup>3</sup> ]
919	$\Phi$	flow coefficient
920	$\Psi$	load coefficient
921	$\Psi_T$	Zweifel's coefficient (ratio between the ideal and real tangential load of a blade)
922	$\omega$	speed of revolution [rad/s]
923		

924 Subscripts

925	0	total value (stagnation)
926	1, 2, 3, 4	referred to sections 1, 2, 3, 4 (figure 4)
927	act	actual
928	bl	blade loading
929	cl	clearance
930	D	diffuser
931	df	disk friction
932	f	friction
933	h	hub
934	i	incidence
935	id	ideal
936	ke	kinetic energy
937	le	leading edge
938	m	meridional (flow rate component)
939	N	nozzle
940	opt	optimal
941	p	profile
942	R	rotor
943	r	relative, radial (referring to clearance)
944	s	isentropic (Nozzle or rotor, h-s diagram)
945	s	shroud (referred to diameter)
946	ss	double isentropic (Nozzle + Rotor, h-s diagram)
947	t	tip

948 te trailing edge

949 u peripheral

950

951 Acronyms

952

953 IFR turbine with Radial blades at rotor inlet (Radial Inlet Flow)

954 IFG turbine with General rotor shape at inlet (General Inlet Flow)

955 ORC Organic Rankine Cycle

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974 **References**

975

976 [1] Invernizzi, C., Iora, P., Silva, P., 2007, “Bottoming micro-Rankine cycles for micro-gas  
977 turbines”, *Applied Thermal Engineering*, 27, pp. 100–110

978 [2] Chacartegui, R., Sánchez, D., Muñoz, J.M., Sánchez, T. , 2009, Alternative ORC bottoming  
979 cycles FOR combined cycle power plants, *Applied Energy*, 86 , pp. 2162–2170

980 [3] Al-Sulaiman, F.A., Dincer, I., Hamdullahpur, F., 2010, “Exergy analysis of an integrated solid  
981 oxide fuel cell and organic Rankine cycle for cooling, heating and power production”, *Journal of*  
982 *Power Sources*, 195 , pp. 2346–2354

983 [4] Vaja, I., Gambarotta, A., 2010, “Internal Combustion Engine (ICE) bottoming with Organic  
984 Rankine Cycles (ORCs)”, *Energy*, 35 , pp. 1084–1093

985 [5] GE Energy Announces Industrial Waste-Heat Recovery Innovation for Onsite Power Plants,  
986 <http://www.genewscenter.com/content/detail.aspx?releaseid=7229&newsareaid=2>, 6 July, 2009.

987 [6] “Turboden Combined Heat And Power Orc Units For The Pellet Industries”, 2008,  
988 [http://www.turboden.eu/en/public/press/Turboden\\_ORC\\_for\\_pellets\\_english.pdf](http://www.turboden.eu/en/public/press/Turboden_ORC_for_pellets_english.pdf)

989 [7] Schuster, A., Karellas, S., Kakaras, E., Spliethoff, H., 2009, “Energetic and economic  
990 investigation of Organic Rankine Cycle applications”, *Applied Thermal Engineering*, 29, pp. 1809–  
991 1817

992 [8] Zhai, H., Dai, Y.J., Wu, J.Y., Wang, R.Z. , 2009, “Energy and exergy analyses on a novel  
993 hybrid solar heating, cooling and power generation system for remote areas”, *Applied Energy*, 86 ,  
994 pp.1395–1404

995 [9] Fiaschi, D., Lifshitz, A., Manfrida, G., “ Fuel-Assisted Solar Thermal Power Plants with  
996 supercritical ORC cycle”, Proceedings of ECOS 2010, Lausanne.

997 [10] Di Pippo, R., 2006, “Geothermal Power Plants: Principles, Applications and Case Studies”.  
998 Elsevier Advanced Technology, London, UK.

999 [11] Heberle, F., Brüggemann, D., 2006, “Exergy based fluid selection for a geothermal Organic  
1000 Rankine Cycle for combined heat and power generation”, *Applied Thermal Engineering* 30, pp.  
1001 1326-1332

- 1002 [12] Lentz, A., Almanza, R., 2006, "Solar-geothermal hybrid system", *Applied Thermal*  
1003 *Engineering*, 26, pp. 1537-1544
- 1004 [13] Whitfield A., Baines N.C., 1990."Design of radial turbomachines", Longman, New York.
- 1005 [14] Dixon S.L., 1998, "Fluid Mechanics and Thermodynamics of Turbomachinery", Butterworth.
- 1006 [15] Cohen H., Rogers G.F.C., Saravanamuttoo H.I.H.,1996"Gas Turbine Theory", Longman, New  
1007 York.
- 1008 [16] Glassman A.J.,"Turbine Design and Application", *Scientifical and technical information*  
1009 *program-1994*, National Aeronautics and Space Administration (NASA) *Washington, DC*.
- 1010 [17] Rohlik Harold E., "Analytical Determination of Radial Inflow Turbine Design Geometry for  
1011 Maximum Efficiency",National Aeronautics and Space Administration (NASA), Washington,  
1012 D.C., 1968.
- 1013 [18] Meitner Peter L., Glassman Arthur J., " Computer Code for Off-Design Performance Analysis  
1014 of Radial-Inflow Turbines With Rotor Blade Sweep", National Aeronautics and Space  
1015 Administration (NASA), Scientifical and Technical Information Branch, 1983.
- 1016 [19] Whitfield, A., 1990, "The Preliminary Design of Radial Inflow Turbines", ASME J. of  
1017 Turbomachinery, Vol. 112, pp. 51-57.
- 1018 [20] Rohlik, H. E., 1975, "Radial Inflow Turbines," NASA SP 290, Vol. 3, Ch. 10.
- 1019 [21] Ainley, D.G. and Mathieson, G.C.R. (1951). "A method of performance estimation for axial  
1020 flow turbines", ARC.R. and M.2974.
- 1021 [22] Wiesner, F.J: "A Review of Slip Factors in Centrifugal Impellers ". J. Eng. Power, vol 89, no.  
1022 4, Oct 1967, pp 558-572.
- 1023 [23] Zweifel, O. (1945). "The spacing of turbomachine blading, especially with large angular  
1024 deflection. " *Brown Boveri Rev.*, 32,12.
- 1025 [24] Benson S. Rowland. " A Review of Methods For Assessing Loss Coefficients In Radial Gas  
1026 Turbines". *Int. J. mech. Sci.* Pergamon Press. 1970. Vol 12, pp 905-932.
- 1027 [25] Hiatt G.F. and Johnston I.H., Proc. Inst. Mech. Engrs 178, part 31 (1963-64).
- 1028 [26] Balje O.E. " A Contribution to the Problem of Designing Radial Turbomachines ", *Trans.*  
1029 *A.S.M.E.* 741, 451 (1952).

- 1030 [27] Bridle E.A., Boulter R.A., " A Simple Theory for the Prediction of Losses in the Rotors of  
1031 Inward Radial Flow Turbines ", Proceedings of the institution of mechanical engineers, conference  
1032 proceedings 1964-1970 (vols 178-184), 1967.
- 1033 [28] Futral Samuel M., Jr., Wasserbauer Charles A., " Off Design Performance Prediction with  
1034 Experimental Verification for a Radial-Inflow Turbine ", *Lewis Research Center Cleveland, Ohio*,  
1035 National Aeronautics and Space Administration, Washington, D.C. February 1965.
- 1036 [29] Hiatt, G. F. and Johnston, I. H. " Experiments Concerning the Aerodynamic Performance of  
1037 Inward Flow Radial Turbines ", Paper 7, *Thermodynamics and Fluid Mechanics Conv., Proc. Instn*  
1038 *mech. Engrs 1963-64 178 (Pt 3 I (ii), 28.*
- 1039 [30] Futral Samuel M., Jr., and Holeski Donald E., " Experimental Results of Varying the Blade-  
1040 Shroud Clearance in a 6.02-Inch Radial-Inflow Turbine ", *Lewis Research Center Cleveland, Ohio*,  
1041 National Aeronautics and Space Administration, Washington, D.C. January 1970.
- 1042 [31] Atlas Copco Gas and Process Division, "Driving Expander Technology",  
1043 [http://www.atlascopco-gap.com/download\\_file.php?id=457](http://www.atlascopco-gap.com/download_file.php?id=457)
- 1044 [32] GE Energy Oil & Gas, TurboExpanders/Generators for Natural Gas Applications,  
1045 [http://www.geoilandgas.com/businesses/ge\\_oilandgas/en/prod\\_serv/prod/turboexpanders/e](http://www.geoilandgas.com/businesses/ge_oilandgas/en/prod_serv/prod/turboexpanders/en/downloads/turboexpanders.pdf)  
1046 [n/downloads/turboexpanders.pdf](http://www.geoilandgas.com/businesses/ge_oilandgas/en/prod_serv/prod/turboexpanders/en/downloads/turboexpanders.pdf)
- 1047 [33] Pratt&Whitney Power Systems, "Organic Rankine Cycle Technology",  
1048 [http://www.pw.utc.com/StaticFiles/Pratt%20%26%20Whitney%20New/Media%20Center/As](http://www.pw.utc.com/StaticFiles/Pratt%20%26%20Whitney%20New/Media%20Center/Assets/1%20Static%20Files/Docs/pwps_orc_brochure_2010.pdf)  
1049 [sets/1%20Static%20Files/Docs/pwps\\_orc\\_brochure\\_2010.pdf](http://www.pw.utc.com/StaticFiles/Pratt%20%26%20Whitney%20New/Media%20Center/Assets/1%20Static%20Files/Docs/pwps_orc_brochure_2010.pdf)
- 1050 [34]<http://www.fchart.com/ees/>
- 1051 [35]<http://www.fchart.com/ees/ees-refprop.php>, <http://www.nist.gov/srd/nist23.cfm>
- 1052 [36] Manfrida, G., Tempesti, D., 2011, "Maximum exergy control of micro-CHP systems operating  
1053 with geothermal and solar energy", ICAE 2011 Conference, Perugia.
- 1054 [37] Daniele Fiaschi, Giampaolo Manfrida and Francesco Maraschiello, "Thermo-fluid dynamics  
1055 preliminary design of turbo-expanders for ORC cycles", *Applied Energy*, Volume 97, September  
1056 2012, Pages 601-608.
- 1057 [38] Balje O. E.: "Turbomachines. A Guide to Design, Selection and Theory", 1981, Wiley.

- 1058 [39] Deng QingHua, Niu JiuFang & Feng ZhenPing, " Study on Leakage Flow Characteristics of  
1059 Radial Inflow Turbines at Rotor Tip Clearance ", 2008, *Science in China Series E: Technological*  
1060 *Sciences*, Springer.
- 1061 [40] Rodgers, C., Geiser, T., 1987, Performance of a High-Efficiency Radial/Axial Turbine, ASME  
1062 J. of Turbomachinery, 109, pp. 151-154.
- 1063 [41] Idelc'ijk, 1978, " Memento des pertes de charge ", Eyrolles
- 1064 [42] Runstadler, P.W., Dolan, F.X., Dean, R.C., 1974, " Diffuser Data Book ", CREARE Inc.
- 1065